TIDAL COMPUTATIONS IN RIVERS AND COASTAL WATERS

J. J. DRONKERS

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BY

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PREFACE

The foundations of tidal theory were laid in the period that saw the rise of the classical natural sciences — the years between the work of Copernicus and the publication of Newton's Principia Mathematica in 1687. Tidal theory became increasingly important for navigational purposes in the course of the nineteenth century. As a result, the initially qualitative approaches gave way to more quantitative treatment following the work of Laplace, e.g. the introduction of harmonic analysis to describe and predict the tides.

The last decades have seen the start of a great number of major projects relating to dredging schemes, sewage works and the closing of river branches. In each case, a study of tidal effects either by computations or with models, is one of the basic criteria in the planning of these projects.

Because technical projects can only be carried out in waters of limited depth, the contents of this book are mainly concerned with tidal computations in coastal waters and rivers. Ocean tides are not discussed but the reader is referred to the recent publication, "The Sea", edited by M.N. Hill (nrs. 77, 78, Bibliography Part II) and the book, "Physical Oceanography" by A. Defant (nr. 16, Bibliography Part I).

It should be noted that the term "tidal computations" includes both harmonic analysis of the tides and computations of the propagation of tides.

Tidal studies in the Netherlands received much stimulus from the investigations of the Zuiderzee Committee under the presidence of the great physicist H.A. Lorentz (1918-1926) (nr. 39, Bibliography Part I). After that the most thorough investigation of long waves including tides in rivers and coastal waters has been in J.C. Schönfeld's thesis (1951) (nr. 55, Bibliography Part I). As an introduction to tidal hydraulics G.B. Pillsbury's book (nr. 49, Bibliography Part I), is recommended.

In Schönfeld's thesis, and in the publication "Tidal computations in shallow water" by J.J. Dronkers and J.C. Schönfeld (1955) (nr. 26, Bibliography Part III), a review was given of the methods of tidal computation up to 1955. Since that time a rapid development in the use of electronic computers has taken place. This has influenced the execution of tidal computations. The reader will find a discussion of this development in the book, although the theoretical basis is still in its early stages.

Tidal research has been a very important subject in Holland and the experience gained by the closing of the Zuiderzee and the Deltaworks forms a vital part of the theory and examples used throughout the book.

In the outline, on page 1 and 2, the contents are further discussed.

The author has tried to write a book that is of value to readers who are not already experts in the field. To this end, a part of the book can be considered as a textbook, dealing with the principles of hydrodynamics, hydraulics, harmonic analysis and propagation of the tides. Much of the rest is devoted to recent progress in the field, including the author's own contributions.

Although some basic knowledge of mathematics and physics is required, it was also the aim of the author to frame the book in such a way that it could be followed by readers who have had relatively little training in mathematics. However, the more advanced part requires some graduate training.

I wish to express my gratitude to Messrs. F. Gerritsen and H.M. Oudshoorn of the Rijkswaterstaat, who read parts of the book and made many valuable suggestions. My thanks are due also to Mr. G.H. Ghotankar, first research officer of the Central Water and Power Research Station at Poona (India) for his information concerning the physical aspects of the motion of the bore in the river Hooghly, and to Mr. H.J. Stroband for some practical computations.

Furthermore, I wish to thank my collaborators at the Rijkswaterstaat for their help during the preparation of the manuscript and the execution of computations, dealt with in this book. Special mention is due to Mr. H.J. van Dienst, who checked the formulae.

I am also very grateful to Mrs. Dra. G. Bieger, who kindly revised the English text and made valuable comments.

My appreciation is extended to those of the Delta Department of the Rijkswaterstaat who drafted the illustrations and Mr. F.J. van der Laan for his corrections and typing of the manuscript.

ERRATA

Page 197, 9th line, *read* : p. 173 *instead of* p. 193.

Page 265, 2nd line, *read* : Q_0 or a_0, H_p, and $Q_p (p = 1, ..., n)$.

Page 296, last line, *read* : note that in this figure the minimum is negative.

Page 382, 8th line, *read* : $h_2(t)$ *instead of* $h_2'(t)$.

Page 401, 15th line from the bottom, *read* : of the components of the wind, $\alpha_0 U_0$ and $\alpha_0 V_0$.

CONTENTS

PART ONE

THE GENERATION AND HARMONIC ANALYSIS OF THE TIDES

PART TWO

BASIC HYDRODYNAMICS AND HYDRAULICS

PART FOUR

PRACTICAL APPLICATIONS

OUTLINE OF THE BOOK

This book is devoted to those tidal computations in rivers and coastal waters which are of great use in the planning and construction of engineering works in tidal waters. The derivation of the formulae for these computations will be discussed from a physical as well as a mathematical point of view.

In Chapter I, the reader will find a historical review of the development of the theory of tides. In addition, the purposes of tidal computation will be considered in order to provide an understanding of the methods for the practical application to a theoretical problem. Further discussion includes the means developed in the course of time for dealing with tidal problems, i.e., tidal computations and tidal models.

The other chapters of the book are divided into four parts.

Part one deals with the theory of the generation of the tides in an ocean. Harmonic analysis is then discussed as a method of representing the observed tide mathematically and for forecasting the tides.

Part two refers to the basic hydrodynamic and hydraulic theories which provide the principles for the study of the propagation of the tides in coastal waters and rivers. Moreover, a comparison is made between the principles of short-wave and long-wave theory; tidal theory is contained in the latter. Because the tidal motion in a tidal river shades off gradually into a steady flow in the upper parts of the river, a short chapter is devoted to the theory of steady flow.

Part three deals with the various methods of tidal computation in rivers and estuaries proper; the harmonic method, the use of characteristics and numerical methods are considered.

Finally, in part four, the reader will find practical considerations and applications for tidal motion: measurements, schematization of a river system, practical results of tidal computations, also for closing gaps, storm surges, etc.

It must be emphasized that, because tidal waves belong to the extensive group of long waves, most mathematical methods dealt with in part three, may also be applied to long waves, e.g. high tides and flood waves.

In applied mathematics, theory is based on the physical properties of the phenomena and influenced by the practical purposes. Therefore, remarks will often be made which are based on practical experience.

The mathematical methods, which are the basis of the tidal computations, are introduced in the beginning of each chapter by means of the solution of the simplified equations. In nrs. 10, 11, 32, 65 of part two and 7, 8, 40, 58, 78 and 80 of part three of the Bibliography, textbooks are mentioned for basic mathematics and physics.

At the beginning of each part, the reader will find a short description and justification of the contents. Whenever useful, the contents of a chapter or section are also summarized at the beginning of that chapter or section.

A list of symbols and definitions which are used throughout the book, and an index of terms are added.

For a long time many symbols have already been in use for special terms in the theory of harmonic analysis and the propagation of tides. In this book this use has been followed. Consequently, it was not possible to maintain an unambiguous notation throughout the book. Therefore the symbols are separately mentioned for the various parts of the book. To prevent confusion the meaning of the various symbols is often repeated in the sections.

CHAPTER I

GENERAL CONSIDERATIONS

1. The phenomenon of the tides

Tides are produced by the attractions of the sun and moon on the waters of the ocean and are observable as a regular rise and fall of water within a range of several feet. They generally occur twice a day; the interval between one high water period and the next averaging twelve hours and twenty-five minutes.

Even the most elementary observations made at most sea shores show that the tides in the seas are more strongly affected by the moon than by the sun. The time of high and low water appears closely related to the moon's position, i.e. to its passage over the meridian which occurs, on the average, fifty minutes later on each succeeding day. However, between the moon's passage over the meridian and high water, there is a time lapse which differs from place to place and also differs for the same place depending on the propagation of the tide from the ocean into the coastal waters.

The height to which the water rises varies from day to day. Spring tide occurs during new moon when the sun and moon are on the same side of the earth, as well as full moon when they are diametrically opposite. At full moon, the tidal range — equalling the difference in height between high and low water — is often greatest. During the moon's first or third quarter, the range is smallest and we have neap tide. Besides these regular changes in the ranges, the tides at some places are subject to various irregularities. The ranges of two successive tides are usually not the same at any given place; this is called the diurnal inequality of the tide, and the extent of this effect differs from place to place. When the moon is in perigee (i.e., nearest the earth), the range of the tide is greater (by nearly 20 per cent) than when it is in apogee. The maximum range occurs during the new or full phases when the moon is in perigee.

In some seas, namely in parts of the Gulf of Mexico, Gulf of St Lawrence, the Indonesian Archipel, and the Chinese Sea, the tides are still more irregular: there the water levels rise and fall only once daily during a part of each month. In such diurnal tides, however, the tidal fluctuations are relatively small compared to the so-called semi-diurnal tides.

It is clear that the tidal phenomenon is a complicated one. No general theory can explain all of the irregularities in the tide and its propagation; the available theory, however, does enable us to establish a practical method for the determination of the tides at points along the coast and in estuaries, and numerical computations can be used to determine the coefficients which fix the tides at any given point in detail.

There are two other effects, one caused by periodic winds and the other by variations in atmospheric pressure, which are so closely associated with tides that all three must be considered together in any tidal study. In addition, the air is also affected by the attractions of the sun and moon, as seen in the regular rise and fall of the height of the barometer; this may be called the atmospheric meteorological tide (see Kertz [34])). The same attractions act on the solid earth, and since the earth is not rigid, this produces an alternating change in its shape which is called the tide of the solid earth (see Tomaschek [62])). In this book, the tides in seas and rivers relative to the solid earth will be considered, while the atmospheric tide will be left out of consideration because its influence on the tides of the seas is negligible.

For an introduction to the theory of tides, reference is made to Darwin [13]) and regarding more modern methods and theories, to the books of Proudman [51]), Defant [16]) and [16a]) and Hill [27a]). See also Doodson [23a]).

2. Historical review

Almost two thousand years ago, naturalists like Strabo and Pliny suspected that a relation existed between the celestial bodies and the motion of the sea. Kepler realized that the attracting forces of the moon and the sun were involved, but Newton was the first to discuss the origin of the tides from a mathematical point of view. In his master work, *Philosophiae Naturalis Principia Mathematica*, (London, 1687), he dealt with the problem of the tides by assuming the entire earth to be covered with water, and showed that the surface of the water would then become a spheroid as a result of the pull of the moon (or sun). The fact that the main axis of the spheroid would be directed toward the moon explained the twice-daily occurrence of ebb and flood. From the positions of the sun and moon, he could derive the phenomenon of the spring and neap tides. He also demonstrated the effect of the difference in the distance between the earth and the sun, respectively the moon.

Newton's development was qualitative and, therefore, unsuitable for practical application. In 1738, the French Academy of Science held a competition for the best tidal theory, and one of the prize-winners was Bernoulli[5]) who extended Newton's theory concerning the course of the tides at the equator. Both Newton and Bernoulli postulated a so-called equilibrium surface which the water mass on the earth assumes under the influence of the pull of a celestial body, and which changes continuously such that the main axis is constantly directed toward the celestial body. Neither considered the forces of inertia, and consequently, the motion of the particles themselves; in fact, Newton's hypothesis is incompatible with the actual motion of the particles, which is a subject for hydrodynamic theories.

Laplace attempted to improve Newton's work in this respect. Volume IV of his important work, *La Mécanique céleste*[37]), is devoted to the theoretical and practical study of the oscillations of the sea. The last part of Volume IV and almost all of Volume XIII deal with the tides at Brest, but the theory is not adequate for consideration of the many variations inherent in terrestrial conditions. Nevertheless, the results of this theory which describe the general principles of the correspondence between the periodic forces and the motion of the sea, are very useful for showing how tidal observations must be analysed for theoretical and practical purposes. These principles form the foundation of the harmonic method, which was developed by Thomson (Lord Kelvin) in about 1868 and later. While Laplace combined all periodic terms into one formula, Thomson based his method on the development of a sum of periodic terms, the so-called tidal constituents. Since the amplitude and the phase of each tidal constituent may be determined from observations, and since the period is known from the theory of harmonic analysis, the tides can be forecasted at any given place.

After Thomson, the theory of harmonic analysis, especially the methods for numerical computation of the harmonic constituents, was extended by Darwin and other investigators. A review of these investigations is given in Chapter III.

The course of the tide along a coast may be determined approximately