Modern Physics

A TEXTBOOK FOR ENGINEERS

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PREFACE

The aim of this book is to present to engineers those parts of twentieth-century physics which are of greatest importance in engineering. The research of the last fifty years has not only seriously changed our understanding of physics but has also produced from this understanding devices and processes vital to present-day engineering. This book describes the modern physics of electrons, atoms, and nuclei and applies this basic physics to problems of engineering interest. The two most dramatic applications are to the transistor and to the nuclear power reactor, both developments of the last dozen years. There are many additional and equally important applications of modern physics in engineering, such as, for example, applications to the electrical, thermal, mechanical, and magnetic properties of solids and to the electrical and chemical properties of surfaces.

This book has developed from notes used in a one-semester course in atomic, solid-state, and nuclear physics for engineering undergraduates which has been taught at Cornell since 1950 and has been taught several times extramurally to engineers in nearby industries. The purpose of this course is to provide an analytical introduction to modern physics and its applications. Some of the students use the course as a prerequisite for advanced engineering courses; for others it is a "terminal course" in physics. In any case, it serves as a base on which to build an understanding of engineering devices. Training in the *devices* is frequently carried out by industrial employers, but training in the *physics* underlying them is an indispensable part of an engineering education. For example, a chemical or an electrical engineer working in the fluorescent-lamp industry may well be trained "on the job" in the engineering development, manufacture, and economics of fluorescent lamps. But he should have acquired as an undergraduate an understanding of thermionic emission, electronic processes in gases, and luminescence in solids.

Many modern physics experiments have produced instruments and measurement methods of importance in engineering. The developments of such tools as the mass spectrometer, radiation and resistance thermometry, nuclear-particle detectors, and devices for X-ray inspection and crystallography have profoundly affected engineering. An attempt has been made in this book to apply the experiments of modern physics to engineering instrumentation wherever possible. For example, black-body radiation is discussed primarily as one of the fundamental experiments upon which quantum physics is based. But after the role of this experiment in modern physics is explained, the application of the understanding of black-body radiation to pyrometric instruments is discussed.

The order in which modern physics is discussed here may require some explanation. The first chapter presents descriptions of the particles present in atoms and nuclei; these particles are the "ingredients" of the subject. Chapter 2 develops the concept of a distribution function, which is needed in several places later in the book. It also produces evidence on the size of atoms and shows that any system (such as an atom or solid) encountered will always be in nearly the lowest possible energy state for the system. Chapter 3 presents the demonstration that atomic physics can be split into two parts: the study of the extranuclear structure of the atom and the study of nuclear physics. The separate treatments of the two subjects are possible because of the small size and large binding energy of the nucleus relative to the scales of sizes and energies involved in experiments on the extranuclear structure of the atom.

The heart of modern physics is quantum mechanics, and the central part of this book is devoted to treating it. Chapter 4 presents the experiments which show the *necessity* for quantum theory and which produce much interesting information about atoms. Chapter 5 describes quantum mechanics and some simple, but artificial, examples which illustrate the *method* of using it. The *application* of quantum mechanics begins in Chapter 6 and continues throughout the rest of the book.

Attention could be turned again to nuclear physics after Chapter 6, but the development of molecular and solid-state physics follows more naturally (Chapters 7 to 11). Chapter 12 on physical electronics is also interposed, and finally the subject of nuclear physics is taken up again at the point where it was left at the end of Chapter 3. The nuclear physics discussion in Chapter 13 can thus use not only quantum mechanics but also a few topics in Chapters 10 and 12 which help to explain the instrumentation of nuclear experiments. The teacher who objects to the dispersal of nuclear physics may decide to interpose Chapter 13 between Chapters 6 and 7.

A detailed picture of the structure of this book can be obtained by reading the "Introduction" sections of each chapter. The student would be well advised to read all these sections before embarking upon

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I

FUNDAMENTAL PARTICLES

1-1 Introduction

This book presents the basic physics underlying a large and growing part of engineering. This basic physics has been developed during the last few decades, and the applications to engineering, although already considerable, are expanding rapidly. The study of modern physics leads to new devices and energy sources, to more convenient and accurate instruments, to the development of new materials of construction, and to a clearer understanding of the existing materials.

This book is primarily concerned with physical laws and processes, but engineering applications will be described frequently. Television camera tubes, transistors, nuclear reactors, and other devices will be analyzed as part of the application of the physical processes. But most of the applications of the modern physics which is presented here will be found in other engineering courses and in engineering practice. Many of the applications of modern physics are to instrumentation; as the trend toward "automation" continues, the engineer becomes more and more an expert who devises and operates instruments and control systems.

The plan of this book is as follows: We shall first present the properties of elementary particles, which are the "ingredients" of modern physics. The variation of mass with velocity and the famous Einstein $E = Mc^2$ relation (which is basic to the whole field of "atomic energy") will be presented in conjunction with these properties in the present chapter. In later chapters we shall study the interactions of these particles, analyze experiments which show that new physical laws govern the behavior of particles in atoms and nuclei, and state and illustrate these laws. At the end of Chapter 6 we shall be in a position to apply the new understanding of "quantum" physics to molecules, solids, and nuclei. As we apply this understanding in

Chapters 7 to 13 we shall find that the quantum physics provides explanations in many problems of engineering interest, such as the properties of solids, the characteristics of electron tubes, and the operation of transistors. Furthermore, we shall find that the applications of quantum physics in many areas, such as metallurgy and nuclear energy, are developing rapidly, and much rewarding work remains to be done by engineers in these fields.

1-2 The Electron

(a) Source. The usual source of electrons in the laboratory is a hot filament in a vacuum tube. The physical process of emission of electrons from this source will be discussed in Chapter 12. Such emission is called "thermionic emission" and produces electrons with small initial kinetic energies.

Another source of electrons is the emission from radioactive nuclei; this process will also be discussed later. Such electrons are emitted with a wide range of energies extending up to very high energies. Since this source was discovered before the emitted particles were identified, they were not called electrons but were called "β rays" and later "β^- particles." The name persists, and so high-energy electrons are frequently called "β^- particles."

(b) Size. No experiments capable of measuring the size or shape of the electron have been performed. An *upper limit* to the size of the electron can be obtained, however, from experiments in which electrons at very high energies are used as projectiles and nuclei are used as targets. The *maximum* size that an electron could have and be consistent with these experiments is about 10^{-14} m.* (distance across the electron, or diameter if it were a sphere). This distance is so small compared to the other distances we shall be concerned with that we can consider the electron as a *mass point* with zero extension in space. Such a mass point is called a "particle" in mechanics.

(c) Charge. The charge of the electron is negative, and its magnitude is

$$e = 1.602 \times 10^{-19} \text{ coulomb}$$

Throughout this book we use the symbol e for the absolute magnitude of the electronic charge. The charge of the electron is therefore $-e$.

The electronic charge can be measured by the Millikan "oil-drop experiment." The apparatus used is shown schematically in Fig. 1-1. A pair of horizontal, parallel condenser plates is mounted inside an

* "m." will be used as an abbreviation for meter. The symbol m will be used later for the mass of the electron.

enclosure. The enclosure prevents drafts and permits varying the pressure. Except in very precise work (where the pressure must be varied in order to determine small corrections), the chamber is filled with ordinary air at atmospheric pressure. An atomizer permits spraying fine drops of a non-evaporating oil into the space between the plates. A telescope with horizontal hairs ("fiducial lines") permits the observation of a single drop and the measurement of the vertical velocity of a drop. The velocity is determined by measuring the

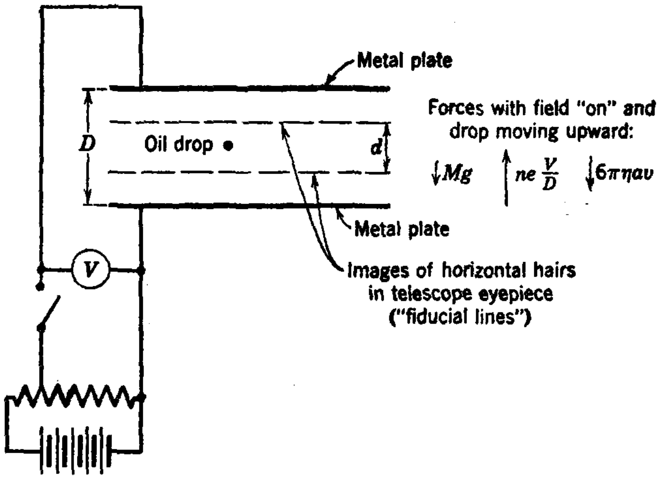

Fig. 1-1. Geometry of the oil-drop experiment for determining e. The forces diagrammed are on the assumption that the charge on the drop is negative.

time required for the drop to rise or fall the fixed distance d between the images of the hairs. This distance is usually considerably less than D, the spacing between the plates.

A source of ionizing radiation which can be turned on or off is provided. This source can be an X-ray tube or an ultraviolet arc. The *process* of ionization will be considered in detail in later chapters; at this point, all we need to know is that it is possible to remove an electron from an atom by X-rays or by ultraviolet light. If this atom is a gas atom, a positive ion and an electron are provided, either one of which may be captured by the oil drop. If this atom is one of the atoms making up the oil drop, the oil drop will attain a positive charge. Thus, while the ionizing radiation is turned on, the oil drop can have its charge either increased or decreased but always by an *integral number of electronic charges.*

A falling drop of the size used in this experiment reaches its "terminal velocity" very quickly ($\ll$1 sec). This is the velocity such that the resistance of the air to the motion of the drop equals the negative ot the applied force. In other words, the "drag" caused by the viscosity of air is equal in magnitude and opposite in sign to the other forces (gravitational or gravitational plus electrical) acting on the drop. For spherical drops this viscous force has been found by ordinary hydrodynamics experiments to be

$$F = -6\pi\eta a v \tag{1-1}$$

where η is the viscosity of the medium, a is the radius of the sphere,

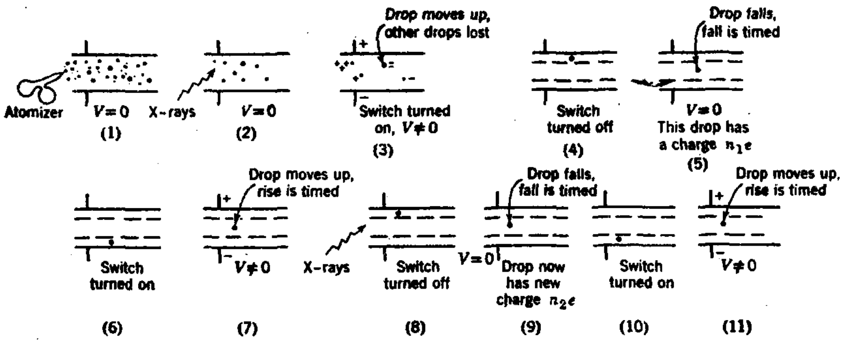

Fig. 1-2. Procedure of the oil-drop experiment. Steps 1–3 produce many drops and select a suitable drop. Steps 4–11 are repeated many times with the same drop.

and v is its terminal velocity. Equation 1-1 is called "Stokes' law" (it requires a correction of a few per cent for the very small drops used to measure e, but this correction can be found by varying the pressure of the air in the chamber). Drops of the size encountered in this experiment are exactly spherical because of the action of the surface-tension force.

The procedure of this experiment is illustrated in Fig. 1-2. A drop is selected, its time t_0 of fall through a distance d is measured with no electric field, and its terminal velocity is then determined from the relation $v_0 = -d/t_0$ (velocities upward are considered positive). Since the drop is not accelerating, the sum of the forces on it must be zero. Therefore the sum of the gravitational force $-Mg$ (downward) and viscous force F (upward) must be zero, and

$$Mg = -6\pi\eta a v_0 = 6\pi\eta a(d/t_0) \tag{1-2}$$

where g is the acceleration of gravity (9.80 m./sec^2) and M is the mass of the drop. Since the drop is spherical,

$$M = \tfrac{4}{3}\pi a^3\rho \tag{1-3}$$

where ρ is the density of the oil. Equations 1-2 and 1-3 are two equations with two "unknowns" (M and a), and so M and a can be determined from them. Because the drop is so small, neither M nor a can be measured directly.

Next a short burst of X-rays produces some charge on the drop, the electric field V/D is applied in the correct sense (usually upper plate positive) to move the drop upward, and the time t_1 of rise is measured. The velocity of rise is d/t_1, and, if there are n_1 electronic charges on the drop,

$$n_1e(V/D) - Mg = 6\pi\eta a(d/t_1) \tag{1-4}$$

The drop is again allowed to fall without an electric field, another burst of ionization changes the charge to n_2e, and a new time t_2 of rise is observed:

$$n_2e(V/D) - Mg = 6\pi\eta a(d/t_2) \tag{1-5}$$

The subtraction of eq. 1-4 from eq. 1-5 yields:

$$(n_2 - n_1)e = \frac{6\pi\eta aDd}{V}\left(\frac{1}{t_2} - \frac{1}{t_1}\right)$$

This procedure is repeated over and over again. By making the bursts of ionization short enough, the differences $(n_2 - n_1)$, $(n_3 - n_2)$, etc., can be kept small. These differences are therefore small integers (like 3, -2, 4, 1, $\cdots$). A table of values $(n_{i+1} - n_i)e$ can be prepared, and the integers and e can be determined from this table. Hundreds of rise times have been measured for a single drop, and never has a change in charge smaller than 1.6×10^{-19} coulomb been observed. This fact constitutes evidence that the fundamental quantity of charge is the charge of the electron, and that all electrical processes (e.g., ionization) involve the transfer of an integral number of electronic charges.

It should be noted that the Millikan oil-drop experiment permits the determination of a quantity of atomic size (the charge of the electron) by measurements of quantities of ordinary laboratory size (length, time, potential difference). This result is accomplished by the use of an oil drop which is large enough to be seen and to move slowly, yet which is small enough so that its motion is appreciably affected by a change in charge of only 1.6×10^{-19} coulomb. It should also be noted that e can be measured indirectly by combining the results of other experiments.

(d) e/m. The ratio of the electron's charge to its mass m is

$$e/m = 1.759 \times 10^{11} \text{ coulombs/kg}$$

and therefore the mass m is

$$m = 9.11 \times 10^{-31} \text{ kg}$$

We shall see in Sec. 1-6 that at velocities near the velocity c of light the mass of a particle depends on its velocity. The values of e/m and of m given above are the values for velocities very much less than c and are, strictly speaking, the values e/m_0 and m_0 appropriate to zero velocity. Since in almost all of this book we shall be dealing with velocities very much less than c, we shall usually omit the subscript. Where both m and m_0 appear in an equation, m will be the actual mass and m_0 will be the mass of the electron at rest.

The e/m of the electron is much larger than the similar ratio for any other particle or aggregate of particles. This fact and the relative ease with which electrons can be obtained from solids are responsible for the great usefulness of electrons in vacuum-tube devices. Particles with smaller charge-to-mass ratios are more sluggish in electric and magnetic fields. If such particles were used in electron-tube devices, the tubes could be used only at low frequencies and the space-charge-limited currents would be much smaller.

The e/m of the electron can be measured by the deflection of an electron beam in electric and magnetic fields or by the use of an electric field and a measurement of a time of flight. There is a wide variety of possible ways of making such measurements. We shall illustrate the principles of possible measurements by an example.

The vacuum tube illustrated in Fig. 1-3 is constructed with nonferromagnetic materials. It is placed in a uniform magnetic field with the magnitude of the magnetic induction equal to $\mathcal{B}$. A thin, flat beam of electrons is emitted from the hot cathode and enters the deflection region at the center of the tube. The force on a charge $-e$ moving with velocity $\mathbf{v}$ (a vector) is *

$$\mathbf{F} = -e\mathbf{v} \times \boldsymbol{\mathcal{B}} \qquad (1\text{-}6)$$

The direction of this force is at right angles to both $\mathbf{v}$ and $\boldsymbol{\mathcal{B}}$, and its sense is illustrated in Fig. 1-3. Since $\mathbf{v}$ is perpendicular to $\boldsymbol{\mathcal{B}}$ in this experiment, the magnitude F of the force is

$$F = |\mathbf{F}| = ev\mathcal{B}$$

* The vector "cross product" $\mathbf{v} \times \boldsymbol{\mathcal{B}}$ is defined as follows: Its *magnitude* is $v\mathcal{B} \sin \theta$, where v and $\mathcal{B}$ are the magnitudes of $\mathbf{v}$ and $\boldsymbol{\mathcal{B}}$, respectively, and where θ is the smaller of the two angles between $\mathbf{v}$ and $\boldsymbol{\mathcal{B}}$. Its *direction* is at right angles to the plane of $\mathbf{v}$ and $\boldsymbol{\mathcal{B}}$ and pointed in the direction a right-hand screw would travel if turned from $\mathbf{v}$ to $\boldsymbol{\mathcal{B}}$ through the angle θ.

where v is $|\mathbf{v}|$ (the "speed") and $\mathcal{B}$ is $|\mathcal{B}|$. This force produces a deflection toward the lower plate; hence the electron beam does not pass through the "exit slit" at the right, and no current is measured in the meter I.

Next an electrostatic deflection is produced by introducing the potential difference V_d, and V_d is adjusted until the beam passes through the exit slit and a current I is observed. The electrostatic

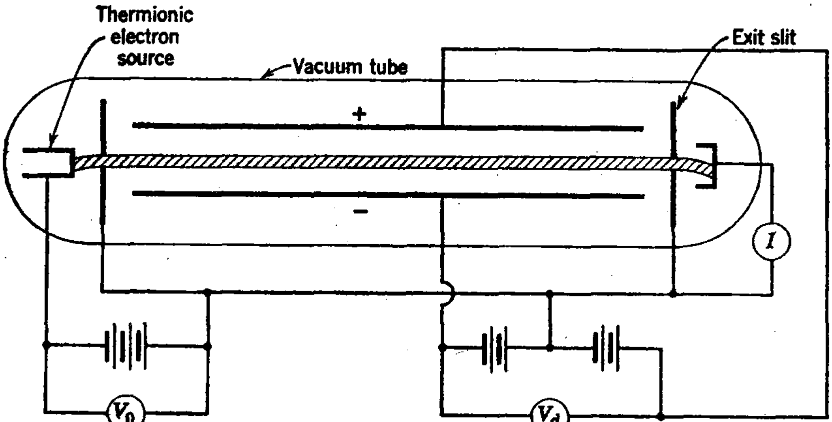

Fig. 1-3. Vacuum tube for measuring e/m. There is a magnetic induction $\mathcal{B}$ directed into the paper.

force eV_d/d must be just equal in magnitude (but opposite in direction) to the magnetic force:

$$eV_d/d = ev\mathcal{B} \tag{1-7}$$

This vacuum tube is therefore a "velocity selector," since only electrons with a velocity satisfying eq. 1-7 can traverse the tube.

The potential difference V_0 through which the electrons have been accelerated is also measured. The work done by this potential difference on one electron equals the increase in kinetic energy of the electron (which started with nearly zero kinetic energy at the cathode):

$$eV_0 = \tfrac{1}{2}mv^2 \tag{1-8}$$

The combination of eqs. 1-7 and 1-8 gives

$$\frac{e}{m} = \frac{v^2}{2V_0} = \frac{V_d^2}{2V_0 d^2 \mathcal{B}^2} \tag{1-9}$$

Therefore e/m can be determined from the observable quantities. This method is capable of high precision, but careful attention must be paid to edge effects of the deflecting plates and to precision of construction.

A rough measurement of e/m can be made with an ordinary oscilloscope cathode-ray tube and with the earth's field for $\mathcal{B}$. Equation 1-9 is replaced by a somewhat more complicated equation in this case, since the magnetic force is present throughout the tube but the electrostatic deflection force is present for only a short distance. The position of the bright spot on the fluorescent screen is used to indicate the condition of zero net deflection of the beam.

Measurements of e/m have been very useful in identifying electrons, since no particle has been discovered with an e/m at all close to the electron's. In the experiments to be discussed in the following chapters any doubt as to the nature of the particles participating can usually be removed by measuring their e/m.

(e) Other properties. The behavior of an electron is such that it must possess a definite angular momentum. It acts as if it were spinning about its center with this angular momentum. The principal experiments indicating this fact are the observations of the details of the line spectra of light emitted by atoms, which will be described briefly in Chapter 6. This property is usually referred to as the "spin angular momentum" (or just "spin") to distinguish it from any angular momentum the electron may have because of its motion from one point in space to another. (The angular momentum of a *rigid body* is represented as a vector which can be computed from the angular velocity vector and the moments and products of inertia; since the "size" and "shape" of an electron are not properties which can be found by experiment, it is useless to inquire about the angular velocity and moments of inertia of an electron.)

The electron is also known to have a definite magnetic moment. This fact, too, is demonstrated chiefly by experiments on line spectra. The magnetic moment of a bar magnet is usually thought of as the product of pole strength and pole separation, but these quantities have limited physical significance even for the bar magnet. Like angular velocity and moment of inertia, these quantities have no significance for the electron.

These two properties (spin and magnetic moment) are not so obvious or so often mentioned as the charge and mass, because they do not play a dominant role in experiments like those already described. The forces between electrons (or between electrons and external magnetic fields) because of their magnetic moments are usually much less than the forces present because of the charge and velocity of the electron. Spin and magnetic moment are nevertheless important attributes of the electron, as we shall discover when considering electrons in atoms and in solids.

1-3 The Proton

(a) Source. Protons are usually obtained by ionizing hydrogen atoms. The hydrogen atom consists of a proton and an electron. If an electrical discharge is operated in a hydrogen atmosphere, electrons gain kinetic energy from the electric field. When an electron with sufficient energy collides with a hydrogen atom it can remove the electron from the atom, leaving the proton. A hydrogen discharge can therefore be used as a source of protons. A proton is also called the "hydrogen nucleus" or "hydrogen ion."

(b) Size. The proton is so small that it can be considered as a particle (that is, zero dimensions) in all the experiments we shall consider. An effective radius can be experimentally determined from experiments in which high energy protons collide with other protons. This radius is about 3×10^{-15} m.

(c) Charge. The charge of the proton is positive and exactly equal in magnitude to the electron's charge e. This statement follows from the fact that the hydrogen atom is electrically neutral. Presumably an oil-drop experiment could be performed using protons, but it would be more difficult to provide them than to provide electrons.

(d) Ratio of charge to mass. This ratio for the proton is

$$e/M = 9.58 \times 10^{7} \text{ coulombs/kg}$$

This leads to a mass M of the proton:

$$M \doteq 1.672 \times 10^{-27} \text{ kg}$$

which is 1836 times the electron mass. (The symbol M is not reserved exclusively for the proton but is used for any mass other than the electron mass.)

The methods described in Sec. 1-2(d) could be used to measure e/M of the proton, but they would not be very accurate. The difficulty is that practical sources provide protons with an appreciable spread of energies. Therefore eq. 1-8 does not hold accurately, since it assumes that the initial energy of the particle is practically zero. A more precise method is to utilize the principle of the cyclotron, which is explained in Sec. 1-8. The cyclotron itself could be used for this measurement, but it is more convenient and accurate to use a special tube. The measurement of e/M is reduced to the measurement of a magnetic field strength and of the frequency of a radio-frequency oscillator.