

Statistics for technology

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A Course in Applied Statistics

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'It's perfectly intelligible,' the captain said, in an offended tone, 'to anyone that understands such things.'

Preface

This book provides an introduction to statistics, with particular emphasis on applications in the applied sciences and engineering. The book may be used as a text for a basic course in statistics or for self-tuition. Although the book was originally intended for 'service' courses to scientists and engineers, I have been pleased to find that the book has also been widely used for applied statistics courses for students in mathematics and statistics departments. Although 'technology' was an 'in' word when the book was written, it may be that a better title today would be 'A Course in Applied Statistics'.

The book is divided into three sections. Part One includes an introduction and some material on descriptive statistics. Part Two deals with the theory of probability and statistics, while Part Three considers some applications, including the design and analysis of experiments, quality control and life-testing. The reader who is specifically interested in one of these last two topics can proceed directly from Chapter 7 to Chapter 12 or 13 as appropriate.

The favourable reaction to the first and second editions has prompted me to make relatively few changes for the third edition. The references have been updated throughout, and Sections 2.1 and 6.3 have been rewritten. In the second edition Sections 12.11 and 13.4 were rewritten. I have also clarified and updated the text with many minor alterations, but without changing the page numbering up to page 346. This should be helpful to teachers who are used to earlier editions. It should also keep costs down and avoid introducing new typographical errors.

Appendix C, giving further references, has been completely rewritten again. A new Appendix D was introduced in the second edition describing some miscellaneous topics which were omitted from the first edition. In this edition I have added a new Section D.4 on stem-and-leaf plots and box plots. I have also added a completely new Appendix E in which I make some general comments on how to tackle statistical problems. I strongly recommend that the reader reads this Appendix

after taking a conventional introductory course. It includes remarks on topics such as processing the data, preliminary data analysis, computer programs, using a library, and writing a report. These vital topics are often omitted from conventional courses.

I have kept the mathematics throughout the book as simple as possible; an elementary knowledge of calculus plus the ability to manipulate algebraic formulae are all that is required. I have tried to introduce the theory of statistics in a comprehensible way without getting too involved in mathematical details. A few results are stated without proof where this is unlikely to affect the student's comprehension. However, I have tried to explain carefully the basic concepts of the subject, such as probability and sampling distributions; these the student must understand. The worst abuses of statistics arise in the 'cook-book' approach to the subject where scientists try to analyse their data by substituting measurements into statistical formulae which they do not understand.

Many readers will have access to a computer, or at least to a microcomputer or sophisticated pocket calculator. The statistical software which is readily available has taken much of the drudgery out of statistics. There are some remarks on computer programs in Appendix E.5. I have amended the text in a number of places to make it clear that the reader need no longer worry too much about some computational details, although there is much to be said for working through some of the methods in detail at least once in order to understand fully what is going on.

I am grateful to many people for constructive comments on earlier editions and I am always pleased to hear from any reader with new ideas.

I am indebted to *Biometrika* trustees for permission to publish extracts from *Biometrika Tables for Statisticians* (Appendix B, Table 6) and to Messrs Oliver and Boyd Ltd, Edinburgh for permission to publish extracts from *Statistical Tables for Biological, Agricultural and Medical Research* (Appendix B, Table 7).

The quotations are, of course, from the works of Charles Lutwidge Dodgson, better known as Lewis Carroll.

CHRISTOPHER CHATFIELD
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December, 1982

Part one

Introduction

'Surely,' said the governor, 'Her Radiancy would admit that ten is nearer to ten than nine is – and also nearer than eleven is.'

Chapter 1

Outline of statistics

Statistics is an increasingly important subject which is useful in many types of scientific investigation. It has become the science of collecting, analysing and interpreting data in the best possible way. Statistics is particularly useful in situations where there is *experimental uncertainty* and may be defined as 'the science of making decisions in the face of uncertainty'. We begin with some scientific examples in which experimental uncertainty is present.

Example 1

The thrust of a rocket engine was measured at ten-minute intervals while being run at the same operating conditions. The following thirty observations were recorded (in newtons $\times 10^5$):

999.1	1003.2	1002.1	999.2	989.7	1006.7
1012.3	996.4	1000.2	995.3	1008.7	993.4
998.1	997.9	1003.1	1002.6	1001.8	996.5
992.8	1006.5	1004.5	1000.3	1014.5	998.6
989.4	1002.9	999.3	994.7	1007.6	1000.9

The observations vary between 989.4 and 1014.5 with an average value of about 1000. There is no apparent reason for this variation which is of course small compared with the absolute magnitude of each observation; nor do the variations appear to be systematic in any way. Any variation in which there is no pattern or regularity is called *random variation*. In this case if the running conditions are kept uniform we can predict that the next observation will also be about a thousand together with a small random quantity which may be positive or negative.

Example 2

The numbers of cosmic particles striking an apparatus in forty consecutive periods of one minute were recorded as follows.

3	0	0	1	0	2	1	0	1	1
0	3	4	1	2	0	2	0	3	1
1	0	1	2	0	2	1	0	1	2
3	1	0	0	2	1	0	3	1	2

The observations vary between zero and four, with zero and one observed more frequently than two, three and four. Again there is experimental uncertainty since we cannot exactly predict what the next observation would be. However, we expect that it will also be between zero and four and that it is more likely to be a zero or a one than anything else. In Chapter 4 we will see that there is indeed a pattern in this data even though individual observations cannot be predicted.

Example 3

Twenty refrigerator motors were run to destruction under advanced stress conditions and the times to failure (in hours) were recorded as follows.

104.3	158.7	193.7	201.3	206.2
227.8	249.1	307.8	311.5	329.6
358.5	364.3	370.4	380.5	394.6
426.2	434.1	552.6	594.0	691.5

We cannot predict exactly how long an individual motor will last, but, if possible, we would like to predict the pattern of behaviour of a batch of motors. For example we might want to know the over-all proportion of motors which last longer than one week (168 hours). This problem will be discussed in Chapter 13.

When the scientist or engineer finishes his education and enters industry for the first time, he must be prepared to be faced frequently with situations which involve experimental uncertainty. The purpose of this book is to provide the scientist with methods for treating these uncertainties. These methods have proved to be very useful in both industry and research.