FRACTAL GROWTH PHENOMENA

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Tamás Vicsek
Research Institute for Technical Physics
H-1325 Budapest
P O Box 76
Hungary

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Foreword

The discovery of fractal growth phenomena has been, among all the scientific developments of recent years, one of the most rewarding. As a complete surprise, rules of random growth which are so simple as to seem trivial are in fact capable of generating an unimaginably rich variety of fractal shapes! The finding has immediately fired many skillful and ardent investigators worldwide, and soon proved to involve a fascinating interface between the roughness associated with fractals and the smoothness associated with the Laplace equation. By now, we have moved beyond the necessary and even unavoidable first stage, when the main task was to sort out the newly discovered riches. For example, some critical features of diffusion limited aggregates have now been explained. But the topic is beautifully alive, and will stay with us for a long time. Thus, a study born in the context of carbon deposits on the cylinder walls of Diesel engines promises to become part of a great new stream of mathematical physics.

What has been lacking is a broad survey of the field. It is our very good fortune that this gap is being beautifully filled by the present book. Professor Vicsek has taken full advantage of the unusual combination of circumstances that surrounds his life and work. He is in close professional interaction with all other workers in the field, therefore the book is very much up to date. On the other hand, he has achieved a degree of objectivity, detachment and balance that was indispensable, but might have been hard to achieve in the scientific "downtowns". It is good, also, that the book is self contained, thanks to its introductory chapters. But the author stresses, and I agree, that the surveys in these chapters do not claim to provide a complete introductory treatise.

Personally, I have taken no direct part in this great adventure, though, of course, I have provided in advance its tools, namely the fractal sets and the multifractal measures! Hence, I feel free to express my enthusiasm.

The present book will hasten the pace of progress in the study of fractal aggregates, and will help interest in this topic to spread everywhere.

Benoit B. Mandelbrot
Physics Department, IBM T.J. Watson Research Center,
Yorktown Heights, NY 10598 and
Mathematics Department, Yale University, New Haven, CT 06520

Preface

Even bearing in mind that we live in an era of explosive advances in various areas of science, the investigation of phenomena involving fractals has gone through a spectacular development in the last decade. Many physical, technological and biological processes have been shown to be related to and described by objects with non-integer dimensions – an idea which was originally proposed and beautifully demonstrated by Benoit B. Mandelbrot in his classic books on fractals.

The physics of far-from-equilibrium *growth phenomena* represents one of the main fields in which fractal geometry is widely applied. During the past couple of years considerable experimental, numerical and theoretical information has accumulated about such processes, and it seemed reasonable to bring together most of this knowledge into a *separate book*, in addition to the numerous conference proceedings and reviews devoted to irreversible growth.

My intention was to provide a book which would summarize the basic concepts born in the studies of fractal growth as well as to present some of the most important new results for more specialized readers. Thus, it is hoped that the book will be able to serve as a textbook on the *geometrical aspects of fractal growth* and will also treat this area in sufficient depth to make it useful as a reference book. It follows from the nature of this approach that the emphasis is on presenting results in a reproducable manner rather than on briefly reviewing a large number of contributions. Obviously, the field of fractal growth phenomena is too broad to enable all of the related topics to be included. Among the important aspects not treated are, for example, cellular automata or the physical properties (elasticity, conductivity, etc.) of growing fractals.

Collaboration with many colleagues has greatly helped me in gaining an insight into the processes discussed in this book. For the last ten years my closest colleague and friend János Kertész and I have worked together at the Institute for Technical Physics of the Hungarian Academy of Sciences on a number of problems related to fractals. We have had many stimulating di cussions in the past and we have a welth of interesting new ideas to study together in the future. A considerable amount of my activity in the field of aggregation was realized during my visit to Emory University, Atlanta, where I was working with Fereydoon Family. With him, and with Paul Meakin of du Pont, Wilmington, or fruitful cooperation has become regular and now spans the ocean. I am also grateful to Á. Buka, D. Grier, V. Horváth, L. J. Montag, H. Nakanishi, D. Platt, Z. Rácz, G. Radnóczy, L. M. Sander, A. Szalay, B. Taggett, T. Tél and Y. Zhang for their kind collaboration.

I should also like to thank a number of colleagues who greatly stimulated my work by showing interest in my investigations. Discussions with Gene Stanley and Dietrich Stauffer have helped me to be involved in the most interesting current problems of the physics of fractals. I have learned much about fractals from long conversations with Benoit Mandelbrot. At the suggestion of György Marx and Péter Szépfalusy became involved into the teaching of growth phenomena. I thank Tamás Geszti and Tivadar Siklós for their numerous helpful suggestions.

It was Len Sander who encouraged me to write this book. János Kertész and Tamás Tél read parts of the preliminary version and I am grateful for their useful comments and suggestions. My thanks to Harvey Shenker for a number of last-minute linguistic corrections. Many of the figures were reproduced from works by other authors and here I thank these colleagues for granting me the necessary permissions and for providing the corresponding originals. The rest of the illustrations were made and reproduced by Mihály Hubai and Sára Tóth.

Finally, this Preface provides me with a good opportunity to express my gratitude to my wife, Mária Strehó. She, a specialist in numerical analysis, has helped me in many ways in the writing of this book.

Tamás Vicsek

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PART II. CLUSTER GROWTH MODELS

Chapter 1

INTRODUCTION

During the last decade it has widely been recognized by physicists working in diverse areas that many of the structures common in their experiments possess a rather special kind of geometrical complexity. This awareness is largely due to the activity of Benoit Mandelbrot (1977, 1979, 1982, 1988), who called attention to the particular geometrical properties of such objects as the shore of continents, the branches of trees, or the surface of clouds. He coined the name *fractal* for these complex shapes to express that they can be characterized by a *non-integer* (fractal) *dimensionality*. With the development of research in this direction the list of examples of fractals has become very long, and includes structures from microscopic aggregates to the clusters of galaxies.

An important field where fractals are observed is that of far-from-equilibrium growth phenomena which are common in many fields of science and technology. Examples for such processes include dendritic solidification in an undercooled medium, viscous fingering which is observed when a viscous fluid is injected into a more viscous one, and electrodeposition of ions onto an electrode. Fig. 1. demonstrates the complexity of possible patterns growing under a wide variety of experimental conditions. In the experiments leading to the structures shown in Fig. 1. quasi two-dimensional samples were used and the motion of the interfaces was determined by the spatial distribution

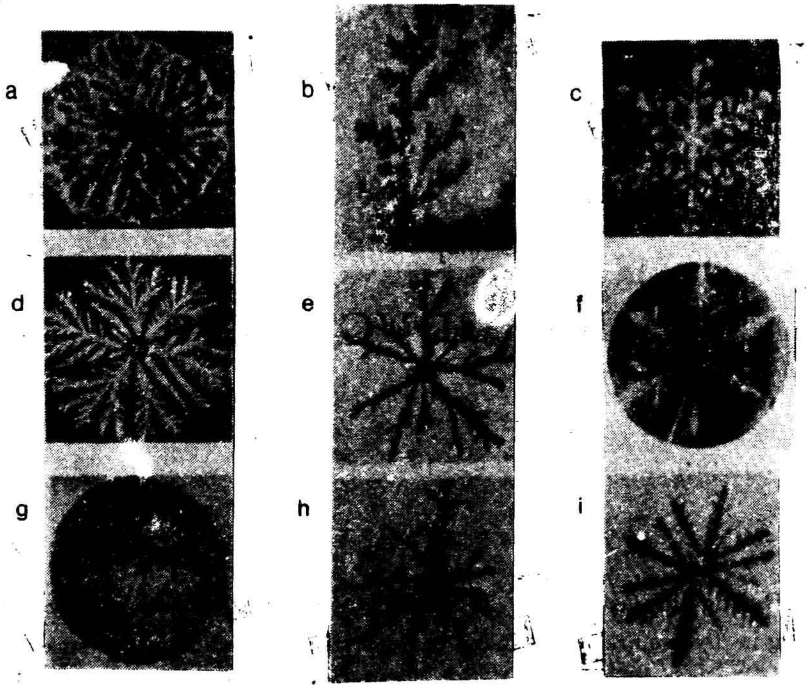

Figure 1. Examples for complex geometrical structures observed in various experiments on growth of unstable interfaces. The three major types of patterns found in the experiments on i) crystallization (a, b and c), ii) viscous fingering (d, e and f), and iii) electrodeposition of zinc (g, h and i) are grouped in separate columns. The fractal dimension of the structures shown in the middle column is close to 1.7. (This set of pictures is reproduced from Vicsek and Kertész (1987). The individual pictures are from: (a) Ben-Jacob *et al* (1986), (b) Radnóczy *et al* (1987), (c) Bentley and Humpreys (1962), (d) Buka *et al* (1986), (e) Daccord *et al* (1986), (f) Ben-Jacob *et al* (1985), (g and i) Sawada *et al* (1986) and (h) Matsushita *et al* (1984); For details see References to Part III.)

of a quantity which satisfies the Laplace equation with moving boundary conditions.

In addition to interfacial growth, *aggregation* of similar particles represents another important class of growth phenomena producing complicated geometrical objects. Aggregation may take place particle by particle, while in other cases (for example during the formation of aerogels) the aggregates themselves are also mobile and are joined together to form larger clusters during their motion.

A broad class of growing patterns is characterized by an open branching structure as is illustrated by the middle column of Fig. 1. Such objects can be described in terms of fractal geometry. In the present case this means that the growing structures are *self-similar* in a statistical sense and the volume $V(R)$ of the region bounded by the interface scales with the increasing linear size R of the object in a non-trivial way

$$V(R) \sim R^D. \tag{1}$$

Here $D < d$ is typically a non-integer number called the *fractal dimension* and d is the Euclidian dimension of the space the fractal is embedded in. Naturally, for a real object the above scaling holds only for length scales between a lower and an upper cutoff.

There are a number of reasons for the recent rapid development in the research of fractal growth. The interest is greatly motivated by the fact that fractal growth phenomena are closely related to many processes of practical importance. Here we shall mention only two examples. The internal texture of alloys due to the dendritic structures developing during their solidification is largely responsible for most of their mechanical properties. Another area of application is secondary oil recovery, where water pumped into the ground through one well is used to force the oil to flow to the neighbouring wells. The effectiveness of this method is influenced by the fractal structure of viscous fingers corresponding to the water-oil interface.

The internal evolution of physics as a discipline has also given rise to an increased interest in the investigation of structures growing under far-from-equilibrium conditions. In the 1970's most of the researchers working in the field of statistical mechanics were involved in problems related to *phase transitions* in equilibrium systems. These studies led to many important theories and methods including renormalization based on the scale invariance of thermodynamical systems at their critical point.

Since growing fractals are also *scale-invariant* objects (this property is equivalent to their self-similarity), the knowledge which had accumulated during the investigations of second order phase transitions was particularly useful in making a step forward and investigating scaling in growth processes. Thus the fields involving fractals were developing fast and it has become evident that *multifractal scaling*, which is the generalization of simple scaling, represents an important characteristic of many growth phenomena.

Most of the already large amount of new results on fractal growth (and fractals in general) can be found in conference and school proceedings (Family and Landau 1984, Shlesinger 1984, Stanley and Ostrowsky 1985, Pynn and Skjeltorp 1985, Boccara and Daoud 1985, Pietronero and Tosatti 1986, Engelman and Jaeger 1986). Some aspects of growth phenomena are discussed in the recent books by Jullien and Botet (1987) on aggregation, and by Feder (1988) on fractals. Finally, a number of review papers have been published recently about processes related to fractal growth (e.g., Herrmann 1986, Sander 1986, Witten and Cates 1986, Meakin 1987a, Meakin 1987b, Jullien 1987).

Here, I concentrate on the geometrical aspects of fractal growth. My intenton was to give a balanced account of the most important results in a pedagogical style. The material is divided into three parts, viz. I: Fractals, II: Cluster Growth Models; III: Fractal Pattern Formation. References are provided at the end of each part.

Part I. introduces the basic definitions and concepts related to *fractal geometry* in general. The major types of fractals are discussed and a few useful rules for the estimation of fractal dimensions are given. Fractal measures are treated in a separate chapter since many recent results demonstrate their important role in physical processes. The last chapter of the first part contains a collection of methods which are commonly used to determine fractal dimensions of various objects including experimental samples and computer generated clusters. Throughout this part examples are given to illustrate the principles introduced in the text.

Computer models based on growing clusters made of identical particles have proven to be a particularly useful tool in the investigation of fractal growth. The main advantage of such models is that they provide a possibility to determine the most relevant factors affecting the geometrical properties of objects developing in a given kind of growth phenomenon. Accordingly, in Part II. important results concerning a wide variety of it cluster growth models are discussed. First those models (called local) are examined in which the probability of adding a particle to the growing cluster depends only on the immediate environment of the given position. In the models of diffusion-limited growth (Chapter 6.) the probability of adding a particle to the cluster is determined by the structure of the whole cluster; consequently, these processes are truly non-local. In some cases both local and non-local models may lead to compact structures with self-affine surfaces which are treated in Chapter 7. The last chapter of Part II. discusses results obtained in the numerical and experimental studies of cluster-cluster aggregation. An important aspect of the aggregation of clusters is that the time is well defined in such processes. This fact allows for the development of a dynamic scaling theory for the cluster size distribution.

Part III. deals with *fractal pattern formation*, where the term pattern formation is used for interfacial growth phenomena in which the motion of the unstable interfaces is dominated by the surface tension. Diffusion-limited growth processes may lead to a variety of structures (see Fig. 1.), and in a number of cases it is still not known which are the conditions for the development of a given type of the possible interfaces. The answer to the questions about the relevance of the factors affecting the growth of complicated patterns could unambiguously be provided if it were possible to solve directly the corresponding non-linear equations. However, this approach does not seem to be feasible at present, because of the instability and the extreme complexity of the solutions. Thus, fractal pattern formation has mainly been studied by numerical simulations and model experiments which are reviewed in the two chapters of this part.

The field of fractal growth phenomena is still growing quickly, and there are many new results which could not be included into this book. Those readers who are interested in the developments not treated here are advised to consult the already mentioned literature or to look for the numerous conference proceedings and reviews which are currently in a preparatory stage.

PART I

FRACTALS