

LNCS 3843

Patrick Healy  
Nikola S. Nikolov (Eds.)

# Graph Drawing

13th International Symposium, GD 2005  
Limerick, Ireland, September 2005  
Revised Papers



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Limerick, Ireland, September 12-14, 2005  
Revised Papers



Springer

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Library of Congress Control Number: 2005938800

CR Subject Classification (1998): G.2, F.2, I.3, E.1

LNCS Sublibrary: SL 1 – Theoretical Computer Science and General Issues

ISSN	0302-9743
ISBN-10	3-540-31425-3 Springer Berlin Heidelberg New York
ISBN-13	978-3-540-31425-7 Springer Berlin Heidelberg New York

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[springer.com](http://springer.com)

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Printed in Germany

Typesetting: Camera-ready by author, data conversion by Scientific Publishing Services, Chennai, India  
Printed on acid-free paper SPIN: 11618058 06/3142 5 4 3 2 1 0

# Preface

The 13th International Symposium on Graph Drawing (GD 2005) was held in Limerick, Ireland, September 12-14, 2005. One hundred and fifteen participants from 19 countries attended GD 2005.

In response to the call for papers the Program Committee received 101 submissions, each detailing original research or a system demonstration. Each submission was reviewed by at least three Program Committee members; each referee's comments were returned to the authors. Following extensive discussions, the committee accepted 38 long papers, 3 short papers and 3 long system demos, each of which were presented during one of the conference's 12 sessions. Eight posters were also accepted and were on display throughout the conference.

Two invited speakers, Kurt Mehlhorn and George Robertson, gave fascinating talks during the conference. Prof. Mehlhorn spoke on the use of minimum cycle bases for reconstructing surfaces, while Dr. Robertson gave a perspective, past and present, on the visualization of hierarchies.

As is now traditional, a graph drawing contest was held during the conference. The accompanying report, written by Stephen Kobourov, details this year's contest. This year a day-long workshop, organized by Seok-Hee Hong and Dorothea Wagner, was held in conjunction with the conference. A report on the "Workshop on Network Analysis and Visualization," written by Seok-Hee Hong, is included in the proceedings.

We are indebted to many people for the success of the conference. The Program Committee and external referees worked diligently to select only the best of the submitted papers. The Organizing Committee under the co-chairmanship of Nikola Nikolov worked tirelessly in the months leading up to the conference. In particular, a big debt is owed to Aaron Quigley for his Herculean fund-raising efforts, to Alex Tarasov for his system maintenance, to Karol Lynch for his web page development, and to Gemma Swift and Nuala Kitson for their administrative support and constant good humor. Thanks are also due to Vincent Cunnane, who opened the conference. Last, but not least, we thank Peter Eades, who provided valuable direction and kept a steady head throughout.

The conference received assistance from Science Foundation Ireland (Benefactor); Intel Corp., Microsoft Corp. and Tom Sawyer Software (Gold Sponsors); National ICT Australia, Enterprise Ireland, Fáilte Ireland, ILOG Inc., AbsInt Angewandte Informatik GmbH (Silver Sponsors); Lucent Technologies, Jameson Irish Whiskey and Dell Inc.

The 14th International Symposium on Graph Drawing (GD 2006) will be held September 18-20, 2006 in Karlsruhe, Germany, co-chaired by Michael Kaufmann and Dorothea Wagner.

October 2005

Patrick Healy  
Nikola S. Nikolov  
Limerick

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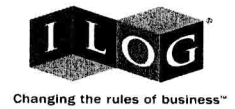
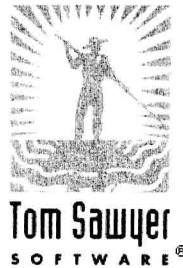
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# Table of Contents

## Papers

Crossings and Permutations <i>Therese Biedl, Franz J. Brandenburg, Xiaotie Deng</i> .....	1
Morphing Planar Graphs While Preserving Edge Directions <i>Therese Biedl, Anna Lubiw, Michael J. Spriggs</i> .....	13
Dynamic Spectral Layout of Small Worlds <i>Ulrik Brandes, Daniel Fleischer, Thomas Puppe</i> .....	25
Exact Crossing Minimization <i>Christoph Buchheim, Dietmar Ebner, Michael Jünger, Gunnar W. Klau, Petra Mutzel, René Weiskircher</i> .....	37
On Embedding a Cycle in a Plane Graph <i>Pier Francesco Cortese, Giuseppe Di Battista, Maurizio Patrignani, Maurizio Pizzonia</i> .....	49
On Rectilinear Duals for Vertex-Weighted Plane Graphs <i>Mark de Berg, Elena Mumford, Bettina Speckmann</i> .....	61
Bar $k$ -Visibility Graphs: Bounds on the Number of Edges, Chromatic Number, and Thickness <i>Alice M. Dean, William Evans, Ellen Gethner, Joshua D. Laison, Mohammad Ali Safari, William T. Trotter</i> .....	73
Drawing $K_n$ in Three Dimensions with One Bend Per Edge <i>Olivier Devillers, Hazel Everett, Sylvain Lazard, Maria Pentcheva, Stephen K. Wismath</i> .....	83
Small Area Drawings of Outerplanar Graphs <i>Giuseppe Di Battista, Fabrizio Frati</i> .....	89
Volume Requirements of 3D Upward Drawings <i>Emilio Di Giacomo, Giuseppe Liotta, Henk Meijer, Stephen K. Wismath</i> .....	101
How to Embed a Path onto Two Sets of Points <i>Emilio Di Giacomo, Giuseppe Liotta, Francesco Trotta</i> .....	111



Upward Spirality and Upward Planarity Testing <i>Walter Didimo, Francesco Giordano, Giuseppe Liotta</i> .....	117
Graph Treewidth and Geometric Thickness Parameters <i>Vida Dujmović, David R. Wood</i> .....	129
Stress Majorization with Orthogonal Ordering Constraints <i>Tim Dwyer, Yehuda Koren, Kim Marriott</i> .....	141
Fast Node Overlap Removal <i>Tim Dwyer, Kim Marriott, Peter J. Stuckey</i> .....	153
Delta-Confluent Drawings <i>David Eppstein, Michael T. Goodrich, Jeremy Yu Meng</i> .....	165
Transversal Structures on Triangulations, with Application to Straight-Line Drawing <i>Éric Fusy</i> .....	177
A Hybrid Model for Drawing Dynamic and Evolving Graphs <i>Marco Gaertler, Dorothea Wagner</i> .....	189
Two Trees Which Are Self-intersecting When Drawn Simultaneously <i>Markus Geyer, Michael Kaufmann, Imrich Vrt'o</i> .....	201
C-Planarity of Extrovert Clustered Graphs <i>Michael T. Goodrich, George S. Lueker, Jonathan Z. Sun</i> .....	211
Non-planar Core Reduction of Graphs <i>Carsten Gutwenger, Markus Chimani</i> .....	223
An Experimental Comparison of Fast Algorithms for Drawing General Large Graphs <i>Stefan Hachul, Michael Jünger</i> .....	235
Hierarchical Layouts of Directed Graphs in Three Dimensions <i>Seok-Hee Hong, Nikola S. Nikolov</i> .....	251
Layout Effects on Sociogram Perception <i>Weidong Huang, Seok-Hee Hong, Peter Eades</i> .....	262
On Edges Crossing Few Other Edges in Simple Topological Complete Graphs <i>Jan Kynčl, Pavel Valtr</i> .....	274

On Balloon Drawings of Rooted Trees <i>Chun-Cheng Lin, Hsu-Chun Yen</i> .....	285
Convex Drawings of Plane Graphs of Minimum Outer Apices <i>Kazuyuki Miura, Machiko Azuma, Takao Nishizeki</i> .....	297
Energy-Based Clustering of Graphs with Nonuniform Degrees <i>Andreas Noack</i> .....	309
A Mixed-Integer Program for Drawing High-Quality Metro Maps <i>Martin Nöllenburg, Alexander Wolff</i> .....	321
Crossing Number of Toroidal Graphs <i>János Pach, Géza Tóth</i> .....	334
Drawing Graphs Using Modular Decomposition <i>Charis Papadopoulos, Constantinos Voglis</i> .....	343
Applications of Parameterized <i>st</i> -Orientations in Graph Drawing Algorithms <i>Charalampos Papamantou, Ioannis G. Tollis</i> .....	355
Complexity Results for Three-Dimensional Orthogonal Graph Drawing <i>Maurizio Patrignani</i> .....	368
On Extending a Partial Straight-Line Drawing <i>Maurizio Patrignani</i> .....	380
Odd Crossing Number Is Not Crossing Number <i>Michael J. Pelsmajer, Marcus Schaefer, Daniel Štefankovič</i> .....	386
Minimum Depth Graph Embeddings and Quality of the Drawings: An Experimental Analysis <i>Maurizio Pizzonia</i> .....	397
No-bend Orthogonal Drawings of Series-Parallel Graphs <i>Md. Saidur Rahman, Noritsugu Egi, Takao Nishizeki</i> .....	409
Parallel-Redrawing Mechanisms, Pseudo-Triangulations and Kinetic Planar Graphs <i>Ileana Streinu</i> .....	421
Proper and Planar Drawings of Graphs on Three Layers <i>Matthew Suderman</i> .....	434

Incremental Connector Routing  
*Michael Wybrow, Kim Marriott, Peter J. Stuckey* . . . . . 446

An Application of Well-Orderly Trees in Graph Drawing  
*Huaming Zhang, Xin He* . . . . . 458

Software Demonstrations

GEOMI: GEOMetry for Maximum Insight  
*Adel Ahmed, Tim Dwyer, Michael Forster, Xiaoyan Fu, Joshua Ho, Seok-Hee Hong, Dirk Koschützki, Colin Murray, Nikola S. Nikolov, Ronnie Taib, Alexandre Tarassov, Kai Xu* . . . . . 468

WhatsOnWeb: Using Graph Drawing to Search the Web  
*Emilio Di Giacomo, Walter Didimo, Luca Grilli, Giuseppe Liotta* . . . . . 480

Drawing Clustered Graphs in Three Dimensions  
*Joshua Ho, Seok-Hee Hong* . . . . . 492

Posters

BLer: A Boundary Labeller for Technical Drawings  
*Michael A. Bekos, Antonios Symvonis* . . . . . 503

D-Dupe: An Interactive Tool for Entity Resolution in Social Networks  
*Mustafa Bilgic, Louis Licamele, Lise Getoor, Ben Shneiderman* . . . . . 505

A New Method for Efficiently Generating Planar Graph Visibility Representations  
*John M. Boyer* . . . . . 508

SDE: Graph Drawing Using Spectral Distance Embedding  
*Ali Civril, Malik Magdon-Ismail, Eli Bocek-Rivele* . . . . . 512

MultiPlane: A New Framework for Drawing Graphs in Three Dimensions  
*Seok-Hee Hong* . . . . . 514

Visualizing Graphs as Trees: Plant a Seed and Watch It Grow  
*Bongshin Lee, Cynthia Sims Parr, Catherine Plaisant, Benjamin B. Bederson* . . . . . 516

On Straightening Low-Diameter Unit Trees  
*Sheung-Hung Poon* . . . . . 519

Mixed Upward Planarization - Fast and Robust <i>Martin Siebenhaller, Michael Kaufmann</i> .....	522
--	-----

## Workshop on Network Analysis and Visualisation

Network Analysis and Visualisation <i>Seok-Hee Hong</i> .....	524
--	-----

## Graph Drawing Contest

Graph-Drawing Contest Report <i>Christian A. Duncan, Stephen G. Kobourov, Dorothea Wagner</i> .....	528
--	-----

## Invited Talks

Minimum Cycle Bases and Surface Reconstruction <i>Kurt Mehlhorn</i> .....	532
--	-----

Hierarchy Visualization: From Research to Practice <i>George G. Robertson</i> .....	533
--	-----

<b>Author Index</b> .....	535
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# Crossings and Permutations<sup>\*</sup>

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**Abstract.** We investigate crossing minimization problems for a set of permutations, where a crossing expresses a disarrangement between elements. The goal is a common permutation  $\pi^*$  which minimizes the number of crossings. This is known as the Kemeny optimal aggregation problem minimizing the Kendall- $\tau$  distance. Recent interest into this problem comes from application to meta-search and spam reduction on the Web.

This rank aggregation problem can be phrased as a one-sided two-layer crossing minimization problem for an edge coloured bipartite graph, where crossings are counted only for monochromatic edges.

Here we introduce the max version of the crossing minimization problem, which attempts to minimize the discrimination against any permutation. We show the NP-hardness of the common and the max version for  $k \geq 4$  permutations (and  $k$  even), and establish a  $2-2/k$  and a 2-approximation, respectively. For two permutations crossing minimization is solved by inspecting the drawings, whereas it remains open for three permutations.

## 1 Introduction

One-sided crossing minimization is a major component in the Sugiyama algorithm. The one-sided crossing minimization problem has gained much interest and is one of the most intensively studied problems in graph drawing [8, 15]. For general graphs the crossing minimization problem is known to be NP-hard [13]. The NP-hardness also holds for bipartite graphs where the upper layer is fixed, and the graphs are dense with about  $n_1 n_2 / 3$  crossings [10], or alternatively, the graphs are sparse with degree at least four on the free layer [17]. The special case with degree 2 vertices on the free layer is solvable in linear time, whereas the degree 3 case is open.

The rank aggregation problem finds a consensus ranking on a set of alternatives, based on preferences of individual voters. The roots for a mathematical

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<sup>\*</sup> The work of the first author was supported by NSERC, and done while the author was visiting Universität Passau. The work of the second and third authors was partially supported by a grant from the German Academic Exchange Service (Project D/0506978) and from the Research Grant Council of the Hong Kong Joint Research Scheme (Project No. G-HK008/04).

investigation of the problem lie in voting theory and go back to Borda (1781) and Condorcet (1785). Rank aggregations occur in many contexts, including sport, voting, business, and most recently, the Internet. "*Who is the winner?*" In gymnastics, figure skating or dancing this is decided by averaging or ranking the points of the judges. In Formula 1 racing and similarly at the annual European Song Contest the winner is who has the most points. Is this scheme fair? Why not deciding the winner by the majority of first places?

Also, the organizers of GD2005 are confronted with our crossing minimization problem. They have to make many decisions. For example, which beer (wine, food) shall be served at the GD conference dinner? What is the best choice for the individual taste of the participants? Or, more specific: *which beer is the best?*

In their seminal paper from the WWW10 conference, Dwork et al. [9] have used rank aggregation methods for web searching and spam reduction. A search engine is called *good* if it behaves close to the aggregate ranking of several search engines. Besides experimental results they have investigated the theoretical foundations of the rank aggregation problem. One of the main results is the NP-hardness of computing a so-called Kemeny optimal permutation of just four permutations, here called PCM-4. However, the given proof has some flaws, and is repaired here. In addition, we show a relationship to the feedback arc set problem and establish a  $2\text{-}2/k$  approximation, which is achieved by the best input permutation.

The common rank aggregation methods take the sum of all disagreements over all permutations. Here we introduce the *maximum version*,  $\text{PCM}_{\max-k}$ , which expresses a fair aggregation and attempts to avoid a too severe discrimination of any participant or permutation. With the optimal solution, nobody should be totally unhappy. We show the NP-hardness of  $\text{PCM}_{\max-k}$  for all  $k \geq 4$  and establish a 2-approximation, which is achieved by any input permutation. This parallels similar results for the Kemeny aggregation problem [1, 9] and for the Coherence aggregation problem [5]. The case  $\text{PCM}_{\max-2}$  with two permutations is efficiently solvable, whereas the case  $k = 3$  remains open.

Besides the specific results, this work aims to bridge the gap between the combinatorics of rank aggregations and crossing minimizations in graph drawing, with a mutual exchange of notions, insights, and results.

In Section 2 we introduce the basic notions from graph drawing and rank aggregations, and show how to draw rank aggregations. In Section 3 we state the NP-hardness of the crossing minimization problems for just four permutations, and prove the approximation results, and in Section 4 we investigate the special cases with two and three permutations.

## 2 Preliminaries

Given a set of alternatives  $U$ , a *ranking*  $\pi$  with respect to  $U$  is an ordering of a subset  $S$  of  $U$  such that  $\pi = (x_1, x_2, \dots, x_r)$  with  $x_i > x_{i+1}$ , if  $x_i$  is ranked higher than  $x_{i+1}$  for some total order  $>$  on  $U$ .

For convenience, we assign unique integers to the items of  $U$  and let  $U = \{1, \dots, n\}$ . We call  $\pi$  a (*full*) *permutation*, if  $S = U$ , and a *partial permutation*, if  $S \subseteq U$ . A permutation is represented by an ordered list of items, where the rank of an item is given by its position in the ordered list, with the highest, most significant, or best item in first place.

The rank aggregation or the *crossings of permutations problem* is to combine several rankings  $\pi_1, \dots, \pi_k$  on  $U$ , in order to obtain a common ranking  $\pi^*$ , which can be regarded as the compromise between the rankings. The goal is the best possible common ranking, where the notion of ‘better’ depends on the objective. It is formally expressed as a cost measure or a penalty between the  $\pi_i$  and  $\pi^*$ ; the *common* version takes the sum of the penalties, the *max* version is introduced here. Several of these criteria have a correspondence in graph drawing.

A prominent and frequently studied criterion is the Kendall- $\tau$  distance [3, 5, 9, 16]. The *Kendall- $\tau$  distance* of two permutations over  $U = \{1, \dots, n\}$  measures the number of pairwise disagreements or inversions,  $K(\pi, \tau) = |\{(u, v) \mid \pi(u) < \pi(v) \text{ and } \tau(u) > \tau(v)\}|$ . This value is invariant under renaming, or the application of a permutation  $\sigma$  on both  $\pi$  and  $\tau$ , and such that  $\tau$  becomes the identity. For a set of permutations  $P = \{\pi_1, \dots, \pi_k\}$  this generalizes by collecting all disagreements,  $K(P, \pi^*) = \sum_{i=1}^k K(\pi_i, \pi^*)$ .

The value  $K(P, \pi^*)$  can be expressed in various ways. For every pair of distinct items  $(u, v)$ , the *agreement*  $A_P(u, v)$  is the number of permutations from  $P$  which rank  $u$  higher than  $v$ , and the *disagreement* is  $D_P(u, v) = k - A_P(u, v)$ . Clearly, the agreement on  $(u, v)$  equals the disagreement on the reverse ordering  $(v, u)$ . For every (unordered) pair of items, let  $\Delta(u, v) = |k - 2A_P(u, v)|$  express the difference between the agreement and the disagreement of  $u$  and  $v$ .

There is an established lower bound for the number of unavoidable crossings for the permutations of  $P$ , which is the sum over the least of the agreements and disagreements,

$$LB(P) = \sum_{u < v} \min\{A_P(u, v), D_P(u, v)\}.$$

Then the disagreement against a common permutation  $\pi^*$  is

$$K(P, \pi^*) = LB(P) + \sum_{\pi^*(u) < \pi^*(v) \text{ and } D_P(u, v) > A_P(u, v)} \Delta(u, v).$$

Thus  $\Delta(u, v)$  is added as a penalty if  $\pi^*$  disagrees with the majority of the permutations. If there is a tie for the ranking of  $u$  and  $v$  in  $P$ , then just the term from the lower bound is taken into account.

Recall that for the crossing minimization problem of two layered graphs the agreement and disagreement of two free vertices  $u$  and  $v$  is the crossing number of the edges incident with  $u$  and  $v$  and placing  $u$  left of  $v$ , or vice versa. The so obtained lower bound is often ‘good’ and close to the optimum value [14].

Another popular measure for the distance between permutations is the *Spearman footrule distance*, which accumulates the linear arrangement or the length between two permutations over  $\{1, \dots, n\}$  by  $f(\pi, \tau) = \sum_i |\pi(i) - \tau(i)|$ . Again this extends to a set  $P$  of permutations by summation  $f(P, \pi^*) = \sum_{j=1}^k f(\pi_j, \pi^*)$ .

These measures can be scaled by individual weights, and they can be extended to partial permutations  $\pi_1, \dots, \pi_k$ , where each permutation operates on its subset of the universe, see [9].

Given a set of (full or partial) permutations  $P = \{\pi_1, \dots, \pi_k\}$  on a universe  $U = \{1, \dots, n\}$ , the *crossing number* of  $P$  is the number of crossings against the best permutation  $\pi^*$  with respect to the Kendall- $\tau$ -distance, i.e.,  $CR(P) = \min_{\pi^*} K(P, \pi^*)$ . The crossing minimization problem is finding such a permutation  $\pi^*$ . We will refer to the crossing minimization problem of  $k$  permutations as the PCM- $k$  problem.

A new cost measure is the *max crossing number*, which attempts to minimize the number of crossings for any permutation. For a set of  $k$  permutations  $P$  and a target permutation  $\pi^*$  let  $K_{max}(P, \pi^*) = \max\{K(\pi_i, \pi^*) \mid \pi_i \in P\}$  and define the max crossing number of  $P$  by  $CR_{max}(P) = \min_{\pi^*} K_{max}(P, \pi^*)$ . The permutation  $\pi^*$  giving the value  $CR_{max}(P)$  is a solution to the max crossing minimization problem. This problem is referred to as the PCM<sub>max</sub>- $k$  problem. One could similarly consider a maximum version for the Spearman footrule distance; we have not investigated the latter further.

The following fact is readily seen.

**Lemma 1.** *For a set of  $k$  permutations  $P = \{\pi_1, \dots, \pi_k\}$ ,*

$$CR_{max}(P) \leq CR(P) \leq k \cdot CR_{max}(P).$$

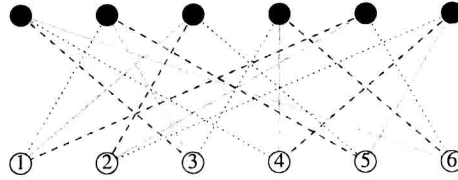
The crossing number represents an aggregation, which is the best compromise for the given lists of preferences and minimizes the number of disagreements. The minimal number of crossings does not necessarily distribute them uniformly among the given permutations; one can construct examples where  $CR_{max}(P) \geq \lceil CR(P)/2 \rceil$  and not  $CR_{max}(P) = \lceil CR(P)/k \rceil$  as one would hope. The latter equation holds for  $k = 2$ . The objective behind the max crossing number is an aggregation, which is fair and treats every permutation equally well and minimizes the discrimination of each participant. Clearly, both objectives can be combined to the best possible permutation  $\pi^*$  which minimizes the sum of crossings and then balances their distribution.

## 2.1 Drawing Permutations

We now translate rank aggregations to graph drawing. Two permutations  $\pi$  and  $\tau$  on a universe  $U = \{1, \dots, n\}$  are drawn as a two-layer bipartite graph with the vertices  $1, \dots, n$  on each layer in the order given by  $\pi$  and  $\tau$  and a straight-line edge between the two occurrences of each item  $v$  on the two layers.

A set of  $k$  permutations  $\pi_1, \dots, \pi_k$  and a common permutation  $\pi^*$  are represented by a sequence of pairs of permutations, where the lower layer is fixed in all drawings. For convenience, we let the lower layer be the identity with  $\pi^*(i) = i$ . We can merge the permutations into the *coloured permutation graph*  $G$ , which is a bipartite graph with  $k$  edge colours, such that there are vertices  $1, \dots, n$  on each layer. There is an edge in the  $i$ -th colour between  $u$  on the upper layer and  $j$  on the lower layer if and only if  $\pi_i(u) = j$ . See also Fig. 1.





**Fig. 1.** Coloured permutation graph for  $\pi_1 = (6, 3, 1, 4, 2, 5)$  (green and solid),  $\pi_2 = (3, 5, 2, 6, 1, 4)$  (blue and dashed), and  $\pi_3 = (4, 1, 5, 3, 6, 2)$  (red and dotted)

Obviously, for two full or partial permutations  $\pi$  and  $\tau$ , the Kendall- $\tau$  distance  $K(\pi, \pi^*)$  is the number of edge crossings in a straight-line drawing of their bipartite graph. It ranges between 0 and  $n(n-1)/2$  and can be efficiently computed either by accumulating for every  $i$  the number of items, which are greater than  $i$  and occur to the left of  $i$  in  $\pi$ , provided  $\pi^*$  is the identity, or by techniques from counting crossings in two-layer graphs in [21].

**Lemma 2.** *The Kendall- $\tau$  distance  $K(\pi, \pi^*)$  of two permutations over  $U = \{1, \dots, n\}$  can be computed in  $O(n \log n)$  time.*

## 2.2 Penalty Graphs

There is a direct relationship between the crossing minimization problem and the *feedback arc set problem*, which has been established at several places. Recall that the feedback arc set problem is finding the least number of arcs  $F$  in a directed graph  $G = (V, E)$ , such that every directed cycle contains at least one arc from  $F$ , i.e., the graph  $G' = (V, E - F)$  is acyclic. In the more general weighted case, the objective is a set of arcs with least weight. In the two-layer crossing minimization problem, the *penalty graph* has arcs with weights corresponding to the difference between the number of crossings among the edges incident with two vertices  $u$  and  $v$ , if  $u$  is placed left of  $v$ , or vice versa.

In their seminal paper, Sugiyama et al. [20] have introduced the penalty digraph for the two-layer crossing minimization problem, and in [2] it is used for voting tournaments. Demetrescu and Finocchi [6] have used this approach for the two-sided crossing minimization problem and have tested several heuristics. Recently, Ailon et al. [1] have established improved randomized approximations for aggregation and feedback arc set problems.

For the crossing minimization problem for permutations, the penalty graph can be applied in the same spirit, but we use the difference in the majority counts  $\Delta(u, v)$  as edge weights. Thus, for a set of permutations  $P$  over  $\{1, \dots, n\}$  the *penalty digraph* of  $P$  is a weighted directed graph  $H = (V, A, w)$  with a vertex for each item  $u$  and an arc  $(u, v)$  with weight  $\Delta(u, v)$  if and only if a strict majority of permutations rank  $u$  higher than  $v$ , i.e., if  $(u - v) \cdot (D_P(u, v) - A_P(u, v)) < 0$ . Let  $w(\text{FAS}(P))$  denote the *weight* of the optimum feedback arc set in the penalty digraph.