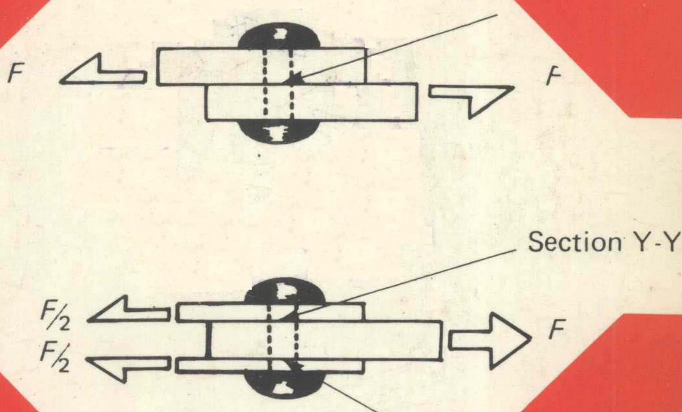
Peter A. Lilley

Mechanical Science Level III



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Mechanical Science Level III

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M&E TECBOOKS

The primary need of students taking Business and Technician Education Council (BTEC) courses is for books which will reflect the new methodology and syllabus requirements. In presenting M & E TECBOOKS, we believe that careful collaboration between subject editors and authors has resulted in the very best, "tailor-made" texts which could be devised for the student – and which are likely to be of guidance to the lecturer also. The aim has been to provide books based on BTEC's own objectives, concisely and authoritatively presented, priced as closely as possible to the student budget.

Preface

The aim of "Mechanical Science Level III" is to extend the student's understanding of Mechanical Engineering and to develop an analytical approach to the solution of problems associated with deformation of materials, dynamics, and fluid flow.

The text is written according to the new standard unit TEC U82/040 in Mechanical Science Level III which replaces unit U75/058.

Each chapter opens by quoting the TEC objectives and the areas dealt with directly reflect each item within the stated objectives. The general objectives of the book are realistically achieved through clear emphasis of basic principles rather than the blunt quoting of final results and formulae. There are continuous worked examples that enhance comprehension and illustrate application. The plentiful line drawings and clear explanatory style serve to lead the reader through the text with a confident understanding of the basic concepts. Each chapter concludes with a summary of essential formulae dealt with in that chapter.

The symbols and abbreviations used are in accordance with British Standards and are also approved by TEC.

The author would like to record his grateful acknowledgement of the advice and guidance offered by Mr George A. Woolvet and to the publishers, Macdonald and Evans, for their friendly co-operation. Appreciation is due also to my wife for her patience and forbearance during my cloistered periods spent in the preparation of this book.

1983 PAL

Symbols and Units

BASIC SI UNITS:

mass	kg	(kilogramme)
length	m	(metre)
time	s	(second)

Symbol	Quantity	SI unit
A	area	m^2
	amplitude	m
a	acceleration	m/s^2
d, *D*	diameter	m
E	modulus of elasticity (Young's modulus)	N/m^2
f	frequency	Hz
f_n	natural frequency	Hz
F	force	N
G	modulus of rigidity (shear modulus)	N/m^2
g	acceleration due to gravity (9.81)	m/s^2
I	second moment of area (statics)	m^4
	moment of inertia (dynamics)	kg m^2
J	polar second moment of area	m^4
k	radius of gyration	m
M	bending moment	N m
m	mass	kg
$\dot{m}$	mass flow rate	kg/s
p	pressure	N/m^2
$\dot{Q}$	volume flow rate	m^3/s
s	linear displacement	m
	displacement around arc	m
t	time	s
T	torque	N m
	period	s
v	velocity	m/s
V	volume	m^3
x	linear displacement	m

Greek Symbols

α (alpha)	angular acceleration	rad/s^2
	coefficient of linear expansion	/°C
γ (gamma)	shear strain	rad
δ (delta)	small change in a quantity	—
ϵ (epsilon)	direct strain	—
θ (theta)	angle	rad
μ (mu)	coefficient of friction	—
π (pi)	ratio of circle diameter to circumference (3.14159)	—
ϱ (rho)	density	kg/m^3
σ (sigma)	direct stess	N/m^2
Σ (capital sigma)	sum of a quantity of terms	—
τ (tau)	shear stress	N/m^2
ϕ (phi)	diameter on drawings	m
ω (omega)	angular velocity	rad/s
	circular frequency	rad/s
ω_n	natural circular frequency	rad/s

Multiples of quantities are expressed in powers of 1000 i.e. 10^3. Preferred multiples are given below.

Prefix	Symbol	Multiplication factor
giga	G	10^9 = 1 000 000 000
mega	Ma	10^6 = 1 000 000
kilo	k	10^3 = 1 000
milli	m	10^{-3} = 0.001
micro	μ	10^{-6} = 0.000 001

Contents

CHAPTER ONE

Stress and Strain: Elasticity

CHAPTER OBJECTIVES

After studying this chapter you should be able to:

* calculate the stresses and deformations of composite bars resulting from uniaxial loads at uniform temperature;
* calculate the stress resulting from the natural expansion or contraction of a bar being wholly or partially restricted;
* calculate the stresses and deformations of composite bars resulting from uniform temperature change;
* state that stresses due to external load and temperature change can be combined provided the nature of the stresses is taken into account;
* define shear stress, shear strain and modulus of rigidity (shear modulus), and solve associated problems.

INTRODUCTION

When an external force acts on a body internal forces per unit area (known as *stresses*) are set up in the material. The intensity of these internal forces is known as *stress* and the magnitude of the stress depends on the applied force and the area over which it acts, limited by the material itself, the size and shape of the body, and the way in which the force is applied.

As the material making up the body is deformed so the forces between the molecules of the material increase so as to oppose the external forces. This deformation is described by a quantity known as *strain*; all bodies undergo some strain when subjected to a force. The stresses and strains in a body which result from the application of an external force are related by a quantity known as the *elasticity* of the material.

A clear understanding of the nature of applied forces, the size of internal stresses, and the resulting deformations in materials is of fundamental importance to the Engineer.

FORCES

Forces are complex in nature but may be simply defined as those actions which change the state of rest or uniform motion of a body, or deform it. A force may be measured only by its effect on a body, e.g. in *statics* where bodies are at rest, the deformations that occur due to forces are examined; and in *dynamics*, where bodies are moving, the change in velocity or energy of the body is examined.

To completely specify a force four quantities are required: its magnitude; its point of application; its line of action; and its sense. Thus force is a vector quantity and its simplest form is known as a *direct* force. Direct forces acting on solid stationary bodies are either *tensile* or *compressive* in nature and are uniaxial. There is a third type of force known as *shear* which results from the application of two equal and opposite parallel forces which are *not* acting uniaxially.

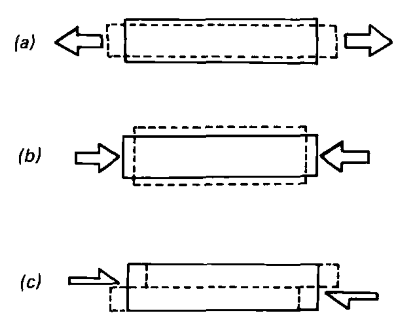

FIG. 1. *Deformations resulting from* (a) *tensile,* (b) *compressive and* (c) *shear forces.*

Figure 1 shows these forces and the types of deformation they tend to cause.

Forces may be applied locally to a body or they may be applied without direct contact. Examples of localised forces are the compressive action within a column supporting a load and the tension within a crane cable. An example of a force that occurs between systems that are not in direct contact but are physically separated is the force experienced by a body in a gravitational or magnetic field.

Direct Stress

When a body is acted upon by an external force then resisting forces are set up *within* the material. The intensity of these internal forces of cohesion are known as *stresses*. Equilibrium usually exists between the applied forces and the internal stresses.

For a uniformly distributed direct force (tensile or compressive) the stress is given by the externally applied force F, divided by the area A over which it is distributed. Thus direct stress σ is given by

$$\sigma = \frac{\text{force}(F)}{\text{area } (A)}$$

i.e.

$$\sigma = \frac{F}{A} \quad (1)$$

When force is measured in newtons (N) and area in square metres (m^2) the basic unit of stress is the newton per square metre (N/m^2) or multiples thereof. The sign convention usually adopted for forces, and consequently stresses, is to make tensile forces positive and compressive forces negative.

Direct Strain

If a load, however small, is applied to a body and that body resists the load then some deformation will take place. A measure of this deformation, in the direction of the applied load, is known as *strain*. There is a type of strain associated with each type of stress. In the case of a rod loaded in tension the actual extension of the rod depends on the load and its original length. Tensile and comprehensive strains ϵ, are given by the change in length divided by the original length, thus

$$\text{direct strain, } \epsilon = \frac{\text{change in length}(\delta L)}{\text{original length}(L)}$$

i.e.

$$\epsilon = \frac{\delta L}{L} \quad (2)$$

Strain is a ratio of lengths and therefore has no units.

Elasticity

A material is said to be perfectly elastic when the strain due to loading disappears when the load is removed. Most metals are elastic over a limited range of stress known as the elastic range. Generally elastic materials obey Hooke's law, which states that strain is directly proportional to stress. Thus the ratio of stress and strain for a particular material is constant, within limits. This constant is known as the *modulus of elasticity* or *Young's modulus*, E, where

$$\text{Young's modulus, } E = \frac{\text{direct stress}(\sigma)}{\text{direct strain}(\epsilon)}$$

i.e.

$$E = \frac{\sigma}{\epsilon} \quad (3)$$

A typical stress-strain graph for a ductile material is shown in Fig. 2 where the elastic limit represents the greatest stress that can be applied without the material suffering permanent deformation. Most engineering components are kept within the elastic range by incorporating a *factor of safety* at the design stage,

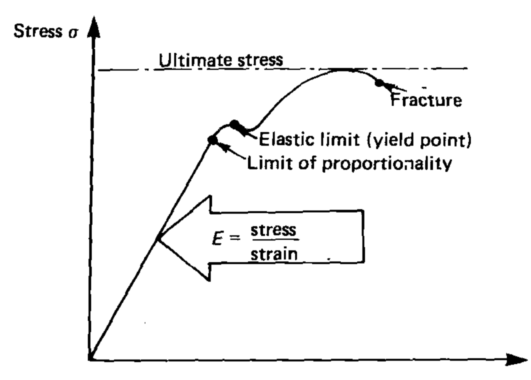

FIG. 2. *Typical stress-strain graph for a ductile material.*

TABLE I.
MODULUS OF ELASTICITY FOR SOME COMMON MATERIALS

Material	*Modulus of elasticity* E(GN/m^2)
Steel (mild)	210
(hardened)	200
Copper	120
Brass (70/30)	100
Glass (crown)	70
Aluminium	70
Timber (ash, along grain)	16
Concrete	10
Perspex	6

where

$$\text{factor of safety} = \frac{\text{ultimate stress}}{\text{working stress}} \quad (4)$$

Approximate values of E for some common materials are given in Table I. It should be noted that values of E are typical rather than absolute as variations occur due to material purity, heat treatment, etc. Furthermore, certain materials, such as rubber and cast iron, do not obey Hooke's law and their E values are subject to wide variations. Note that the units of E are those of stress because strain has no units.

Example 1
A square bar with 5 mm sides is 400 mm in length. When subjected to a tensile load of 10 kN the bar extends by 2 mm. Find (a) the stress, (b) the strain, and (c) the modulus of elasticity of the material.

(a) From equation (1)

direct stress $\sigma = \dfrac{F}{A}$

where $F = 10\text{ kN} = 10 \times 10^3\text{ N}$

and $A = 5\text{ mm} \times 5\text{ mm} = 25 \times 10^{-6}\text{ m}^2$

$$\therefore \quad \sigma = \frac{10 \times 10^3}{25 \times 10^{-6}} \frac{\text{N}}{\text{m}^2} = 0.4 \times 10^9\text{ N/m}^2 = 0.4\text{ GN/m}^2$$

(b) From equation (2)

direct strain $\epsilon = \dfrac{\delta L}{L}$

where $\delta L = 0.002\text{ m}$

and $L = 0.4\text{ m}$

$$\therefore \quad \epsilon = \frac{0.002}{0.4} \frac{\text{m}}{\text{m}} = 0.005$$

(c) From equation (3)

modulus of elasticity $E = \dfrac{\sigma}{\epsilon}$

$$E = \frac{0.4}{0.005}\text{ GN/m}^2 = 80\text{ GN/m}^2$$

Shear Stress

Shear stress tends to make one face of the material slide over the adjacent face as shown in Fig. 3. A typical case of shear is when a pair of scissors cuts a piece of paper.

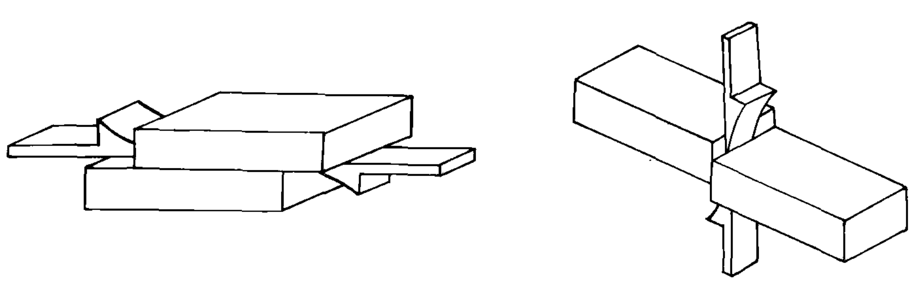

FIG. 3. *Shear stress.*

In exactly the same way as we considered direct stress we may also define shear stress τ as the external force F divided by the area A resisting shear. Thus

shear stress $$\tau = \frac{\text{force}(F)}{\text{area}(A)}$$

i.e. $$\tau = \frac{F}{A}$$

The basic unit of shear stress is the same as for direct stress, i.e. N/m^2.

Example 2
Single shear and double shear
Figure 4 represents a rivet in single and double shear. Single shear occurs when the tendency for shearing takes place on one face only, e.g. on cross-sectional area XX. Double shear occurs when the tendency for shearing occurs on two parallel faces and the stress level is usually half that in single shear, as in double shear the shearing force is resisted by the shear stress set up on the two cross-sectional areas YY and ZZ.

A rivet of 40 mm diameter is used to join two plates which are being pulled apart by a shear force of 20 kN. Estimate the shear stress in the rivet assuming a simple lap joint.

In the simple lap joint the rivet is subjected to single shear and the shear stress is given by

$$\tau = \frac{F}{A}$$

where $$F = 20 \times 10^3 \text{ N}$$

and $$A = \frac{\pi(40 \times 10^{-3})^2}{4} \text{ m}^2$$

$\therefore$ $$\tau = \frac{F}{A}$$
$$= 15.9 \times 10^6 \text{ N/m}^2$$

i.e. shear stress in rivet
$$= 15.9 \text{ MN/m}^2$$

Example 3
The coupling shown in Fig. 5 uses eight equally spaced bolts on a pitch circle of diameter 10 cm. The maximum torque to be transmitted is 2000 Nm. If the ultimate shear stress of the bolt material is 320 N/mm^2 and there is a factor of safety of 4, estimate a minimum appropriate bolt diameter.

$$\text{Factor of safety} = \frac{\text{ultimate shear stress}}{\text{working shear stress}}$$

$\therefore$ working shear stress
$$= \frac{320}{4} \times 10^6 \text{ N/m}^2$$
$$= 80 \times 10^6 \text{ N/m}^2$$

Let d be the diameter of each bolt, then the maximum shear force per bolt

$$= \left(80 \times 10^6 \times \frac{\pi d^2}{4}\right) \text{N}$$

Therefore the total maximum force F to be transmitted by all eight bolts is given by

$$F = 8 \times \left(80 \times 10^6 \times \frac{\pi d^2}{4}\right) \text{N}$$

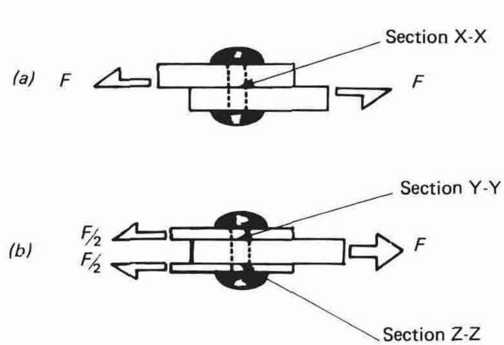

FIG. 4. (a) *Single shear, and* (b) *double shear.*

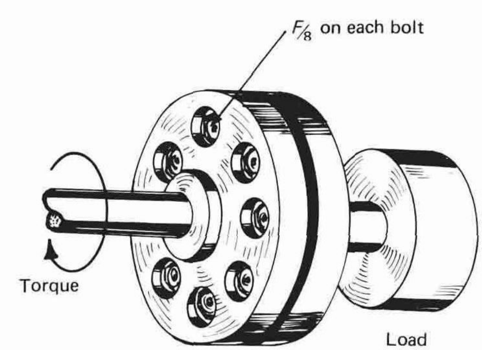

FIG. 5.

Maximum torque T to be transmitted is given by

$$T = F \times r$$

Where r is the radius arm

i.e. $$T = 8 \times \left(80 \times 10^6 \times \frac{\pi d^2}{4}\right) \times 0.05 \text{ N m}$$

As $T = 2\ 000$ Nm then

$$2\ 000 = 8 \times \left(80 \times 10^6 \times \frac{\pi d^2}{4}\right) \times 0.05$$

$$\therefore \quad d^2 = \frac{2\ 000 \times 4}{8 \times 80 \times \pi \times 0.05} \text{m}^2$$

$$\therefore \quad d = 8.9 \text{ mm}$$

Therefore the minimum appropriate diameter is approximately 9 mm.

Example 4
A bridge tie consists of the double lap joint shown in Fig. 6 where the force on the tie is 25 kN. What is the shear stress in each bolt?

For each bolt area A resisting shear is given by

$$A = 2 \times \frac{\pi (0.04)^2}{4} \text{ m}^2$$

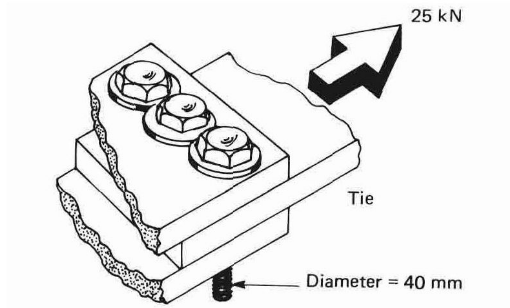

FIG. 6.

$$\therefore \quad \text{shear stress } \tau = \frac{\text{shear force}}{\text{total area resisting shear}}$$

$$\therefore \quad \tau = \frac{25 \times 10^3}{3 \times 2 \times \left[\frac{\pi (0.04)^2}{4}\right]} \text{N/m}^2$$

$$= 3.3 \times 10^6 \text{ N/m}^2$$

i.e. shear stress in each bolt = 3.3 MN/m^2

Example 5
A cross-section of a tensile member is shown in Fig. 7(*a*). If a rivet of diameter 50 mm is used to join the two elements estimate the stresses in the rivet.

The stresses in the rivet are the direct stress, acting along the axis of the rivet, and the shear stress acting along the joint face: these components are shown in Fig. 7(*b*).

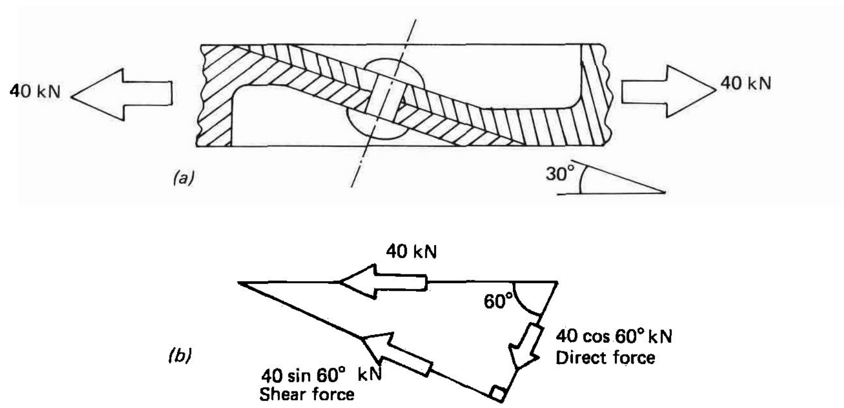

FIG. 7.

The component of the tensile force acting along the axis of the rivet is 40 cos 60° kN, as shown.

$$\therefore \quad \text{direct tensile stress} = \frac{40 \cos 60^\circ}{\dfrac{\pi(0.05)^2}{4}}$$

$$= 10.2 \text{ MN/m}^2$$

The component of the tensile force trying to shear the rivet is 40 sin 60° kN, as shown.

$$\therefore \quad \text{shear stress} = \frac{40 \sin 60^\circ}{\dfrac{\pi(0.05)^2}{4}}$$

$$= 17.6 \text{ MN/m}^2$$

Note that if the direct stress had been significantly higher than the shear stress it would have been preferable to use a bolt because with a high direct stress there is a danger of the rivet heads pulling off.

Shear Strain

The effect of shear force on a block of material of thickness y is shown in Fig. 8. Shear strain may be defined as the ratio of the displacement x to the separation distance y of the two faces which are displaced relative to each other. Since the displacement is very small compared with y the *shear strain* γ may be defined in terms of the angle γ (measured in *radians*).

Thus $\quad$ shear strain $\gamma = \dfrac{x}{y}$ rad $\quad$ (5)

and $\quad$ shear stress $\tau = \dfrac{F}{ab}$ N/m^2

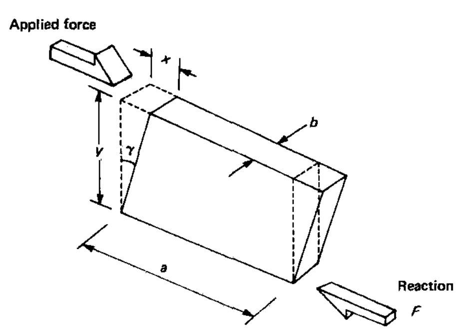

FIG. 8. *Effect of shear force on a block of material.*

Modulus of Rigidity

In the same way as direct stresses and strains were related by the modulus of elasticity so shear stresses and strains are related by the *modulus of rigidity*, or *shear modulus*, G. Thus, within the limit of proportionality,

$$\text{modulus of rigidity } G = \frac{\text{shear stress } \tau}{\text{shear strain } \gamma} \qquad (6)$$

Table II shows approximate values of G for some common materials.

TABLE II.
MODULUS OF RIGIDITY FOR SOME COMMON MATERIALS

Material	*Modulus of rigidity* G(GN/m^2)
Steel (nickel chrome)	82
(mild)	80
Copper	41
Brass	38
Aluminium	27
Timber	0.6–1.0

Example 6

The vibration isolator shown in Fig. 9(a) was found to deflect vertically by 2 mm for an axial load of 400 N. Estimate the shear modulus of the absorber material and the maximum load that can be applied if the vertical deflection is limited to 6 mm.

Force F on each absorber is equal to half of the total load.

$$\therefore \qquad F = 200 \text{ N}$$

For one absorber the area A resisting shear is given by

$$A = 100 \times 25 \text{ mm}^2$$

$$= 2\,500 \text{ mm}^2$$

and $\quad$ shear stress $\tau = \dfrac{F}{A}$

$$= \frac{200}{2\,500} \frac{\text{N}}{\text{mm}^2}$$

$$\therefore \qquad \tau = 0.08 \text{ N/mm}^2$$

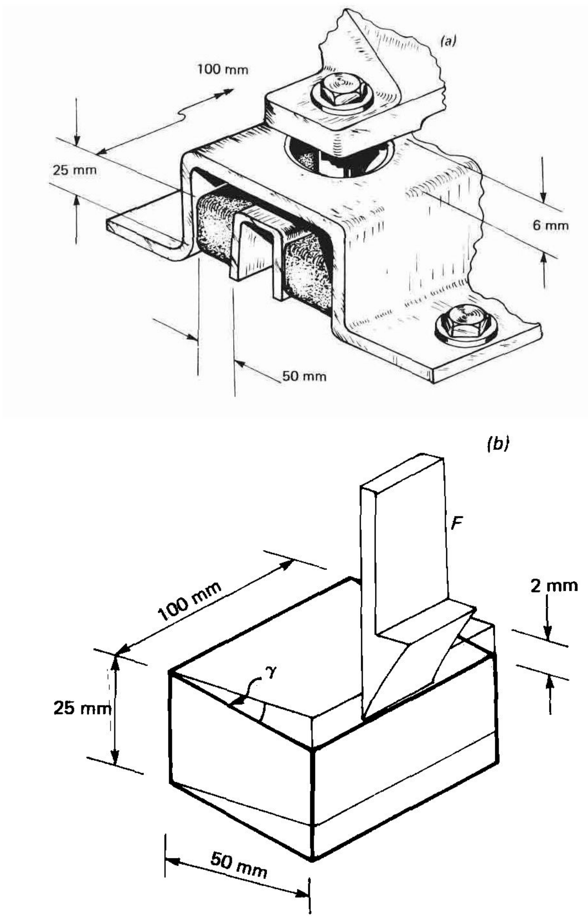

FIG. 9.

From the fact that the shear strain is numerically equal to the angle γ in radians

$$\gamma = \frac{2}{50}$$
$$= 0.04 \text{ rad}$$

$$\text{Shear modulus } G = \frac{\tau}{\gamma}$$
$$= \frac{0.08}{0.04} \frac{\text{N}}{\text{mm}^2}$$
$$= 2 \text{ N/mm}^2$$

i.e. the shear modulus of the absorber material is 2 MN/m^2. If the maximum vertical deflection is to be limited to 6 mm, then by ratios, the maximum force F_{max} is given by

$$\frac{400}{2} \frac{\text{N}}{\text{mm}} = \frac{F_{max}}{6} \frac{\text{N}}{\text{mm}}$$
$$F_{max} = 1\ 200 \text{ N}$$

i.e. the maximum load is 1.2 kN

COMPOSITE BARS

Composites are members that are made up of two or more elements. Composite bars, also known as *compound bars*, employ two or more elements of dissimilar material to give improved load bearing characteristics. The elements are usually rigidly fixed at their ends and loaded symmetrically to avoid bending.

Two conditions are employed to solve problems involving composite bars.

(*a*) *Equilibrium* The total load is the sum of the loads taken by each member.

(*b*) *Compatibility* The extension is the same for each member. (Note that a contraction may be considered as a negative extension.)

Natural Expansion of Composite Bars

Consider the two conditions of *equilibrium* and *compatibility* with reference to the system

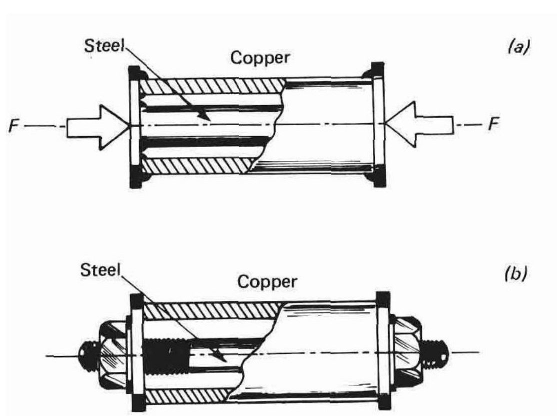

FIG. 10. (a) *Externally applied force F (both members in compression).* (b) *Internally applied force when nuts are tightened (copper tube in compression, steel rod in tension).*

shown in Fig. 10(*a*). For *equilibrium* we have:

total applied force F = force in copper F_c + force in steel F_s

i.e. $$F = F_c + F_s$$

but force = stress × area

$$\therefore \quad F = \sigma_c A_c + \sigma_s A_s \qquad (7)$$

where A is cross-sectional area and the subscripts c and s refer to copper and steel respectively.

For *compatibility* we get:

Strain in copper ϵ_c = strain in steel ϵ_s (since both elements are of the same length),

i.e. $$\epsilon_c = \epsilon_s$$

but $$\epsilon = \frac{\sigma}{E}$$

where E is the modulus of elasticity

$$\therefore \quad \frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s} \qquad (8)$$

Example 7

A concrete column is reinforced by two mild steel bars which are placed symmetrically as shown in Fig. 11. The column is 4 m high and carries an axial load of 700 kN. Determine the stress levels in both materials and the degree of compression of the column.

Let subscripts c and s refer to the concrete and steel respectively.

From Table I $E_c = 10\ \text{GN/m}^2$

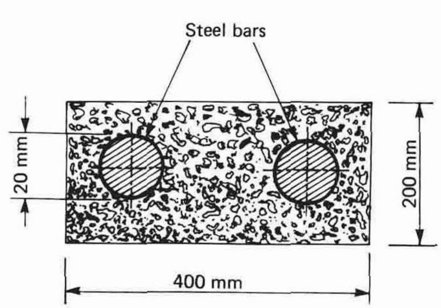

FIG. 11.

and $$E_s = 210\ \text{GN/m}^2$$

$$A_s = 2 \times \frac{\pi(20)^2}{4} = 628\ \text{mm}^2$$

$$A_c = 80\,000 - 628 = 79.37 \times 10^3\ \text{mm}^2$$

From the condition for equilibrium:

total load = load in concrete + load in steel

$$\therefore \quad 700 = \sigma_c(79.37 \times 10^3) + \sigma_s(628) \qquad (i)$$

From the condition for compatibility:

strain in concrete = strain in steel

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

$$\therefore \quad \frac{\sigma_c}{10} = \frac{\sigma_s}{210} \qquad (ii)$$

$$\therefore \quad \sigma_s = \frac{210}{10}\sigma_c$$

Substituting for σ_s in (*i*) we get

$$700 = \sigma_c(79.37 \times 10^3) + \frac{210}{10}\sigma_c \times 628$$

$$\therefore \quad 700 = [(79.37 \times 10^3) + (13.188 \times 10^3)]\,\sigma_c$$

$$\therefore \quad \sigma_c = 7.6\ \text{N/mm}^2 \text{ compressive}$$

Substituting for σ_c in (*ii*) we get

$$\sigma_s = 158.8\text{N/mm}^2 \text{ compressive}$$

To find the compression

$$\epsilon = \frac{\sigma}{E}$$

but $$\epsilon = \frac{\delta L}{L}$$

$$\therefore \quad \delta L = \frac{\sigma L}{E}$$

i.e. $$\delta L = \frac{\sigma_c L}{E_c} \left(\text{or } \frac{\sigma_s L}{E_s}\right)$$

$$\delta L = \frac{7.6 \times 4 \times 1\,000}{10 \times 1\,000}$$

$$= 3 \text{ mm}$$

i.e. the compression of the column is 3 mm.

Restricted Expansion of Composite Bars

If a composite bar is made up of elements whose ends are rigidly fixed to end plates as in Fig. 10(*a*), then forces can be transmitted from one element to another through the end plates. This *restricts* the expansion compared with that of the elements considered individually.

Fig. 10(*b*) shows a composite bar consisting of a copper tube and a threaded steel rod. The end plates, in this case, are not rigidly fixed but forces are still transmitted from the steel rod to the copper tube when the nuts are tightened; the steel rod is in tension and the copper tube is in compression. As the nuts are tightened some of the threaded steel rod is pulled through each nut and the steel is put under tension. The force is transmitted to the copper tube by the end plates and, by Newton's third law is equal and opposite to the tensile force, putting the tube under compression. This is represented in Fig. 12.

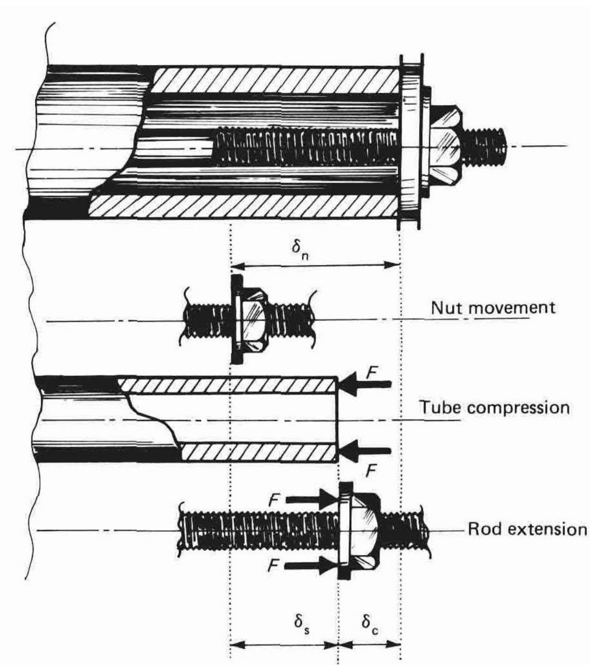

FIG. 12.

Example 8

The composite bar shown in Fig. 10(*b*) consists of a high yield steel rod of diameter 5 mm within a copper tube of inner and outer diameter 10 mm and 14 mm respectively. The original length of the bar is 350 mm and the pitch of the thread is 1 mm.

Calculate the stresses and change in length of the rod and tube when one nut is tightened by half a turn. Take E for steel to be 210 GN/m^2 and for copper to be 120 GN/m^2.

Referring to Fig. 12 and ascribing the subscripts s and c to steel and copper respectively: from the condition for equilibrium (Newton's third law) we get tensile load on steel rod = compressive load on copper tube.

$\therefore$ $$F_s = F_c$$

But $$\text{force} = \text{stress} \times \text{area}$$

$\therefore$ $$\sigma_s A_s = \sigma_c A_c$$

$\therefore$ $$\sigma_s = \frac{A_s \sigma_c}{A_s} \qquad (i)$$

From the condition for compatibility, the absolute movement δ_n of the nut (the axial distance moved along the rod) is made up of the compression of the tube δ_c, and the extension of the rod δ_s.

i.e. $$\delta_n = \delta_c + \delta_s$$

But $$\delta = \epsilon L$$

$\therefore$ $$\delta_n = \epsilon_c L_c + \epsilon_s L_s$$

where $$L_c = L_s = L$$

$\therefore$ $$\frac{\delta_n}{L} = \epsilon_c + \epsilon_s$$

also $$\epsilon = \frac{\sigma}{E}$$

$\therefore$ $$\frac{\delta_n}{L} = \frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} \qquad (ii)$$