

**TIDAL
COMPUTATIONS
IN RIVERS AND
COASTAL WATERS**

J. J. DRONKERS

TIDAL COMPUTATIONS

IN RIVERS AND COASTAL WATERS

BY

J. J. DRONKERS

Netherlands Rijkswaterstaat

(Public Works and Waterways Department)

The Hague, The Netherlands



1964

NORTH-HOLLAND PUBLISHING COMPANY - AMSTERDAM

© 1964 NORTH-HOLLAND PUBLISHING COMPANY
*No part of this book may be reproduced in any form
by print, photoprint, microfilm or any other means
without written permission from the publisher*

PUBLISHERS:

NORTH-HOLLAND PUBLISHING CO. - AMSTERDAM

SOLE DISTRIBUTORS FOR U.S.A.:

INTERSCIENCE PUBLISHERS, a division of

JOHN WILEY & SONS, INC. - NEW YORK

PRINTED IN BELGIUM

PREFACE

The foundations of tidal theory were laid in the period that saw the rise of the classical natural sciences — the years between the work of Copernicus and the publication of Newton's *Principia Mathematica* in 1687. Tidal theory became increasingly important for navigational purposes in the course of the nineteenth century. As a result, the initially qualitative approaches gave way to more quantitative treatment following the work of Laplace, e.g. the introduction of harmonic analysis to describe and predict the tides.

The last decades have seen the start of a great number of major projects relating to dredging schemes, sewage works and the closing of river branches. In each case, a study of tidal effects either by computations or with models, is one of the basic criteria in the planning of these projects.

Because technical projects can only be carried out in waters of limited depth, the contents of this book are mainly concerned with tidal computations in coastal waters and rivers. Ocean tides are not discussed but the reader is referred to the recent publication, "The Sea", edited by M.N. Hill (nrs. 77, 78, Bibliography Part II) and the book, "Physical Oceanography" by A. Defant (nr. 16, Bibliography Part I).

It should be noted that the term "tidal computations" includes both harmonic analysis of the tides and computations of the propagation of tides.

Tidal studies in the Netherlands received much stimulus from the investigations of the Zuiderzee Committee under the presidency of the great physicist H.A. Lorentz (1918-1926) (nr. 39, Bibliography Part I). After that the most thorough investigation of long waves including tides in rivers and coastal waters has been in J.C. Schönfeld's thesis (1951) (nr. 55, Bibliography Part I). As an introduction to tidal hydraulics G.B. Pillsbury's book (nr. 49, Bibliography Part I), is recommended.

In Schönfeld's thesis, and in the publication "Tidal computations in shallow water" by J.J. Dronkers and J.C. Schönfeld (1955) (nr. 26, Bibliography Part III), a review was given of the methods of tidal computation up to 1955. Since that time a rapid development in the use of electronic computers has taken place. This has influenced the execution of tidal computations. The reader will find a discussion of this development in the book, although the theoretical basis is still in its early stages.

Tidal research has been a very important subject in Holland and the experience gained by the closing of the Zuiderzee and the Deltaworks forms a vital part of the theory and examples used throughout the book.

In the outline, on page 1 and 2, the contents are further discussed.

The author has tried to write a book that is of value to readers who are not already experts in the field. To this end, a part of the book can be considered as a textbook, dealing with the principles of hydrodynamics, hydraulics, harmonic analysis and propagation of the tides. Much of the rest is devoted to recent progress in the field, including the author's own contributions.

Although some basic knowledge of mathematics and physics is required, it was also the aim of the author to frame the book in such a way that it could be followed by readers who have had relatively little training in mathematics. However, the more advanced part requires some graduate training.

I wish to express my gratitude to Messrs. F. Gerritsen and H.M. Oudshoorn of the Rijkswaterstaat, who read parts of the book and made many valuable suggestions. My thanks are due also to Mr. G.H. Ghotankar, first research officer of the Central Water and Power Research Station at Poona (India) for his information concerning the physical aspects of the motion of the bore in the river Hooghly, and to Mr. H.J. Strobant for some practical computations.

Furthermore, I wish to thank my collaborators at the Rijkswaterstaat for their help during the preparation of the manuscript and the execution of computations, dealt with in this book. Special mention is due to Mr. H.J. van Dienst, who checked the formulae.

I am also very grateful to Mrs. Dra. G. Bieger, who kindly revised the English text and made valuable comments.

My appreciation is extended to those of the Delta Department of the Rijkswaterstaat who drafted the illustrations and Mr. F.J. van der Laan for his corrections and typing of the manuscript.

ERRATA

Page 197, 9th line, *read* : p. 173 *instead of* p. 193.

Page 265, 2nd line, *read* : Q_0 or a_0 , H_p , and $Q_p (p = 1, \dots, n)$.

Page 296, last line, *read* : note that in this figure the minimum is negative.

Page 382, 8th line, *read* : $h_2(t)$ *instead of* $h_2'(t)$.

Page 401, 15th line from the bottom, *read* : of the components of the wind, $\alpha_0 U_0$ and $\alpha_0 V_0$.

CONTENTS

PREFACE	v
CONTENTS	vii
OUTLINE OF THE BOOK	1
CHAPTER I. GENERAL CONSIDERATIONS	3
1. The phenomenon of the tides	3
2. Historical review	4
3. General considerations on the practical applications of tidal computations	8
4. Tidal computations for civil engineering projects	10
5. General remarks on the physical and mathematical aspects of tidal motion	11
6. The choice of the method of tidal computation	13
7. General considerations on investigations into the tides by means of tidal computations, tidal models, or electric analogy	16
7.1. Tidal computations compared with use of hydraulic models	16
7.2. Comparison with the use of an electric tidal analogue model	18
7.3. Tidal computation with an electronic computer	20
7.4. Some final remarks	22

PART ONE

THE GENERATION AND HARMONIC ANALYSIS OF THE TIDES

INTRODUCTION	27
CHAPTER II. THE DETERMINATION OF THE TIDAL CONSTITUENTS	29
1. The tide-generating forces	29
2. The development of the tidal potential	33
3. The vertical and horizontal components of the tidal forces	37
4. The equilibrium tide	38
5. Some conclusions from the equilibrium tide formula	42
5.1. The height of the lunar tide	42
5.2. The height of the solar tide	43
5.3. Spring tide and neap tide	43
5.4. The tides at mean perigee and mean apogee of the moon	44
6. Further development of the tidal potential	44
7. Discussion of the variation of the equilibrium lunar tidal constituents, with respect to the position on earth and the variations in the declination	47
7.1. The semi-diurnal lunar tidal coefficient	47
7.2. The diurnal lunar tidal coefficient	48
7.3. The long-period tide	48
7.4. The time of high water for the diurnal and semi-diurnal tides	48
7.5. The daily inequality	49
8. Some astronomical terms and formulae concerning the motion of the moon and the sun	50
9. Definition of time	56
10. The further development of the tide-generating potential	58
10.1. The development according to Darwin	59
10.2. The development by Doodson	63

11. The determination of the most important tidal constituents arising from the moon's potential V_2 according to Doodson's method	65
11.1. The harmonic semi-diurnal lunar constituents	66
11.2. Harmonic diurnal lunar constituents	69
11.3. Long-period lunar constituents	70
11.4. The nineteen year variations in the tidal constituents	71
12. The most important constituents which follow from the solar potential V_{s2}	72
12.1. Semi-diurnal solar constituents	73
12.2. Diurnal solar constituents	74
12.3. Long-period solar constituents	75
13. The classification of the harmonic constituents according to Doodson	75
14. Shallow water tides	77
14.1. The generation of the shallow water tides	77
14.2. Classification and relative importance of the shallow water tides	79
15. Types of tides	81
16. Some definitions	82
 CHAPTER III. THE HARMONIC ANALYSIS OF THE TIDE	 88
Introduction	88
1. Basic quantities used in the practical application of harmonic analysis	92
2. The period of observational data from which the harmonic constants can be determined	95
3. The determination of the mean sea level	97
3.1. Mean sea level	97
3.2. Datum planes	100
4. The determination of the harmonic constants by means of the least squares method	102
5. The principles of Darwin's and Børgen's methods of harmonic analysis	105
6. The principles of the Tidal Institute method	108
7. The Admiralty method	110
8. Separation of constituents with nearly the same frequency	119
9. Semi-graphic method	122

PART TWO

BASIC HYDRODYNAMICS AND HYDRAULICS

INTRODUCTION	129
 CHAPTER IV. GENERAL CONSIDERATIONS ON FLUID FLOW	 131
1. Basic equations	131
Introduction	131
1.1. The general equation of continuity	132
1.2. The general equation for the motion of non-viscous fluids	133
1.3. Considerations of the acceleration	134
1.4. The frictional forces for viscous fluids	138
1.5. The equations of motion for viscous incompressible fluids	142
1.6. Introduction to turbulent motion	143
2. The practical application of the theory of turbulence to channel flow	145
2.1. Application of Prandtl's mixing length theory to the velocity distribution in the vertical plane	145
2.2. Comparison between theoretical velocity distribution curves and empirical ones	150

CHAPTER V.	UNIFORM AND NON-UNIFORM STEADY FLOW IN A RECTANGULAR CHANNEL	153
1.	The relation between the mean velocity and the frictional force in open channels in the case of uniform flow	153
2.	Non-uniform steady flow in a rectangular channel	157
2.1.	Notations and assumptions	157
2.2.	The equation of motion	158
2.3.	Solution	160
CHAPTER VI.	A DISCUSSION OF WAVES	162
1.	Classification of waves	162
2.	Approximations which may be applied to the theory of short and long waves, respectively	166
3.	The wave spectrum	168
4.	Waves on rivers	170
4.1.	Surface waves	170
4.2.	Long waves	171
5.	The tidal wave on a river	172
CHAPTER VII.	THE EQUATIONS FOR LONG WAVES IN SHALLOW WATER	177
1.	Notation	177
2.	The equation of continuity for long waves in a sea	177
3.	The equations of motion for long waves in a sea	179
3.1.	General considerations	179
3.2.	The horizontal components of the Coriolis acceleration	181
3.3.	The friction forces which arise from bottom friction and wind influence	184
3.4.	The equations of motion	190
4.	The equations for a long wave in a river or estuary	191
5.	Discontinuities in the water motion of long waves	199
6.	Conditions at river junctions	203
7.	The equations in case of abrupt changes in river bed profiles	205
7.1.	General considerations	205
7.2.	The inflow region, and the flow over a crest	206
7.3.	The outflow region	208
8.	The boundary conditions and the tidal equations which determine the propagation of the tides	211
9.	The Lagrangian equations for one-dimensional flow	212

PART THREE

TIDAL COMPUTATIONS*IN RIVERS AND COASTAL WATERS

INTRODUCTION	219
CHAPTER VIII. THE HARMONIC METHOD	221
Introduction	221
1. The propagation of a simple harmonic wave	225
1.1. The solution of the tidal equations	225
1.2. Computation of the propagation of a simple harmonic wave in a channel	229
1.3. Some examples of the solution of boundary problems	230

1.4. The propagation of a harmonic wave in channels with gradually varying cross-section of special type	245
2. The simple harmonic wave in a two-dimensional rectangular bay	249
Introduction	249
2.1. The propagation of a simple harmonic wave in an infinite channel	250
2.2. The Kelvin wave	258
3. The computation of the propagation of the tide in a river, using Fourier analysis	262
3.1. Introduction to the development of the terms in a Fourier series	262
3.2. The procedure for obtaining the equations for the harmonic components of H and Q	264
3.3. The iteration process to obtain solutions of the equations	265
4. The equations of the main tidal component in a river with run-off discharge	267
4.1. The Fourier developments of the coefficients	267
4.2. The Fourier development of the factor $ Q Q$	271
4.3. The equations which determine the mean water levels and the main tide	275
5. The solution of the equations of the main tidal component	277
5.1. Discussion of the general solution	277
5.2. The solution of the equations of the main tide without run-off discharge	278
5.3. The solution of the equations of the main tide for the case of run-off discharge	282
6. The equations for the main tide and the first over-tide when there is run-off discharge	285
6.1. The Fourier developments of the coefficients of the tidal equations and of $ Q Q$	285
6.2. The equations for the main tide and the first over-tide	290
6.3. The solutions of the tidal equations	291
7. A numerical solution of the tidal equations	294
8. The development of $ Q Q$ using Chebyshev polynomials	296
 CHAPTER IX. CHARACTERISTIC METHODS	305
Introduction	305
1. The propagation of long waves in a uniform shaped river in which the resistance force can be neglected	306
1.1. The equations and the boundary problem	306
1.2. The general solution	307
1.3. Reflection	309
1.4. The solution for initial and boundary conditions	311
2. The characteristic curves of the complete tidal equations	315
3. Formulae for the characteristic curves	319
4. A discussion of the appearance of the characteristics when the flow is subcritical, critical, or supercritical	322
5. The characteristic equations	324
6. The iteration process of Picard and the regions of the solutions	328
7. The change in the water level and discharge along the characteristic curves	334
8. General considerations on the solutions of the characteristic equations	338
9. Computation or graphical determination of water levels and discharges at the points of a grid, when the diagonals approximate the characteristics	340
9.1. Method of computation	340
9.2. Linearization of (9.1) and (9.2), and estimates for the coefficients	342
9.3. The convergence of the characteristic difference processes	344

10. General application of the graphical construction of tidal motion using characteristics	345
11. The propagation of a disturbance along a river	353
12. The theory of the bore	355
Introduction	355
12.1. The formulae for a bore in a river	359
12.2. The computation of the water levels and discharges at both sides of the bore	367
12.3. Computation or construction of the tidal motion when a bore occurs on the river	369
CHAPTER X. FINITE-DIFFERENCE METHODS	372
Introduction	372
1. Finite-difference approximations to the simplified tidal equations	373
1.1. A method when the initial conditions are known	373
1.2. The convergence of the finite-difference method	376
1.3. The stability of the finite-difference method	379
1.4. Difference equations in case of initial and boundary conditions	381
2. Finite-difference approximations to the tidal equations for a river	385
2.1. Finite-difference equations for the general case	385
2.2. Finite-difference approximations for a short narrow part in a river (or for a gap)	391
2.3. Finite-difference approximations at a river junction	393
3. A finite-difference method for one-dimensional unsteady flow, which includes the motion of discontinuities	395
4. Finite-difference approximations to two-dimensional tidal motion	401
4.1. The first difference scheme	402
4.2. A second difference scheme	406
4.3. A mathematical discussion	408

PART FOUR

PRACTICAL APPLICATIONS

INTRODUCTION	413
CHAPTER XI. THE DATA FOR THE APPLICATION OF TIDAL COMPUTATIONS AND SOME EXAMPLES	414
1. Considerations about measurements in tidal waters	414
2. Schematization of an estuary	418
3. The determination of the De Chézy's coefficient	423
4. Formulae for the computation of the tide for the case that the vertical and horizontal tide are known at a place	426
Introduction	426
4.1. The formulae	428
4.2. Computation of currents from vertical tides at both ends of a section	431
4.3. Checking the schematizations	434
5. An application of the harmonic method	435
6. Application of the method of characteristics on flow and water level computations during and after the closure of a sluice	438
7. Final considerations about the application of tidal computations	441

CHAPTER XII. SPECIAL PROBLEMS	445
1. The computation of the tidal flow passing a gap to be closed	445
Introduction	445
1.1. Technical discussion concerning the closing of gaps	445
1.2. The influence of friction on the water motion over a "long weir" .	447
1.3. Critical flow over a weir	449
1.4. The occurrence of subcritical and critical flow during a closure operation	452
1.5. The closure curve for maximum flood and ebb velocity	452
1.6. Examples	455
2. A discussion of extreme high tides	465
2.1. Computation methods	465
2.2. Statistical considerations	468
3. Tidal models	473
SYMBOLS OF PART ONE	476
SYMBOLS OF PARTS TWO, THREE AND FOUR	478
DEFINITIONS	481
BIBLIOGRAPHY AND AUTHOR INDEX	485
SUBJECT INDEX	502

OUTLINE OF THE BOOK

This book is devoted to those tidal computations in rivers and coastal waters which are of great use in the planning and construction of engineering works in tidal waters. The derivation of the formulae for these computations will be discussed from a physical as well as a mathematical point of view.

In Chapter I, the reader will find a historical review of the development of the theory of tides. In addition, the purposes of tidal computation will be considered in order to provide an understanding of the methods for the practical application to a theoretical problem. Further discussion includes the means developed in the course of time for dealing with tidal problems, i.e., tidal computations and tidal models.

The other chapters of the book are divided into four parts.

Part one deals with the theory of the generation of the tides in an ocean. Harmonic analysis is then discussed as a method of representing the observed tide mathematically and for forecasting the tides.

Part two refers to the basic hydrodynamic and hydraulic theories which provide the principles for the study of the propagation of the tides in coastal waters and rivers. Moreover, a comparison is made between the principles of short-wave and long-wave theory; tidal theory is contained in the latter. Because the tidal motion in a tidal river shades off gradually into a steady flow in the upper parts of the river, a short chapter is devoted to the theory of steady flow.

Part three deals with the various methods of tidal computation in rivers and estuaries proper; the harmonic method, the use of characteristics and numerical methods are considered.

Finally, in part four, the reader will find practical considerations and applications for tidal motion: measurements, schematization of a river system, practical results of tidal computations, also for closing gaps, storm surges, etc.

It must be emphasized that, because tidal waves belong to the exten-

sive group of long waves, most mathematical methods dealt with in part three, may also be applied to long waves, e.g. high tides and flood waves.

In applied mathematics, theory is based on the physical properties of the phenomena and influenced by the practical purposes. Therefore, remarks will often be made which are based on practical experience.

The mathematical methods, which are the basis of the tidal computations, are introduced in the beginning of each chapter by means of the solution of the simplified equations. In nrs. 10, 11, 32, 65 of part two and 7, 8, 40, 58, 78 and 80 of part three of the Bibliography, textbooks are mentioned for basic mathematics and physics.

At the beginning of each part, the reader will find a short description and justification of the contents. Whenever useful, the contents of a chapter or section are also summarized at the beginning of that chapter or section.

A list of symbols and definitions which are used throughout the book, and an index of terms are added.

For a long time many symbols have already been in use for special terms in the theory of harmonic analysis and the propagation of tides. In this book this use has been followed. Consequently, it was not possible to maintain an unambiguous notation throughout the book. Therefore the symbols are separately mentioned for the various parts of the book. To prevent confusion the meaning of the various symbols is often repeated in the sections.

CHAPTER I

GENERAL CONSIDERATIONS

1. The phenomenon of the tides

Tides are produced by the attractions of the sun and moon on the waters of the ocean and are observable as a regular rise and fall of water within a range of several feet. They generally occur twice a day; the interval between one high water period and the next averaging twelve hours and twenty-five minutes.

Even the most elementary observations made at most sea shores show that the tides in the seas are more strongly affected by the moon than by the sun. The time of high and low water appears closely related to the moon's position, i.e. to its passage over the meridian which occurs, on the average, fifty minutes later on each succeeding day. However, between the moon's passage over the meridian and high water, there is a time lapse which differs from place to place and also differs for the same place depending on the propagation of the tide from the ocean into the coastal waters.

The height to which the water rises varies from day to day. Spring tide occurs during new moon when the sun and moon are on the same side of the earth, as well as full moon when they are diametrically opposite. At full moon, the tidal range — equalling the difference in height between high and low water — is often greatest. During the moon's first or third quarter, the range is smallest and we have neap tide. Besides these regular changes in the ranges, the tides at some places are subject to various irregularities. The ranges of two successive tides are usually not the same at any given place; this is called the diurnal inequality of the tide, and the extent of this effect differs from place to place. When the moon is in perigee (i.e., nearest the earth), the range of the tide is greater (by nearly 20 per cent) than when it is in apogee. The maximum range occurs during the new or full phases when the moon is in perigee.

In some seas, namely in parts of the Gulf of Mexico, Gulf of St Law-

rence, the Indonesian Archipel, and the Chinese Sea, the tides are still more irregular: there the water levels rise and fall only once daily during a part of each month. In such diurnal tides, however, the tidal fluctuations are relatively small compared to the so-called semi-diurnal tides.

It is clear that the tidal phenomenon is a complicated one. No general theory can explain all of the irregularities in the tide and its propagation; the available theory, however, does enable us to establish a practical method for the determination of the tides at points along the coast and in estuaries, and numerical computations can be used to determine the coefficients which fix the tides at any given point in detail.

There are two other effects, one caused by periodic winds and the other by variations in atmospheric pressure, which are so closely associated with tides that all three must be considered together in any tidal study. In addition, the air is also affected by the attractions of the sun and moon, as seen in the regular rise and fall of the height of the barometer; this may be called the atmospheric meteorological tide (see Kertz³⁴). The same attractions act on the solid earth, and since the earth is not rigid, this produces an alternating change in its shape which is called the tide of the solid earth (see Tomaschek⁶²). In this book, the tides in seas and rivers relative to the solid earth will be considered, while the atmospheric tide will be left out of consideration because its influence on the tides of the seas is negligible.

For an introduction to the theory of tides, reference is made to Darwin¹³) and regarding more modern methods and theories, to the books of Proudman⁵¹), Defant¹⁶) and^{16a}) and Hill^{27a}). See also Doodson^{23a}).

2. Historical review

Almost two thousand years ago, naturalists like Strabo and Pliny suspected that a relation existed between the celestial bodies and the motion of the sea. Kepler realized that the attracting forces of the moon and the sun were involved, but Newton was the first to discuss the origin of the tides from a mathematical point of view. In his master work, *Philosophiae Naturalis Principia Mathematica*, (London, 1687), he dealt with the problem of the tides by assuming the entire earth to be covered with water, and showed that the surface of the water would then become a spheroid as a result of the pull of the moon (or sun). The fact that the main axis of the

spheroid would be directed toward the moon explained the twice-daily occurrence of ebb and flood. From the positions of the sun and moon, he could derive the phenomenon of the spring and neap tides. He also demonstrated the effect of the difference in the distance between the earth and the sun, respectively the moon.

Newton's development was qualitative and, therefore, unsuitable for practical application. In 1738, the French Academy of Science held a competition for the best tidal theory, and one of the prize-winners was Bernoulli⁵⁾ who extended Newton's theory concerning the course of the tides at the equator. Both Newton and Bernoulli postulated a so-called equilibrium surface which the water mass on the earth assumes under the influence of the pull of a celestial body, and which changes continuously such that the main axis is constantly directed toward the celestial body. Neither considered the forces of inertia, and consequently, the motion of the particles themselves; in fact, Newton's hypothesis is incompatible with the actual motion of the particles, which is a subject for hydrodynamic theories.

Laplace attempted to improve Newton's work in this respect. Volume IV of his important work, *La Mécanique céleste*³⁷⁾, is devoted to the theoretical and practical study of the oscillations of the sea. The last part of Volume IV and almost all of Volume XIII deal with the tides at Brest, but the theory is not adequate for consideration of the many variations inherent in terrestrial conditions. Nevertheless, the results of this theory which describe the general principles of the correspondence between the periodic forces and the motion of the sea, are very useful for showing how tidal observations must be analysed for theoretical and practical purposes. These principles form the foundation of the harmonic method, which was developed by Thomson (Lord Kelvin) in about 1868 and later. While Laplace combined all periodic terms into one formula, Thomson based his method on the development of a sum of periodic terms, the so-called tidal constituents. Since the amplitude and the phase of each tidal constituent may be determined from observations, and since the period is known from the theory of harmonic analysis, the tides can be forecasted at any given place.

After Thomson, the theory of harmonic analysis, especially the methods for numerical computation of the harmonic constituents, was extended by Darwin and other investigators. A review of these investigations is given in Chapter III.

The course of the tide along a coast may be determined approximately