CHAOS AND INFORMATION PROCESSING

A Heuristic Outline

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John S. Nicolis
Department of Electrical Engineering
University of Patras

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**CHAOS AND INFORMATION PROCESSING –
A HEURISTIC OUTLINE**

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Preface

This monograph is an attempt to *model* how man *models* the external world, by using as a tool the paradigm of chaotic dissipative dynamics.

How does a biological processor (the human brain for example) impose a partition on the external world thereby splitting it into (hierarchical) sets of discrete (abstract) categories? Why do people display a systematic preference to "$1/f$-music"? What is the dynamics of memory and cognition? What is or might be the mechanism underlying symbolic language?

Until recently ($\sim$ 20 years ago) such "exotic" questions belonged to the domains of jurisdiction charted by epistemology, philosophy and (cognitive) psychology. No "hard" scientist would think of trying — even tentatively — to deal with such topics — much less of attempting to publish a paper on them — without the risk of making a fool of himself.

Recently though a new scientific paradigm has started a low profile widespread revolution (comparable nonetheless with the "bang" provoked by quantum mechanics 70 years earlier); it is the discipline — or rather the interdisciplinary area — of nonlinear dynamics and chaotic dynamics.

Chaotic dynamics has deeply influenced — and changed — the way people used to think about the "immutability" of the physical world — and especially determinism. Traditionally, determinism and predictability were synonyms — in both classical *and* quantum systems; in classical dynamical systems at least, displaying a sensitive dependence on parameters and (or) initial conditions, we know that this is no longer true: sensitive dependence on parameters may provoke instabilities and cascades of bifurcations, i.e. *multiplicity* of solutions or "scenaria"; Sensitive dependence on initial conditions on the other hand may cause "exponential" divergence of nearby trajectories — "scenaria", as a result of which no matter how close you try to reset the initial conditions of an experiment you may not get *repeatability*.

Multiplicity of evolving scenaria then and *non-repeatability* — both coming from deterministic nonlinear systems make us suspect today that the lack of random elements in our systems does not necessarily guarantee predictability. What has this to do with information processing? Information is an *a posteriori* measure of an *a priori* uncertainty, i.e. lack of predictability. Operationally speaking it has to do with the "compressibility" of a long-time series. It is therefore tempting to try to construct nonlinear deterministic "chaotic" models of processors which, e.g. reliably categorise external stimuli from imperfect or distorted inputs and subsequently generate *novelty* out of the ensuing *dynamical* memories; or to try to understand the Linguistic Dynamics at the "Syntactical", the "Semantic" and the "Pragmatic" levels. The treatment will be introductory and heuristic. This is what essentially this book is all about. There is something more: the tantalizing relationship between fractal structure (and fractal function) and chaos is used in order to explore the *raison d'être* of many biological tissues being structurally fractal. In many cases fractal structure (Hardware) gives rise, when properly triggered, to fractal function (Software), displaying the notorious $1/f$-spectrum. What is the functional advantage of a biological tissue "playing" $1/f$-"music"? Does the coexistence of low and high frequency "surplus", afforded by such a spectrum, allow the tissue to adapt with equal comfort to perturbations requiring large *and* small relaxation times? There are some encouraging answers as we are going to see, e.g. from clinical cardiology and electroencephalography.

Unlike my previous books [1], [2] this one is addressed to the "restless" postdoc who is planning an autonomous research on nonlinear dynamical systems as applied to cognitive sciences. The young researcher is supposed to be familiar with the A-B-C's of chaos and information theory.

Nevertheless — for good measure — a lot of such "prerequisite knowledge" is recast in this book by me in a more natural, less "dehydrated" what's-all-about way. In that sense the book can be used as a textbook in a two-semester first-year graduate course on "chaotic dynamics of dissipative systems".

My deep gratitude goes to my secretary Mrs. Balou Kotsopoulos who typed the whole manuscript.

My warm thanks are also due to Dr. K. K. Phua of World Scientific who invited me to write this monograph.

Last but not least a warning — addressed both to the perceptive reader *and* the critic — must be inserted from the very outset; The "style" of the book — at least mathematically speaking — is neither "formal" nor "rigorous": The sole aim is to convey to the young postdoc the excitement of a *new* approach in cognitive sciences — in a more or less liberal and narrative manner.

Contents

Introduction

What this book is about — a potpourri of suggestions

The relation between symbolic language (be it the genetic code, natural language or computer language) and the dynamics of "thinking", e.g. information processing has always been — and to a great extent still remains — enigmatic. Historically, the first scientist who postulated the existence of a bilateral relationship (a functional feedback loop) between language and information processing was the linguist, Lee Benjamin Whorf. In the early fifties Whorf essentially opined that a poor linguistic repertoire drastically restricts the number of categories into which the beholder of such a language splits the external world. For several years this conjecture remained a linguistic curiosity; some structural anthropologists though (working with aborigines) took it seriously and they found that in a number of cases the "Whorfian hypothesis" suggesting essentially a relativity "of categories" — apparently holds true. For example, there is in Liberia an aborigine dialect "the Bassa", in which the seven conventional discrete categories into which most Indo-European languages divide the visible part of the continuous Electromagnetic Spectrum are replaced by just two categories. Those people cannot recognise and *name* but two "colours" although physiologically speaking they do not suffer from colour blindness [3], [4].

Such examples are somewhat encouraging in the following sense: They suggest that a "dynamical" modeling of the language

— about which experimental material from cognitive psychology abounds, can reveal — even in the worst case — the "tip" of the cerebral "iceberg" — that is, the software of biological information processing. We can relegate the dynamics of symbolic language to three different hierarchical[a] levels: the "Syntactical", the "Semantic" and the "Pragmatic".

Let us define these three levels one by one:

a) *The syntactical level*

At this level, given an "alphabet" (e.g., an arbitrary *Markovian* partition on a dissipative flow or map) we are preoccupied with the study of the emerging "syntactical rules" which determine the stochastic sequence of letters or words (once an arbitrary member of the chosen Markovian partition stands for the pause, "a word" is a sequence of symbols between two successive pauses). At the syntactical level, as we are going to see in detail later, two main characteristic macroparameters of the language may be deduced:

1) *The cumulative distribution* $p(x)$ where x is the ranking order of a given word in a long text and $p(x)$ is the *a priori* probability of its appearance and 2) *the transinformation* $I(t)$ which denotes how much information is transferred between two symbols t places apart (or, how much information a given symbol B_i carries to another symbol B_j to appear t places into the future). $I(t)$ stands roughly for the "degree of coherence" of the syntax involved and in a "good" language it should *not* decay with t either too slowly or too quickly. At the syntactical level the language carries *no meaning* whatsoever since no correspondence between "words" and objects of the external world is sought or established.

b) *The semantic level*

At the semantic level we study the correspondence between symbols (words, categories) and stimuli of the external world impinging on the processor's sensors. At that level therefore *meaning*

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[a]In general, a given hierarchical level is considered "higher" or more abstract than another one if the dynamical ongoings at the first are "collective properties" of the deliberations taking place in the second.

— if any — is established. The process of categorization of environmental stimuli onto a set of coexisting attractors can be considered as a decision-making process — in most interesting cases under conditions of uncertainty and conflict. The uncertainty comes as a result of the topology of the basin boundaries — separating individual basins of attraction (subsets of stimuli to be categorised on a given attractor) and the conflict or the ambiguity of classification may come as a result of the "saddle" (semi-stable) character of the attractors involved. In general we call "decision-making" the procedure of relegating a given (cognitive) attractor to a certain subset of environmental stimuli. Traditionally the ongoings at the semantic level have been the subject of cognitive psychology for approximately the last 100 years.

c) *The pragmatic level*

At the pragmatic level we study the correspondence between symbols and the "cognitive patterns" of two conversing partners.[b] The subject is the traditional playground of psychology. A technical person however has discovered a phenomenology which allows him (or her) to bypass the enormous ambiguity associated with psychological thinking: the theory of Games has been invented and developed exactly for this purpose. So at the pragmatic level we study the dynamics of a number of evolving conflicts and establish criteria of convergence that is of asymptotically stable solutions.

In game theory — which involves strategic and *not* tactical games — there is no *a priori* algorithm according to which the contestants exchange their "blows": the algorithms emerge en route via learning. The process starts with a common *motive* which means minimizing a certain cost function. This is essentially an exercise in the calculus of variations and immediately leads to formulating a boundary value problem. From such a formalism a discrete spectrum of eigenfunctions emerge: These eigenfunctions constitute the *attractors* towards which subsets of initial conditions asymptotically

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[b] For example, two parts of the processor.

converge. Once an attractor is reached a "solution" exists and the game comes to an end.

A question which immediately arises is what type of hierarchy holds amongst the *above three* "linguistic" levels (a), (b) and (c). One should be inclined to answer that the syntactical level — being the more abstract of the three — is the higher in hierarchy. However the words and symbols which constitute the variables at that level are *already* prerequisites for the dynamical ongoings on levels (b) and (c). One is then inclined to consider the "ladder" $a \rightarrow b \rightarrow c$ not so much as a "prism" but rather as a "bootstrap" hierarchy "biting its own tail" as it were.

This means that all levels (a), (b), and (c) are dynamically intercoupled via feedback-feedforward loops (Fig. 1). Loosely speaking one should consider a lack of coupling between levels (a) and (b) as the "dyslectic" syndrome and a lack of communication between levels (a) and (c) as the "autistic" syndrome.

We intend to couch the material to be treated in this book in terms of the dynamical deliberations taking place at all three "linguistic" levels $(a) \rightleftarrows (b) \rightleftarrows (c)$ — and possibly suggest something about the nature and dynamics of their mutual dyadic interconnections. Before that however we need to do a lot of "homework". Specifically before embarking on a synthetic task as above we have to understand as clearly as possible first what chaos *is*, how does it *emerge* out of seemingly "innocent", simple but nonlinear dynamics, how it is *characterised* and how it is *related* to computational and information processing problems. Secondly we have to learn the essentials on contemporary information theory and game theory. Thirdly we have to examine the characteristics of dissipative chaos in cases where the modelling of the physical system concerned leads to the coexistence of many (strange) attractors.

The concept of fractality and its relation to chaos comes next. Finally we have to ask ourselves under what circumstances fractal structures triggered by some impinging or self-generated stimulus may give rise to "fractal function" recognised by the ubiquitous

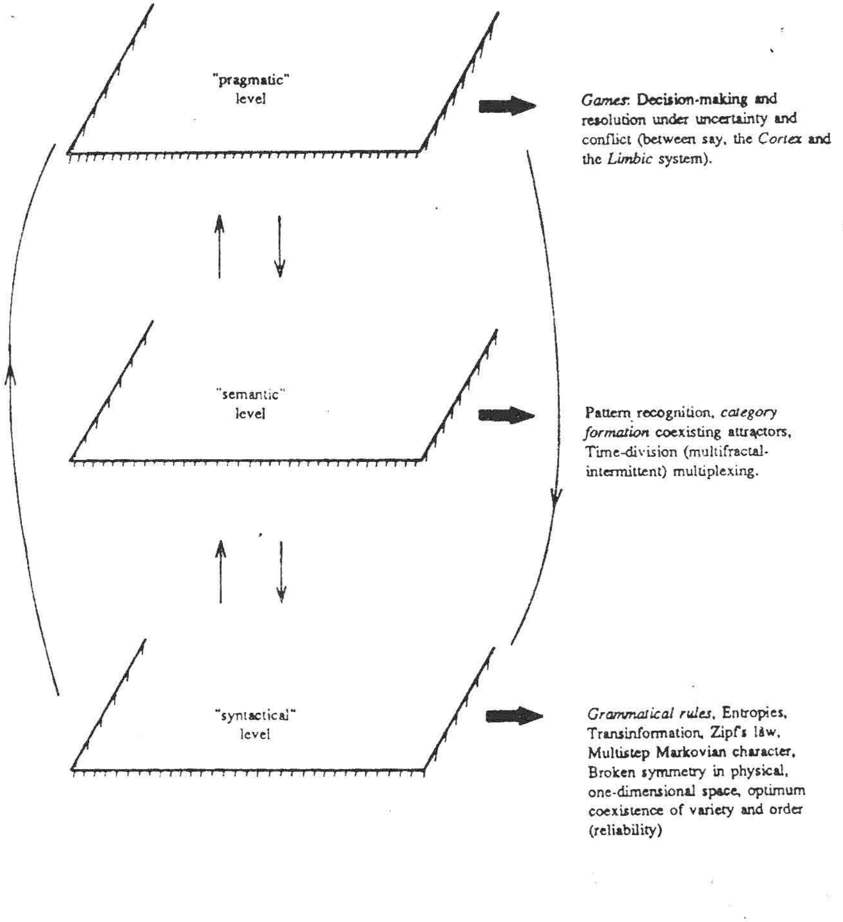

Fig. 1. A layout of the linguistic dynamical processes at *three* hierarchical levels: the syntactical, the semantic and the pragmatic. The exact way these levels are interconnected remains unknown.

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the development of the science of the *complex* had to wait until Man (quite recently) started worrying or rather doubting his abilities to make long-term predictions at the "mesoscopic level" — i.e. the "level of the Laboratory", ("no problem" for Cosmology and Quantum Physics — so far); that is to question his ability to construct algorithms and programs and arrive at conclusions (scenaria) about their possible outcomes long *before* the phenomena under simulation *actually* happen. If the processor cannot perform such a compression in time (Computational Complexity) or in space (Algorithmic Complexity) you find yourself well within the jurisdiction of chaos. What is chaos? In two words: chaos *is deterministic randomness.* It refers to "time series" or to "trajectories" which although they are the outcome of *deterministic* (nonlinear, low dimensional) *dynamics*, they cannot be computed by an algorithm even one bit *shorter* than themselves *and* they cannot be forecasted by any algorithm even one "time unit" *before* they actually get completed. In a nutshell: chaotic time series and trajectories are *incompressible* — both in space and time; the processor which "simulates" them literally disintegrates to the level of a "Xerox machine": The processor does not simulate a chaotic process; it merely mimics it.

How can such a state of affairs (randomness) come up from pure determinism? There is no real conflict actually. "Determinism" implies the *existence* and *uniqueness* of the "orbit" (digital or analog) under consideration, that is the solution of a system of coupled nonlinear differential or difference equations. But existence-uniqueness theorems and criteria seldom are *constructive*: The mere proof that something exists does not necessarily imply that this "something" can actually be found or computed by any algorithmic rule appreciably *simpler* or *shorter* than the real thing itself. We are going to provide at this point a) one characteristic example and b) one characteristic counterexample.

Before that however we have to ask ourselves the question: For what reason exactly impeccably deterministic algorithms (or if you like "laws of motion") may give when repeatedly iterated outcomes