Did You Say Mathematics?

Moscow

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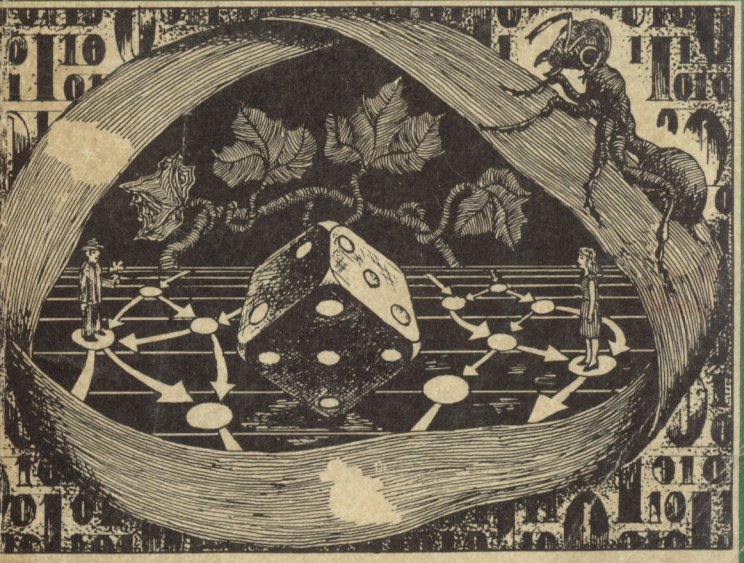

Ya. Khurgin

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TO THE READER

Mir Publishers would be
grateful for your comments
on the content, translation
and design of this book.
We would also be pleased to
receive any other suggestions
you may wish to make.
Our address is: Mir Publishers,
2 Pervy Rizhsky Pereulok, I – 110,
GSP, Moscow, 129820, USSR

На английском языке

First published 1974
Second printing 1984

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НУ И ЧТО?

Издательство
«Молодая гвардия»
Москва

Contents

A Few Words with the Reader

I like to argue and discuss things or just while away the time in friendly conversation. I don't like to write. Talking is better because it's a two-way affair. One gets a response, technically called "feedback".

During the past 25 years I have been involved in discussions with engineers, physiologists, doctors, geologists, and economists—people of different fields, views and talents. I've delivered numberless lectures and talks and I've conducted seminars. These talks deal with the problems and difficulties of various sciences about which I personally haven't the faintest notion most of the time.

I've never liked the idea of merely delivering a lecture—in some cases it amounts to simply reading a text. Nothing could be duller. I make every effort to carry on a conversation with my listeners.

It takes a long time to prepare a two-hour lecture, and even then I'm never sure of what it's actually going to be like because so much depends on the audience.

I imagine TV speakers have a hard time. After all, you can't laugh at your own jokes, and how does one

ask questions without getting so much as a silent nod for an answer?

Writing a book is like that too: there is no feedback. I find it very difficult to speak to an indefinite person, to an unknown reader. So here I'll speak to friends of mine from a variety of fields: physiologists, physicians, engineers, and geologists. We will talk about mathematics. The mathematician (the author, that is) will discuss matters with nonmathematicians. There have been many such conversations and there will be many more in the future. Why? For a very simple reason. A specialist is one who knows a great deal in a narrowly defined field of knowledge. Whereas it would take years for me, a mathematician, to collect the relevant facts of a problem concerning a specialist, the specialist can tell me all about his troubles and problems in a matter of minutes.

And he usually does it with the greatest of pleasure. My curiosity is satisfied and I do not even have to overcome my natural laziness.

In a word, I like conversations with specialists.

Why they come to me, a mathematician, is clear enough. We are in the midst of a mathematization of all sciences, even the descriptive sciences. At least that is what we read in the popular (and not so popular) scientific literature. That is what we hear over the radio and see on television. True, most people have a rather quaint idea of what mathematization is all about. Some think that the mathematician is capable of writing down equations for every imaginable practical situation. Others believe that electronic computers are about to take over and will do most of the thinking instead of human beings. Still others are sober enough to hope only for a certain amount of assistance from the mathematician.

Actually, of course, mathematical methods are no

cure-all for our many problems. But they are certainly applicable in every science if one takes the pains to apply them reasonably and properly.

Using mathematical methods is much like putting meat through a meat grinder—aside from having a good cutter and being able to turn the handle (and in the right direction!), you must put in quality products, otherwise you will grind out nothing but disappointment, in which case do not hurry to blame the theory because the blame lies elsewhere.

It is extremely important that the potential user of mathematical theory get acquainted with it and be capable of applying it appropriately, or at least be able to see when and where it is applicable. Users of mathematical theory will then be in a position to suggest new theoretical trends as they pose fresh problems, and the result will be of mutual benefit to all parties concerned.

The first encounters with specialists are in the nature of a competition, a clash. Each side is more interested in hearing himself than his adversary. Later, as the "battle" heats up, each side manoeuvres to establish its point of view. Then, finally, as a sort of mutual understanding takes place, both sides win.

Mathematicians, and I'm one, delight in such verbal fencing. We enjoy asking provocative questions like, "Now what is the question you are really interested in?" or "What sort of problem is it that's worrying you?" or even mere "So what?"

After a good deal of skirmishing, we finally arrive at a stage when the mathematician can begin to cooperate fruitfully with the specialist. Such joint undertakings are very satisfying and extremely fruitful to both parties.

If the reader finds these discussions exciting and useful, the author will consider his goal achieved.

MATHEMATICIAN AND PHYSIOLOGIST GET TOGETHER IN SEPTEMBER

Autumn is always a fascinating topic for poets, writers and painters. For me, September means young people and the start of the school year. New students, new seminars, new problems.

My first encounter a few years ago was with a young and, so I heard, talented physiologist. He liked his subject and knew it well. He was enthusiastically seeking new pathways, new fields, and he earnestly wanted results. A person to my liking.

Mathematician (me, as usual). What topic are you working on?

Physiologist. I'm studying primary electrical responses of the visual zone of the cortex in the cat caused by flashes of light produced in front of the eye.

[I know what this is about. You insert a wire electrode into the cat's brain and bioelectrical potentials are recorded. The potentials are then fed to an electronic oscillograph where they are displayed or photographed (see the upper curve in Fig. 1). The lower (periodic) curve is the time reference.

Mathematician. Can you be a little more specific?

Physiologist. The stimulus is impressed as a pulse of light, the brigbtness of which can be varied. In the process the magnitude and shape of the positive and negative phases of the induced potential vary too.

Math. So what?

[How little one is able to put across on paper! The very intonation of the question contains a good deal of information. Right now it amounts to mere interest.]

Physiol. Just what do you mean? We have a definite relationship between the intensity of the light flash and all parameters of the electrical response.

[Note the words "definite relationship". What do they mean?]

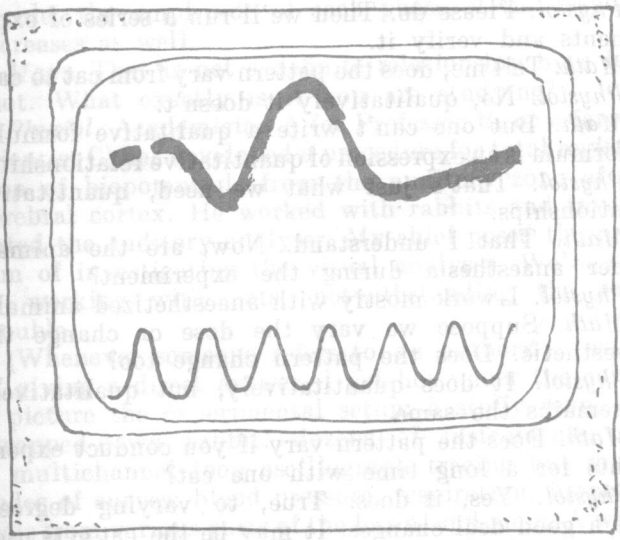

Fig. 1

Math. What kind of a relationship?

Physiol. For instance, with increasing intensity of the light flash the amplitude of the response at first builds up rapidly. then more slowly, and finally remains constant.

Math. That's fine, but where do I come in?

Physiol. I'd like to obtain a mathematical relation.

Math. Why do you need a mathematical relation?

Physiol. What do you mean "why"? Are you against applying mathematics to biology?

Math. Not in the least. I'm very much for it. By a mathematical relation you mean a formula, right?

Physiol. Yes, that's right.

Math. What will you do with the formula if I write it down for you?

Physiol. Please do. Then we'll run a series of experiments and verify it.

Math. Tell me, does the pattern vary from cat to cat?

Physiol. No, qualitatively it doesn't.

Math. But one can't write a qualitative formula. A formula is an expression of quantitative relationships.

Physiol. That's just what we need, quantitative relationships.

Math. That I understand. Now, are the animals under anaesthesia during the experiment?

Physiol. I work mostly with anaesthetized animals.

Math. Suppose we vary the dose or change the anaesthetic. Does the pattern change too?

Physiol. It does quantitatively, but qualitatively it remains the same.

Math. Does the pattern vary if you conduct experiments for a long time with one cat?

Physiol. Yes, it does. True, to varying degrees. But a good deal changes. It may be the cat gets used to it. Then, too, the depth of anaesthesia varies during an experiment.

Math. Why do you say that the relationship between the flash intensity and the duration of the phase is definite?

Physiol. Perhaps I did not put that quite exactly. But you get the idea, I'm sure. What I wanted to say is that a relationship exists.

Math. The point is this. Let's take the law of universal gravitation. It states a very definite relationship between the masses of two bodies, the distance between them, and the force of their attraction. Now in the process you are studying I don't see any definite, clearly defined relationship between the light intensity and the amplitude of the response (electrical reaction of the brain).

Physiol. But if we increase the intensity, then, as

a rule, the amplitude of each phase of the response increases as well.

Math. That is not a definite relationship by a long shot. What exactly is it you are studying?

Physiol. Academician A (or Professor B, or a famous scientist C) has developed a procedure for total registration of biopotentials from the auditory zone of the cerebral cortex. He worked with rabbits and investigated the auditory analyser. My chief posed the problem of investigating the visual analyser. We're used to working with cats, notwithstanding the extra trouble.

[Whenever someone refers to an authority instead of giving a direct answer, I get hot under the collar. I picture the experimental setup: a stuffy room, the strapped-down rabbit, dozens of instruments, and a multichannel loop oscillograph tracing out multitudes of curves: blood pressure, respiration, biopotentials from various parts of the brain, and more. A whole team of scientists carry out this involved many-hour experiment, and wind up, finally, by throwing out the poor little rabbit and also the metres upon metres of tracings—for the simple reason that it was never clear in the first place what they had intended to do with them.]

Math. What is the question that you want an answer to?

Physiol. (By this time also a bit exasperated.) Listen, I've already explained. We would like to know how the parameters of the primary response depend on the intensity of the flash.

Math. Suppose you have that relationship and a formula. How will they help?

Physiol. (Getting more excited.) Look, I told you that Academician A...

Math. O.k. What problem was he working on?