MICROWAVE THEORY AND TECHNIQUES

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This book is dedicated to our patient wives
and to the many scientists and engineers
upon whose work it is based.

PREFACE

This book is intended primarily as a textbook for a senior or first-year graduate course in Microwaves. It may also be useful to research workers and practicing engineers as a reference book. For the benefit of those who have not had a course in Electrodynamics, chapters have been included on the theory of static and dynamic electromagnetic fields, a knowledge of which is essential to the analysis of waveguides and traveling-wave tubes. The arrangement is such, however, that the mathematical treatments contained in the Introduction and in Chapters 1, 2, and the first part of 5 may be omitted by those who are primarily interested in the physical analyses and characteristics of microwave devices. Emphasis has been placed upon the physical principles underlying the operation of microwave amplifiers and oscillators, since a knowledge of these principles may be helpful in suggesting new microwave devices.

The treatment of waveguide and coaxial-line components, measurements, and, to some extent, antennas is largely descriptive and is presented at a relatively low level. The justification for this is that some knowledge of these subjects is essential to work in this field, but that a comprehensive treatment would require far more space than could be assigned to it in a book of reasonable length. Books covering each of these fields are available to those desiring more detailed information.

The term "microwave" has been interpreted by the authors to apply to devices operating at frequencies at which distributed-element circuits are usually used. Thus the frequency range covered by this book extends upward from a lower limit of approximately three or four hundred megacycles per second.

The authors experienced considerable difficulty in the choice of letter symbols, both because of the large number of different quantities to be represented and because of the variety of symbols that have been used by various authors for identical quantities. Standard symbols have been used whenever possible. Where standard symbols do not exist, the authors have tried to choose symbols that agree with the most general usage and that do not lead to confusion. The Index of Symbols lists the pages on which each symbol is defined.

Most of the chapters contain problems that help to emphasize fundamental principles. Some problems, in which the method of attack is suggested, are also used as means of eliminating, in the text, mathematical details that should be familiar or easily determined by students of senior or graduate standing. References to these problems are made at appropriate points in the text. Although many instructors prefer to devise their own laboratory experiments, the authors believe that the experiments provided in this book may at least serve as a useful guide in a laboratory course on Microwaves.

Every effort has been made to include references to papers and books that served as sources for the material presented in this book. Because much of the development in the microwave field took place during World War II, however, credit cannot always be given to those who deserve it.

The microwave field is still sufficiently new so that no book in this field can be completely up-to-date at the end of the time required for publication. The manuscript has been modified frequently in order to include newly available material, and the authors believe the treatment to be reasonably up-to-date at the time of this writing.

New Haven, Conn.
March 1953

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INTRODUCTION

VECTOR ANALYSIS

I-1. Vector Arithmetic.[1] Vectors are quantities that have magnitude and direction. Force is a typical vector quantity—that is, its description requires the specification of its magnitude and of the direction of its action. Other examples are electric intensity, magnetic intensity, velocity, surface, etc. Customarily, a vector is referred to a reference coordinate system. The right-hand reference frame, defined with respect to the right-hand-screw convention, is usually employed. The axes of a right-hand system, shown in Fig. I-1 for the Cartesian system, are oriented so that a right-hand screw rotated from positive x to positive y through the 90° angle will advance in the positive z direction.

With respect to the coordinate system shown in Fig. I-1, the vector **F** which, for example, might represent a force or a surface, can be expressed as

$$\mathbf{F} = \mathbf{1}_x F_x + \mathbf{1}_y F_y + \mathbf{1}_z F_z \tag{I-1}$$

in which F_x, F_y, and F_z are the respective components of **F** that act parallel to the coordinate axes, and $\mathbf{1}_x$, $\mathbf{1}_y$, $\mathbf{1}_z$ are unit vectors that are directed along the respective x, y, z axes. The unit vectors always define the direction of action of the components of the vector.

If **F** is a force, the quantities F_x, F_y, F_z are the magnitudes of the components of the force along the coordinate axes. When **F** represents an open surface such as a sheet of paper or, as in Fig. I-7, **F** is drawn normal to the surface in such a manner that its positive direction is the direction of advance of a right-hand screw turned so as to pass through the surface, and the components F_x, F_y, F_z are the magnitudes of the projected areas of the surface upon the y-z, x-z, and x-y coordinate planes respectively. When **F** represents a small section of a closed surface such

[1]The material in this section is a condensed development of vector mathematics. For more detailed and extensive treatments, the reader may wish to consult other authors. A particularly good treatment from the standpoints of content, pictorial development and detail is that of Guillemin, E. A., *Mathematics of Circuit Analysis*, Chap. 5, John Wiley & Sons, Inc., 1949. Also Page, Leigh, *Introduction to Theoretical Physics*, 3rd Ed., D. Van Nostrand & Co., Inc., 1952, and Skilling, Hugh H., *Fundamentals of Electric Waves*, John Wiley & Sons, Inc., 1942.

as a sphere, the vector is customarily drawn normal to the small section of surface area and outward from the closed surface.

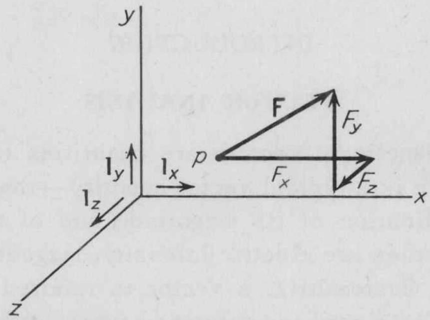

Fig. I-1. Force **F** acting at point p in a right-hand system of Cartesian coordinates.

The magnitude of **F** must evidently be

$$|\mathbf{F}| = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2} \tag{I-2}$$

and $\dfrac{F_x}{|\mathbf{F}|}$, $\dfrac{F_y}{|\mathbf{F}|}$, $\dfrac{F_z}{|\mathbf{F}|}$ must be the cosines of the angles that **F** makes with each of the coordinate axes. These cosines are commonly known as the *direction cosines* of **F**.

The addition of vectors is customarily defined to be the addition of the codirected components of each of the vectors. If

$$\mathbf{F} = \mathbf{1}_x F_x + \mathbf{1}_y F_y + \mathbf{1}_z F_z \tag{I-3}$$

and

$$\mathbf{G} = \mathbf{1}_x G_x + \mathbf{1}_y G_y + \mathbf{1}_z G_z \tag{I-4}$$

then

$$\mathbf{F} + \mathbf{G} = \mathbf{1}_x (F_x + G_x) + \mathbf{1}_y (F_y + G_y) + \mathbf{1}_z (F_z + G_z) \tag{I-5}$$

There are two forms of vector multiplication, the "dot" or scalar product and the "cross" or vector product. The dot product of two vectors is a scalar that is defined by the relation

$$\mathbf{F} \cdot \mathbf{G} = |\mathbf{F}| \; |\mathbf{G}| \cos \theta \Big|_{\mathbf{F}}^{\mathbf{G}} \tag{I-6}$$

where $\theta \Big|_{\mathbf{F}}^{\mathbf{G}}$ is the angle between **F** and **G**. From the definition, it is evident that the dot product of codirected unit vectors is unity and of orthogonal vectors zero; hence

$$\mathbf{1}_x \cdot \mathbf{1}_x = \mathbf{1}_y \cdot \mathbf{1}_y = \mathbf{1}_z \cdot \mathbf{1}_z = 1$$
$$\mathbf{1}_x \cdot \mathbf{1}_y = \mathbf{1}_x \cdot \mathbf{1}_z = \mathbf{1}_y \cdot \mathbf{1}_z = 0 \qquad \text{(I-7)}$$

The dot product of the two vectors **F** and **G** is

$$\mathbf{F} \cdot \mathbf{G} = (\mathbf{1}_x F_x + \mathbf{1}_y F_y + \mathbf{1}_z F_z) \cdot (\mathbf{1}_x G_x + \mathbf{1}_y G_y + \mathbf{1}_z G_z) = F_x G_x + F_y G_y + F_z G_z \qquad \text{(I-8)}$$

As an example of the dot product, the work done when a body is moved a distance $\Delta \ell$ by a force **F** is defined by

$$\text{Work} = |\mathbf{F}| \; |\Delta \boldsymbol{\ell}| \cos \theta \Big|_{\mathbf{F}}^{\Delta \boldsymbol{\ell}} \qquad \text{(I-9)}$$

in which $\theta \Big|_{\mathbf{F}}^{\Delta \boldsymbol{\ell}}$ is the angle between the direction of action of **F** and of the displacement $\Delta \boldsymbol{\ell}$. Work done mechanically can thus be expressed as the dot product of the vectors **F** and $\Delta \boldsymbol{\ell}$.

$$\text{Work} = \mathbf{F} \cdot \Delta \boldsymbol{\ell} \qquad \text{(I-10)}$$

The cross product of two vectors **F** and **G** is defined to be a vector **R**, $\mathbf{F} \times \mathbf{G} = \mathbf{R}$, in which the magnitude of **R** is

$$|\mathbf{R}| = |\mathbf{F} \times \mathbf{G}| = |\mathbf{F}| \; |\mathbf{G}| \sin \theta \Big|_{\mathbf{F}}^{\mathbf{G}} \qquad \text{(I-11)}$$

and in which the direction of **R** is the direction of advance of a right-hand screw rotated through the smaller angle from **F** to **G**.

Since $\sin \theta = -\sin(-\theta)$, the manner of measurement of θ in the cross product must be specified. According to the usual convention, θ is measured as the smaller angle from **F** to **G**—that is, from the first to the second vector in the product: hence the order of the vectors is very important and $\mathbf{F} \times \mathbf{G} = -\mathbf{G} \times \mathbf{F}$. In accordance with the right-hand-screw convention, the following cross products of unit vectors are expressed:

$$\mathbf{1}_y \times \mathbf{1}_z = \mathbf{1}_x = -\mathbf{1}_z \times \mathbf{1}_y$$
$$\mathbf{1}_z \times \mathbf{1}_x = \mathbf{1}_y = -\mathbf{1}_x \times \mathbf{1}_z \qquad \text{(I-12)}$$
$$\mathbf{1}_x \times \mathbf{1}_x = \mathbf{1}_y \times \mathbf{1}_y = \mathbf{1}_z \times \mathbf{1}_z = 0$$

For the two vectors, **F** and **G**, as defined in Eqs. (I-1) and (I-2), the cross product is obtained by complete algebraic multiplication as

$$\begin{aligned}\mathbf{F} \times \mathbf{G} &= (\mathbf{1}_x F_x + \mathbf{1}_y F_y + \mathbf{1}_z F_z) \times (\mathbf{1}_x G_x + \mathbf{1}_y G_y + \mathbf{1}_z G_z) \\ &= \mathbf{1}_x \times \mathbf{1}_x F_x G_x + \mathbf{1}_x \times \mathbf{1}_y F_x G_y + \mathbf{1}_x \times \mathbf{1}_z F_x G_z \\ &+ \mathbf{1}_y \times \mathbf{1}_x F_y G_x + \mathbf{1}_y \times \mathbf{1}_y F_y G_y + \mathbf{1}_y \times \mathbf{1}_z F_y G_z \\ &+ \mathbf{1}_z \times \mathbf{1}_x F_z G_x + \mathbf{1}_z \times \mathbf{1}_y F_z G_y + \mathbf{1}_z \times \mathbf{1}_z F_z G_z\end{aligned} \tag{I-13}$$

When the cross products of the unit vectors are evaluated as in Eq. (I-11), $\mathbf{F} \times \mathbf{G}$ is obtained as

$$\mathbf{F} \times \mathbf{G} = \mathbf{1}_x (F_y G_z - F_z G_y) + \mathbf{1}_y (F_z G_x - F_x G_z) + \mathbf{1}_z (F_x G_y - F_y G_x) \tag{I-14}$$

This product may conveniently be expressed as a determinant in the form

$$\mathbf{F} \times \mathbf{G} = \begin{vmatrix} \mathbf{1}_x & \mathbf{1}_y & \mathbf{1}_z \\ F_x & F_y & F_z \\ G_x & G_y & G_z \end{vmatrix} \tag{I-15}$$

One illustration of this type of multiplication is the computation of torque. When a force acts on a lever arm, the resultant torque acts about a definite axis of rotation (perpendicular to the plane containing the lever arm and the force.) The magnitude of the torque depends upon the products of the magnitude of the force, the length of lever arm, and the sine of the angle between them. Consequently, the torque **T** produced by a force **F** acting on a lever arm $\boldsymbol{\ell}$ can be expressed exactly in vector form as $\mathbf{T} = \mathbf{F} \times \boldsymbol{\ell}$.

I-2. Vector Triple Product and the Scalar Triple Product. The definition of the vector product readily extends to the triple vector product $\mathbf{F} \times \mathbf{G} \times \mathbf{H}$ which has a unique meaning only when the order for obtaining the product is clearly indicated. This order is customarily indicated by association so that $(\mathbf{F} \times \mathbf{G}) \times \mathbf{H}$ is understood to mean that the vector product $\mathbf{F} \times \mathbf{G}$ is formed first, and then the product of this vector with vector **H** is formed. The resultant vector is normal to $(\mathbf{F} \times \mathbf{G})$; consequently the resultant is in the plane containing **F** and **G**. Evidently, $(\mathbf{F} \times \mathbf{G}) \times \mathbf{H} = \mathbf{F} \times (\mathbf{G} \times \mathbf{H})$ and the order of association of vectors in forming the product cannot be interchanged. Since the vectors within parentheses may be commuted without changing the association in forming the triple product,

$$(\mathbf{F} \times \mathbf{G}) \times \mathbf{H} = -(\mathbf{G} \times \mathbf{F}) \times \mathbf{H} = \mathbf{H} \times (\mathbf{G} \times \mathbf{F}) = -\mathbf{H} \times (\mathbf{F} \times \mathbf{G}). \quad \text{(I-16)}$$

Through the use of Eq. (I-15) the vector triple product can be expanded to yield the identity

$$(\mathbf{F} \times \mathbf{G}) \times \mathbf{H} = (\mathbf{F} \cdot \mathbf{H})\,\mathbf{G} - (\mathbf{G} \cdot \mathbf{H})\,\mathbf{F} \quad \text{(I-17)}$$

The combination of a vector and a scalar product of the form $H \cdot (\mathbf{F} \times \mathbf{G})$ is called a scalar triple product. In order to form the scalar triple product, the vector product $\mathbf{F} \times \mathbf{G}$ is first formed, and then the scalar product with $\mathbf{H}$ is obtained. From analogy to Eq. (I-15) it can be shown that the scalar triple product can be expressed in the form

$$\mathbf{H} \cdot \mathbf{F} \times \mathbf{G} = \begin{vmatrix} H_x & H_y & H_z \\ F_x & F_y & F_z \\ G_x & G_y & G_z \end{vmatrix} \quad \text{(I-18)}$$

According to the laws of determinants, interchanging a pair of adjacent rows or a pair of adjacent columns changes only the algebraic sign of the value of the determinant. Therefore, the association in the triple scalar product may be interchanged with appropriate sign changes in the product; thus

$$\mathbf{H} \cdot \mathbf{F} \times \mathbf{G} = -\mathbf{F} \cdot \mathbf{H} \times \mathbf{G} = -\mathbf{H} \cdot \mathbf{G} \times \mathbf{F} = \mathbf{G} \cdot \mathbf{H} \times \mathbf{F} \quad \text{(I-19)}$$

I-3. Vector and Scalar Fields. Most quantities in electromagnetic field theory are functions of position in the coordinate system in which they are expressed. Such quantities are either vectors or scalars. An illustration of a vector function of space, commonly called a vector field, is the force field set up by displaced water in the region of the bottom of a ship. The magnitude and direction of the force exerted at a point on the hull of a ship depend upon the location of the point. On the other hand, an illustration of a scalar function of space, commonly called a scalar field, is the temperature of the walls and interior of a refrigerator. The temperature at a point p within the refrigerator is specified as a magnitude only and depends, among other things, upon the location of the point.*

*Vector fields can often be computed from scalar fields, and *vice versa.* The rate of change of temperature with respect to change in location of the point