Lecture Notes in Mathematics

1544

Michael Schürmann

White Noise on Bialgebras

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Author

Michael Schürmann
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294
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Addresses:

Professor A. Dold
Mathematisches Institut, Universität Heidelberg
Im Neuenheimer Feld 288
W-6900 Heidelberg
Federal Republic of Germany

Professor B. Eckmann
Mathematik, ETH-Zentrum
CH-8092 Zürich, Switzerland

Professor F. Takens
Mathematisch Instituut
Rijksuniversiteit Groningen
Postbus 800
NL-9700 AV Groningen
The Netherlands

Springer-Verlag, Mathematics Editorial
Tiergartenstr. 17
W-6900 Heidelberg
Federal Republic of Germany
Tel.: *49 (6221) 487-410

CONTENTS

Introduction

These notes are a contribution to the field of quantum (or non-commutative) probability theory. Quantum probability can be regarded as an attempt of a unified approach to classical probability and the quantum theory of irreversible processes.

Our special interest lies in a non-commutative theory of processes with independent and stationary increments on a group. Following common practise, such processes will be called *white noise* even though they are actually 'integrated white noise'. The motivating idea is to use non-commutative white noise as a description of a quantum mechanical heat bath to which the quantum mechanical initial system under consideration is coupled. The passage from classical commutative to quantum non-commutative theory is mathematically established by replacing algebras of functions by algebras of linear operators, i.e. by not necessarily commutative algebras. The natural generalization of an algebra of functions on a semi-group (a group), and thus in a way of a semi-group (a group) itself, is a *bialgebra* (a *Hopf algebra*). Recently, *quantum groups* appeared in several fields of non-commutative mathematics; see e.g. [25, 53, 94]. It should be stressed that quantum groups are defined in different ways in the literature but they are Hopf algebras in all cases. We are concerned with the role bialgebras and Hopf algebras play in quantum probability as the 'non-commutative state space' on which our non-commutative white noise is modelled. Since we work with generalizations of probability measures, there is also a need for a positivity structure, and this is how $*$-bialgebras and $*$-Hopf algebras come in. These are bialgebras and Hopf algebras which have an involution compatible both with the algebra and the coalgebra structure. A $*$-Hopf algebra is close to what is called a *matrix pseudo-group* in [94].

The first time $*$-bialgebras appeared in quantum probability was in the paper [92] by W. von Waldenfels where the *non-commutative coefficient algebra of the unitary group* was introduced. After a crucial result on positivity had been established [91, 70], L. Accardi proposed an algebraic framework for the general theory of quantum white noise on graded $*$-bialgebras. This program has been worked out in [5].

One of the main tools in quantum probability is the *quantum stochastic calculus* developed by R.L. Hudson and K.R. Parthasarathy in [41]; see also [15, 16, 51]. The connection between quantum white noise and quantum stochastic differential equations became apparent in [72]. Shortly after that, the question arose as to whether any white noise on a $*$-bialgebra can be realized as a solution of a quantum stochastic differential equation. Under the assumptions of boundedness and of Bose independence of the processes, an answer in the affirmative was given in [29]. Generalizing the results of [75], in these notes we treat the case of arbitrary unbounded processes with 'twisted' independence. We show that a white noise can always be realized on Bose Fock space. Using H. Maassens kernel method [51], we give an explicit formula for the processes. Indeed, the processes are solutions of quantum stochastic differential equations. The form of the equation is governed by the coalgebra structure of the underlying twisted $*$-bialgebra.

Recently, there has been an attempt to generalize both the concepts of convolution semi-groups of states on a $*$-bialgebra (which are closely related to quantum white noise) and of convolution semi-groups of instruments on a group [11] to a theory of convolution semi-groups of positive operator valued maps on a $*$-bialgebra; see [12].

*

The following discussion of four related topics in classical probability theory on groups and in quantum probability is intended to serve as a more detailed introduction to the mathematical problems we are concerned with.

Convolution semi-groups of probability measures. Let $X_t : \Omega \to G$ be a stochastic process indexed by time $t \geq 0$ taking values in some (topological) group G. Suppose that the process X_t is a white noise so that it has independent and stationary increments

$$X_{st} = X_s^{-1} X_t,$$

i.e. the random variables

$$X_{t_1 t_2}, \ldots, X_{t_n t_{n+1}}$$

are independent for all choices of $n \in \mathbb{N}$ and $t_1 < \cdots < t_{n+1}$ and the distribution of X_{st} only depends on the difference $t - s$. Then X_t is determined up to stochastic equivalence by its 1-dimensional distributions φ_t which form a 1-parameter semi-group of probability measures on G with respect to convolution. If φ_t is weakly continuous at 0 we can differentiate $\varphi_t(f)$ for an appropriate class of complex-valued functions f on G to obtain the *generator* of φ_t which in all cases of interest again will determine φ_t and therefore X_t.

For a Lie group G *Hunt's formula* [44] gives a description of all the generators of white noise in terms of left invariant derivations of first and second order and an integral which represents the 'Poisson part' of X_t.

For a compact or locally compact abelian group the generator can be defined on the algebra $\mathcal{R}(G)$ of representative functions (see [34]) of G and the generators of white noise turn out to be the hermitian, conditionally positive linear functionals on $\mathcal{R}(G)$; cf. [35]. The principle of correspondence between 1-parameter semi-groups of positive linear functionals and hermitian, conditionally positive linear functionals goes back to I. Schoenberg [69] and will be called *Schoenberg correspondence*; cf. [17].

Stochastic semi-groups. Let (X_{st}) be a stochastic process indexed by pairs (s, t) of real numbers with $0 \leq s \leq t$, taking values in the space M_n of complex $n \times n$-matrices. Suppose that X_{st} satisfies the evolution equations

$$X_{rs} X_{st} = X_{rt}$$
$$X_{tt} = 1$$

almost everywhere for all $r \leq s \leq t$ and that the 'increments' X_{st} are independent. A process X_{st} with these properties is usually called a stochastic semi-group; see e.g. [81]. Let the increments X_{st} also be stationary and assume that X_{st} converges to 1 in probability for $t \downarrow 0$. (In this case, X_{st} is actually the increment process associated with a classical white noise on the general linear group.) Then there exists a stochastic process $(F_t)_{t \geq 0}$ on the same probability space taking values in M_n such that the following holds. The additive increments $F_{st} = F_t - F_s$ are independent and stationary, F_t converges to

0 in probability for $t \downarrow 0$ and X_{st} is the solution of the operator stochastic differential equation

$$\mathrm{d}X_{st} = X_{st}\mathrm{d}F_t, \quad t \geq s, \tag{1}$$

with the initial condition $X_{ss} = \mathbf{1}$ a.e. Moreover, the sums

$$\sum_{0 \leq t_{k+1} < t} (X_{t_k, t_{k+1}} - \mathbf{1})$$

converge to F_t in probability if the maximal distance between neighbours of the partition $\{0 = t_0 < t_1 < t_2 < \ldots\}$ of $\mathbb{R}_+$ tends to 0; see [81] where the more general case is treated when X_{st} are not necessarily stationary but for each fixed s the process $(X_{st})_{s \leq t}$ is a semi-martingale.

Infinitely divisible representations of groups. A normalized positive definite function f on the group G is called infinitely divisible if for all $n \in \mathbb{N}$ there is a normalized positive definite function f_n on G such that

$$f_n(x)^n = f(x) \text{ for all } x \in G.$$

By Schoenberg correspondence $f = \mathrm{e}^g$ is infinitely divisible if g is hermitian, conditionally positive and g vanishes on the identity element of G. Under certain analytic conditions on f or G each infinitely divisible function on G is of the form e^g; see [63]. The question we are interested in is how to express the Gelfand-Naimark-Segal-representation, see [67], given by f in terms of the following representation naturally associated with g. Divide the vector space $\mathbb{C}G$ formally spanned by the elements of G by the nullspace of the positive sesquilinear form on $\mathbb{C}G$ with

$$(x, y)_g = g(x^{-1}y) - \overline{g(x)} - g(y), \quad x, y \in G,$$

to obtain a pre-Hilbert space D whose completion we denote by H. Denote the canonical mapping from $\mathbb{C}G$ to H by η. Then the equations

$$\rho(x)\eta(y) = \eta(xy) - \eta(x)$$

define a unitary representation ρ of G. Roughly speaking, we must *exponentiate* the representation ρ. The so-called *Araki-Woods embedding theorem* for infinitely divisible functions on a group, see [8, 9, 32, 63, 85], says that the GNS-representation of f lives in the *Bose Fock space* over H. Moreover, it maps exponential vectors to multiples of exponential vectors.

Now quantum probability comes in. The quantum stochastic calculus of Hudson and Parthasarathy is a theory of Ito type integration against operator processes on Bose Fock space over $L^2(\mathbb{R}_+) \otimes H$ where H is some Hilbert space fixing the number of degrees of freedom of the integrators. For $\xi \in H$ and a linear operator R on H the integrators are the *creation process* $A_t^*(\xi)$, the *preservation process* $\Lambda_t(R)$, the *annihilation process* $A_t(\xi)$ and time t, the latter giving the usual Riemann-Bochner integral of operators. It is well known that $A_t^* + A_t$ is the realization of Brownian motion on Bose Fock space; see e.g. [36]. Moreover, $A_t^* + \Lambda_t + A_t + t$ is the realization of a Poisson process on Bose Fock space [41]. So the Hudson-Parthasarathy integral generalizes classical stochastic integration against the Wiener and the Poisson process. (We will see that a much wider class of classical types of white noise is included.) The connection to infinitely divisible representations of G is given by the fact that the GNS-representation π_1 of f can be embedded into the solution of the linear quantum stochastic differential equation

$$\mathrm{d}\pi_t(x) = \pi_t(x)\mathrm{d}F_t(x) \tag{2}$$

with initial condition $\pi_0(x) = \mathrm{id}$ where

$$F_t(x) = A_t^*(\eta(x)) + \Lambda_t(\rho(x) - \mathrm{id}) + A_t(\eta(x^{-1})) + g(x)t.$$

Finally this makes rigorous the idea of exponentiating the representation ρ; cf. [59].

Unification of Bose and Fermi stochastic calculus. It is possible to develop a quantum stochastic calculus on Fermi Fock space [6, 7]; see also the work of C. Barnett, R. Streater and I. Wilde [14] on the Clifford integral. However, it was shown in [43] that the Bose does include the Fermi calculus. This unification is possible because Fermion Brownian motion itself can be interpreted as the Bose quantum stochastic integral

$$\int_0^t \Gamma_\tau(-1)\mathrm{d}A_\tau$$

over the second quantization $\Gamma_t(-1)$ of the *reflexion process* on $L^2(\mathbb{R}_+)$ which is given by

$$f \mapsto -f\chi_{[0,t)} + f\chi_{[t,\infty)}.$$

*

Up to this point we only arranged some known facts of classical probability, mentioned a relation to quantum stochastic calculus and quoted the result on the unification of Bose and Fermi integration. We explain how all this can be seen in a more general non-commutative framework and finally becomes a mathematical theory which goes far beyond the classical results.

For motivation and in order to simplify things, for the moment let us restrict ourselves to the case when G is the group U_n of complex unitary $n \times n$-matrices. Let $\mathcal{U}[n]$ be the $*$-algebra of complex-valued functions on U_n generated by the constant functions and by the coordinate functions x_{kl}, $k, l = 1, \ldots, n$, and their complex-conjugates x_{kl}^*. Since U_n is compact, probability measures on U_n and states, i.e. normalized positive linear functionals, on $\mathcal{U}[n]$ can be identified; see e.g. [35]. The convolution of probability measures becomes the convolution of states φ and ψ given by

$$\varphi \star \psi = (\varphi \otimes \psi) \circ \Delta$$

where Δ denotes the $*$-algebra homomorphism

$$\Delta : \mathcal{U}[n] \to \mathcal{U}[n] \otimes \mathcal{U}[n]$$

given by extension of

$$\Delta x_{kl} = \sum_{m=1}^{n} x_{km} \otimes x_{ml}. \tag{3}$$