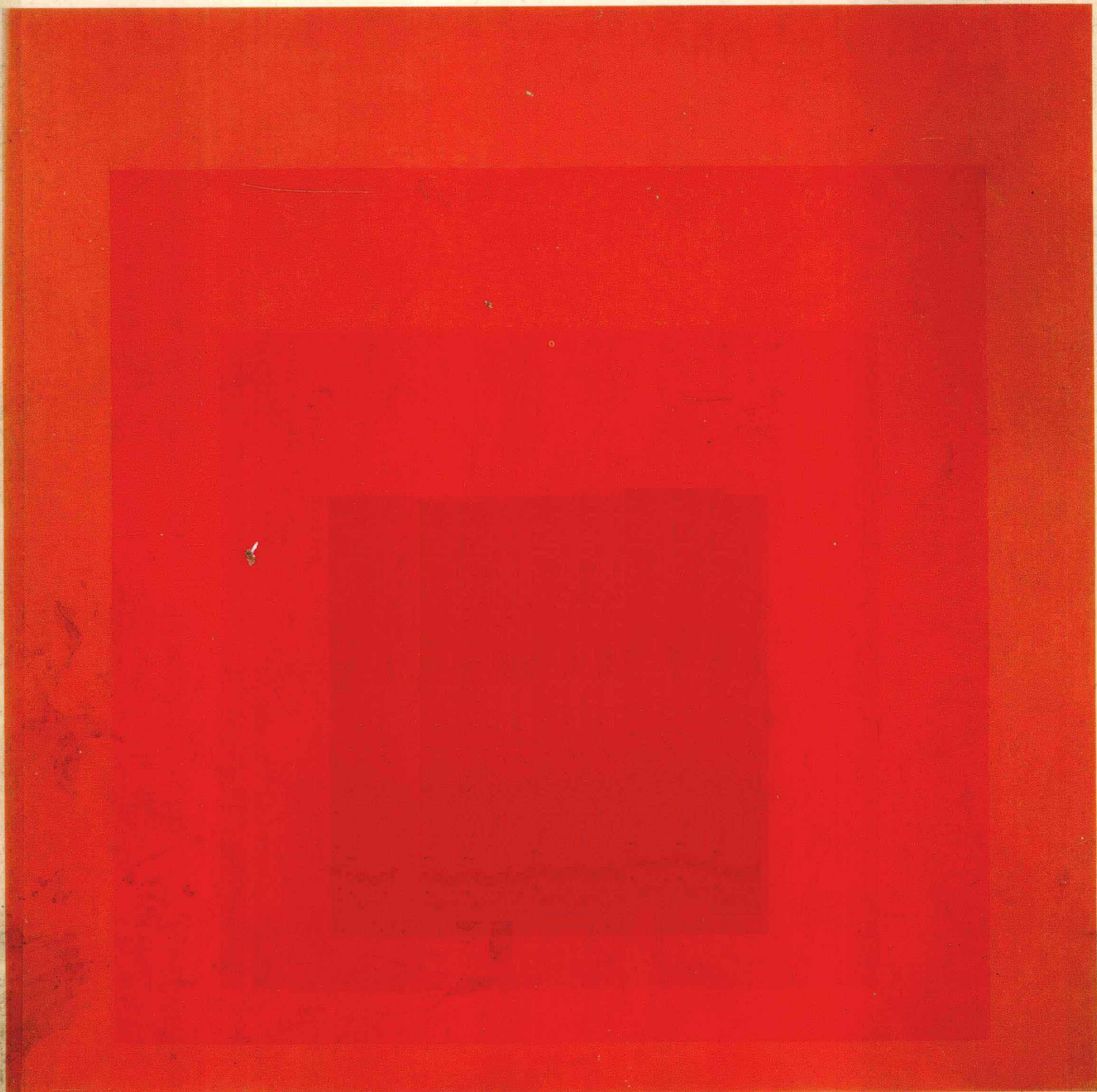


ESSENTIAL MATHEMATICS



MARY KAY BEAVERS

ESSENTIAL MATHEMATICS

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CITY COLLEGE OF SAN FRANCISCO
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1817

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To my father and mother

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PREFACE

TO THE STUDENT

Mathematics is a very exciting subject. If you do not feel this way, perhaps it is because you have had a few bad experiences. I hope that after you complete this book you will want to continue your studies in mathematics. By doing so, you will open up for yourself many new and interesting career options.

Many people view arithmetic as if it were only a bundle of computational skills. Although these computational skills are an important part of arithmetic, concepts, language, and applications are equally important. In this book I have attempted actively to teach you, the student, all four of these aspects. Your main objective should be at all times to understand how to solve the application problems. In so doing, you are putting together conceptual knowledge with computational skills, and in some cases, you are interpreting special language that is related to mathematics.

Mastering computational skills takes a lot of practice and drill. You can make the most of this time, and even enjoy it, if while practicing, you think about the concepts. While you are solving a problem, do not be satisfied with knowing what the next step is, but think about why you are taking that step and why it is permissible to take such a step. After you have arrived at an answer, ask yourself if the answer seems reasonable.

TO THE INSTRUCTOR

This text provides specific help with problems experienced by developmental students enrolling at college level who must learn basic arithmetic.

The main features of the text include:

Easy-to-read, easy-to-grasp examples and explanations, in an adaptable format.

An emphasis on applications, including special help with language.

Extra coverage of common trouble areas.

Built-in repetition and review.

An instructor's manual is available that includes a test bank. In the test bank are six forms of each chapter test and some cumulative review tests. There are also notes to the instructor on how the text fits various types of programs.

This material has been classroom tested at City College of San Francisco during the three years that it was being developed. Each year there were approximately 1600 students and 20 instructors involved.

Mary Kay Beavers

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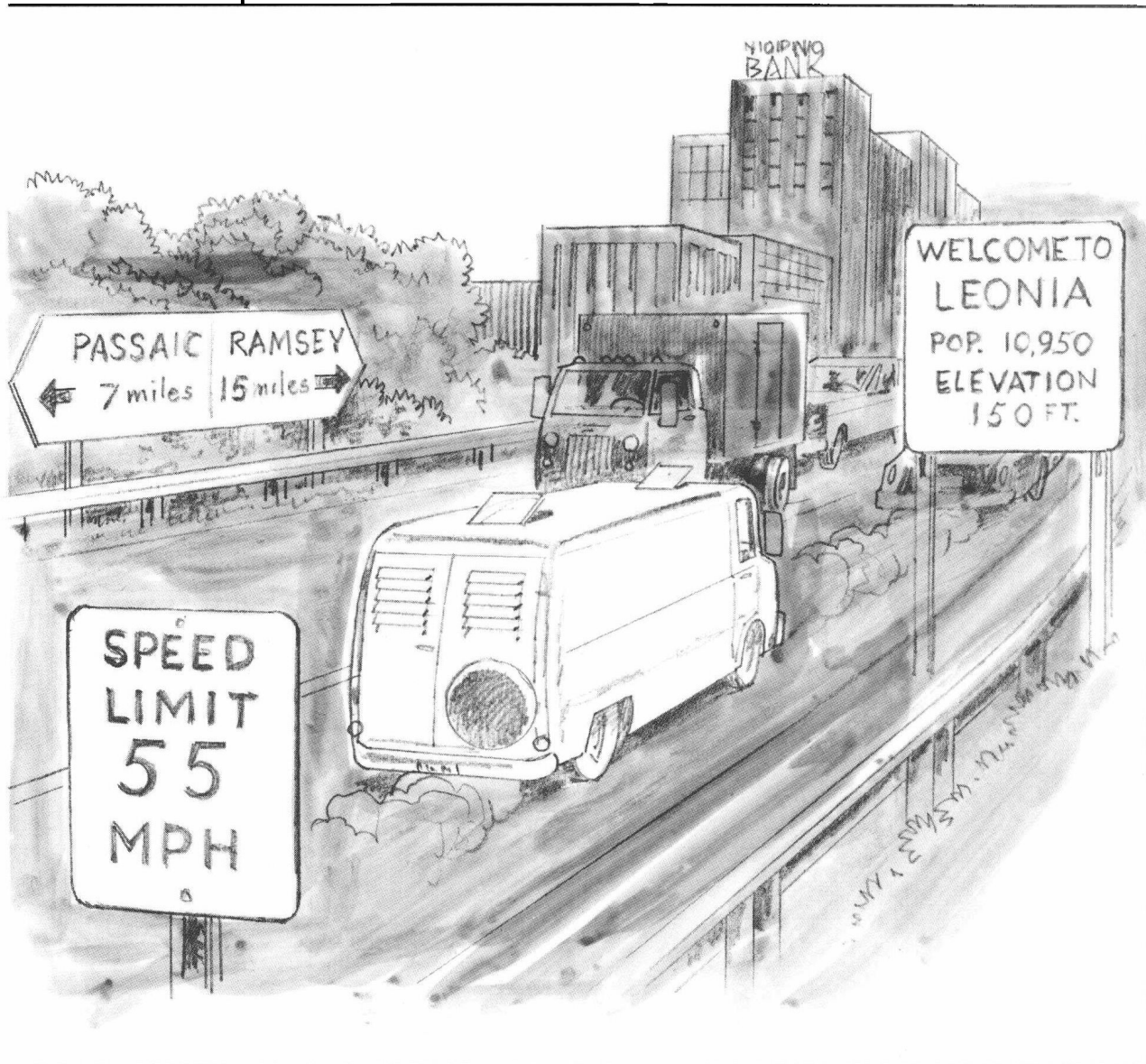
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**WHOLE NUMBERS:
PLACE VALUE,
ADDITION,
AND SUBTRACTION**



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1

1.1 COUNTING, PLACE VALUE, AND COMPARING

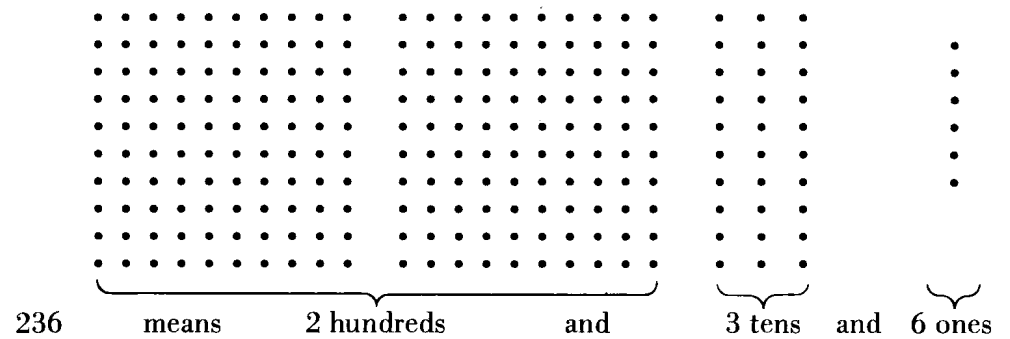
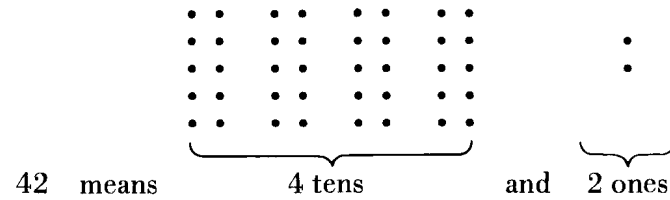
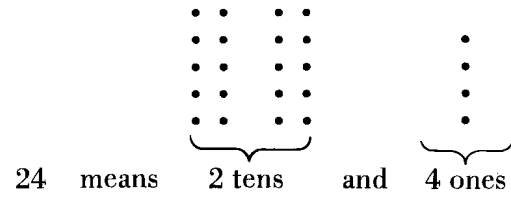
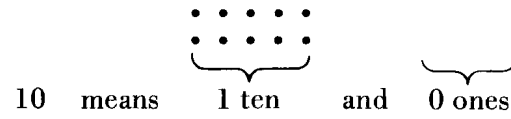
Betty has 26 dollars and Susan has 62 dollars. Which person has more money? To answer this question, you need to know which number is larger, 26 or 62. The question is easy to answer if you understand the place-value system for naming whole numbers.

Each whole number is represented by a symbol called a **whole numeral**. Each numeral consists of one or more of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each of these symbols is called a **digit**.

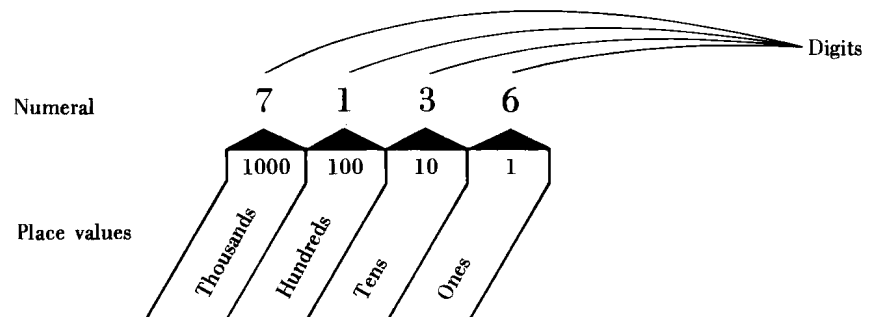
The chart below illustrates the meaning of each digit in terms of "how many" and gives the English name for each.

HOW MANY	DIGIT	ENGLISH NAME
	0	zero
•	1	one
• •	2	two
• • •	3	three
• • • •	4	four
• • • • •	5	five
• • • • • •	6	six
• • • • • • •	7	seven
• • • • • • • •	8	eight
• • • • • • • • •	9	nine

To represent whole numbers larger than 9, we use a place-value system called the **decimal system**. Here are some examples.



Each digit in a whole numeral has a place value.



The place-value system is based on the number ten. Here we show how the place values are related.

one	1
ten	$10 = 10 \text{ ones}$
one hundred	$100 = 10 \text{ tens}$
one thousand	$1\ 000 = 10 \text{ hundreds}$
ten thousand	$10\ 000 = 10 \text{ thousands}$
one hundred thousand	$100\ 000 = 10 \text{ ten thousands}$
one million	$1\ 000\ 000 = 10 \text{ hundred thousands}$

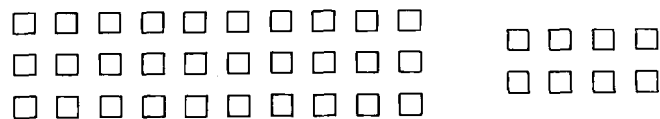
As we move from right to left in a whole numeral, each place value is 10 of the place values immediately before it.

**TRY
THESE
PROBLEMS**



1. How many dots are there? ● ● ● ● ● ●

2. How many squares are there?



3. Count from 78 to 85 by ones.

4. Count from 996 to 1014 by twos.

5. In the numeral 72, give the digit in the tens place.

6. In the numeral 105, give the digit in the ones place.

7. In the numeral 2038, give the digit in the hundreds place.

8. In the numeral 5738, give the place value of the digit 3.

9. Which is larger, 74 or 47?

10. Which is smaller, 200 or 89?

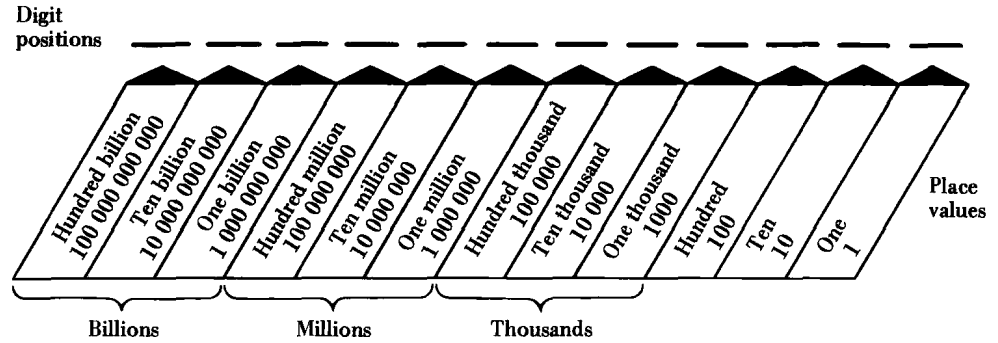
11. Betty earned 26 dollars and Susan earned 62 dollars. Which person earned more money?

12. Last year a small town in Wyoming had a population of 3001 persons. This year the population is 2998 persons. Did the population increase or decrease?

Answers

1. 6 2. 38 3. 78, 79, 80, 81, 82, 83, 84, 85
 4. 996, 998, 1000, 1002, 1004, 1006, 1008, 1010, 1012, 1014
 5. 7 6. 5 7. 0 8. 10 (ten)
 9. 74 10. 89 11. Susan 12. Decreased

The diagram below shows place values for large numbers.



Commas are sometimes placed in whole numerals with more than three digits. If used, the commas are placed every third digit starting from the right. The commas serve as an aid in reading the numeral. For example,

2,460 means 2460

1,360,000 means 1360000

49,505,721 means 49505721

**TRY
THESE
PROBLEMS**



Use the numeral 24107 to answer questions 1 through 5.

1. What digit is in the one thousands place?
2. What digit is in the tens place?
3. What digit is in the ten thousands place?
4. What is the place value of the digit 7?
5. What is the place value of the digit 1?

Use the numeral 37094185 to answer questions 6 through 10.

6. What digit is in the ten millions place?
7. What digit is in the one thousands place?
8. What is the place value of the digit 3?
9. What is the place value of the digit 9?
10. What is the place value of the digit 0?

Write each of these numbers using commas.

11. 5607 12. 24008 13. 102300

14. 7112930 15. 97775 16. 60000000000

17. Which is larger, 2007 or 2700?

18. Warren paid \$308,500 for a condominium in Pacific Heights. Elizabeth bought one in the Marina for \$311,000. Who paid more?

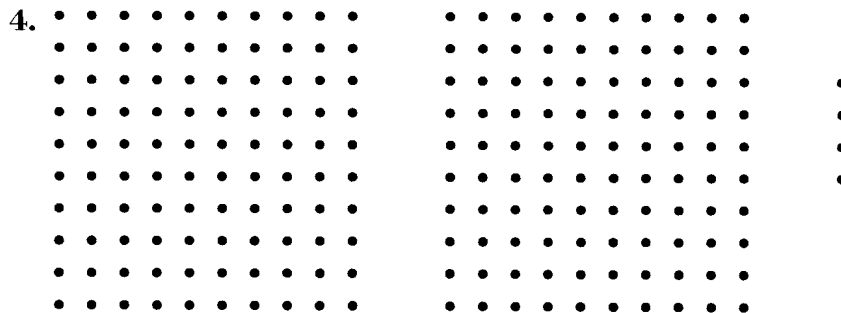
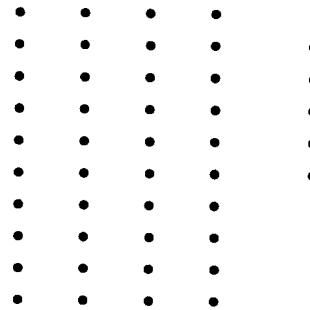
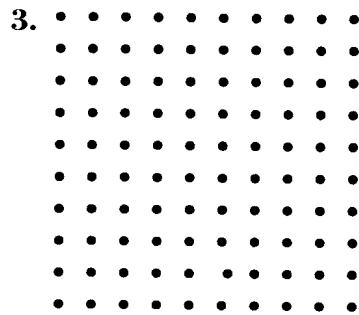
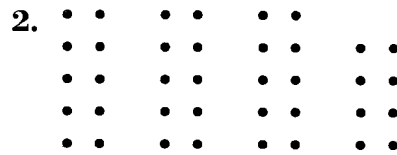
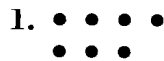
19. The population of Russia is about 240,000,000 and the population of the United States is about 205,000,000. Which country has fewer people?

Answers

- | | | |
|--------------------------|------------------------------------|--------------------|
| 1. 4 | 2. 0 | 3. 2 |
| 4. 1 (one) | 5. 100 (one hundred) | 6. 3 |
| 7. 4 | 8. 10,000,000 (ten million) | |
| 9. 10,000 (ten thousand) | 10. 100,000 (one hundred thousand) | |
| 11. 5,607 | 12. 24,008 | 13. 102,300 |
| 14. 7,112,930 | 15. 97,775 | 16. 60,000,000,000 |
| 17. 2700 | 18. Elizabeth | 19. United States |

EXERCISES 1.1

Write a whole numeral that represents the total number of dots shown.



5. Count from 348 to 360 by ones.
6. Count from 7997 to 8013 by twos.
7. Count from 24,880 to 24,920 by fives.

Use the numeral 78534 to answer questions 8 through 12.

8. What digit is in the hundreds place?
9. What digit is in the ones place?
10. What digit is in the ten thousands place?
11. What is the place value of the digit 3?

12. What is the place value of the digit 8?

Use the numeral 52,730,149,869 to answer questions 13 through 18.

13. What digit is in the one millions place?

14. What digit is in the ten billions place?

15. What digit is in the hundred thousands place?

16. What is the place value of the digit 7?

17. What is the place value of the digit 4?

18. What is the place value of the digit 2?

19. Which is larger, 7983 or 10,000?

20. Which is smaller, 5034 or 5304?

21. List from smallest to largest: 700; 698; 1000

22. Aron is 63 inches tall. Her sister Ann is 36 inches tall. Which one is taller?

23. Last week a head of lettuce was selling for 79 cents. This week a head of lettuce sells for 92 cents. Did the price increase or decrease?

24. Candy has \$500 in her checking account. She writes a check for \$465. Is there enough money in her account to cover the check?

25. A real estate company purchased an apartment building in Chicago for \$1,700,000. A similar building in Houston cost \$1,098,000. Which building was more expensive?

ANSWERS 1.1

- | | | | |
|---|-------------------------|---------------------------|--------|
| 1. 7 | 2. 38 | 3. 145 | 4. 204 |
| 5. 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360 | | | |
| 6. 7997, 7999, 8001, 8003, 8005, 8007, 8009, 8011, 8013 | | | |
| 7. 24,880; 24,885; 24,890; 24,895; 24,900; 24,905; 24,910; 24,915; 24,920 | | | |
| 8. 5 | 9. 4 | 10. 7 | |
| 11. 10 (ten) | 12. 1000 (one thousand) | | |
| 13. 0 | 14. 5 | 15. 1 | |
| 16. 100,000,000 (one hundred million) | | 17. 10,000 (ten thousand) | |
| 18. 1,000,000,000 (one billion) | | 19. 10,000 | |
| 20. 5034 | 21. 698, 700, 1000 | 22. Aron | |
| 23. Increased | 24. Yes | 25. Chicago building | |

1.2

READING AND WRITING NUMERALS

The chart below gives the English names for several whole numerals.

WHOLE NUMERAL	ENGLISH NAME
3	Three
8	Eight