Deborah J. Bennett

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Randomness

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Contents

Chance Encounters

1

> Our brains are just not wired to do probability problems very well.
>
> PERSI DIACONIS
> 1989

Everyone has been touched in some way by the laws of chance. From shuffling cards for a game of bridge, to tossing a coin at the start of a football game, to awaiting the outcome of the Selective Service draft lottery, to weighing the risks and benefits of knee surgery, most of humanity encounters chance daily. The statistics that describe our probabilistic world are everywhere we turn: One-third don't survive their first heart attack. The chance of a DNA match is 1 in 100 billion. Four out of every 10 marriages in America end in divorce. Batting averages, political polls, and weather predictions are pervasive, but an understanding of the concepts underlying these statistics and probabilities is not.

Misconceptions abound, and certain concepts seem to be particularly problematic. To even the mathematically enlightened, some issues in probability are not so intuitive. Despite curriculum reforms that have emphasized the teaching of probability in the schools, most experienced teachers would probably agree with the math teacher who commented, "Teaching statistics and probability well is not easy."[1]

Even in very serious decision-making situations, such as assessing the evidence of guilt or innocence during a trial, most people fail to properly evaluate objective probabilities. The psychologists Daniel Kahneman and Amos Tversky illustrated this with the following example from their research:

> A cab was involved in a hit and run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:
>
> (a) 85% of the cabs in the city are Green and 15% are Blue.
>
> (b) A witness identified the cab as Blue. The court tested the reliability of the witness under the same circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.
>
> What is the probability that the cab involved in the accident was Blue rather than Green?[2]

A typical answer is around 80 percent. The correct answer is around 41 percent. In fact, the hit-and-run cab is more likely to be Green than Blue.

Kahneman and Tversky suspect that people err in the hit-and-run problem because they see the base rate of cabs in the city as incidental rather than as a contributing or causal factor. As other experts have pointed out, people tend to ignore, or at least fail to grasp, the importance of base-rate information because it "is remote, pallid, and abstract," while target information is "vivid, pressing, and concrete."[3] In evaluating the eyewitness's account, "jurors" seem to overrate the eyewitness's likelihood of accurately reporting this specific hit-and-run event, while underrating the more general base rate of cabs in the city, because the latter information seems too nonspecific.

Base-rate misconceptions are not limited to the average person without an advanced mathematics education. Sophisticated subjects have the same biases and make the same mistakes—when they think intuitively. In a study at a prominent medical school, physicians, residents, and fourth-year medical students were asked the following question:

> If a test to detect a disease whose prevalence is one in a thousand has a false positive rate of 5 percent, what is the chance that a person found to have a positive result actu-

4 ally has the disease, assuming you know nothing about the person's symptoms or signs?[4]

Almost half of the respondents answered 95 percent. Only 18 percent of the group got the correct answer: about 2 percent. Those answering incorrectly were once again failing to take into account the importance of the base-rate information, namely, (only) 1 person among 1000 tested will have the disease.

The commonsense way to think mathematically about the problem is this: Only 1 person in 1000 has this disease, as compared with about 50 in 1000 who will get a false positive result (5 percent of 999). It is far more likely that any one person who tests positive will be one of the 50 false positives than the 1 true positive. In fact, the odds are 1 in 51 that any one person who tests positive actually has the disease, and that translates into only a 2 percent chance, even in light of the positive test.

Another way to state the issue is that the chances of having this disease go from 1 in 1000 when one takes the test to 1 in 51 if a person gets a positive test result. That's a big jump in risk, to be sure, but it's a far cry from the 95 out of 100 chance many people erroneously believe they have after a positive test.

False positives are not human errors or lab errors. They happen because screening tests are designed to be overly sensitive in picking up people who deviate from some physiological norm, even though those people do not have the disease in question. In order to be sensitive enough to pick up most people who have tuberculosis, for example, skin tests for TB infection will always yield a positive result for around 8 percent of people who do not have the infection but who have other causes for reaction to the test; if 145 people are screened, roughly 20 will test positive. Yet only 9 of these 20 will turn out to have TB infection.[5]

The rate of false positives can be reduced by making screening tests less sensitive, but often this just increases the percentage of false negatives. A false negative is a test result that indicates no disease in a person who actually has the disease. Because false negatives are usually considered more undesirable than false positives (since people who get a false negative will not receive prompt treatment), the designers of screening tests settle on a compromise—opting for a very small percentage of false negatives and a somewhat larger percentage of false positives than we might prefer. In the case of tuberculosis, whereas roughly 7.5 percent of people tested will receive a false positive result, only 0.69 percent (roughly 1 person out of every 145 screened) will get a false negative test result. In other words, out of 145 people

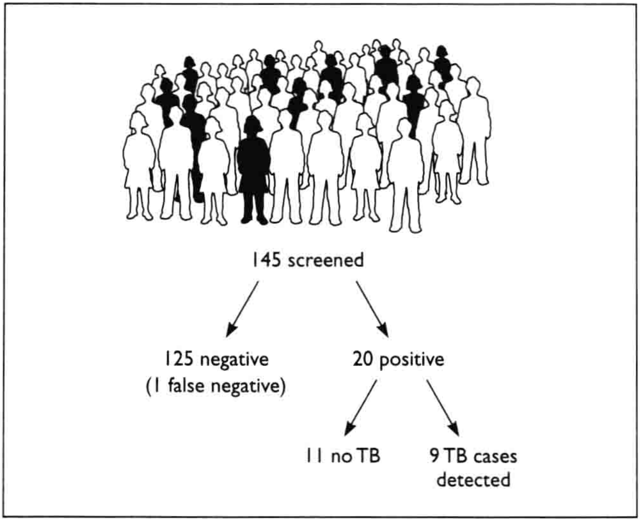

FIGURE 1 Really well, or really sick? If a person has a routine screening test for tuberculosis, she or he has a 10 in 145 (about 7 percent) chance of having the infection at the time of the test. If the result comes back positive, the patient's odds of having TB go up to 9 in 20 (45 percent). If the result comes back negative, the patient still has a 1 in 125 chance of having the disease (about 0.8 percent); the original risk has been drastically reduced, but not eliminated, by the doctor's "good news."

screened for TB using this method, 9 cases of the disease will be detected and 1 case will remain undetected (see Figure 1).

Considering that even highly educated medical personnel can make errors in understanding probabilistic data of this kind, we should not be at all surprised that probability often seems to be at odds with the intuitive judgments of their patients and other ordinary people.

In addition to base-rate misconceptions, psychologists have shown that people are subject to other routine fallacies in evaluating probabilities, such as exaggerating the variability of chance and overattending to the short run versus the long run.[6] For example, the commonly held notion that, on a coin toss, a tail should follow a string of heads is erroneous. Children seem particularly susceptible to this fallacy. Jean Piaget and Barbel Inhelder, who studied the development of mathematical thinking in children and whose work will be described frequently in the following chapters, pointed out that "by contrast with [logical and arithmetical] operations, chance is gradually discovered."[7]

One would think that the experiences acquired over a lifetime ought to solidify some correct intuitions about statistics and probability. Intuitive ideas about chance do seem to precede formal ideas, and, if correct, are an aid to learning; but if incorrect, they can hinder the grasp of probabilistic concepts. Kahneman and Tversky have concluded that statistical principles are not learned from everyday experience because individuals do not attend to the detail necessary to gain such knowledge.[8]

Not surprisingly, over the course of our species' history, acquiring an understanding of chance has been extremely gradual, paralleling the way an understanding of randomness and probability develops in an individual (if it does). Our human dealings with chance began in antiquity, as we will see in Chapters 2 and 3. Archaeologists have found dice, or dice-like bones, among the artifacts of many early civilizations. The practice of drawing lots is described in the writings of ancient religions, and priests and oracles foretold the future by "casting the bones" or noting whether an even or odd number of pebbles, nuts, or seeds was poured out during a ceremony. Chance mechanisms, or randomizers, used for divination (seeking divine direction), decision making, and games have been discovered throughout Mesopotamia, the Indus valley, Egypt, Greece, and the Roman Empire. Yet the beginnings of an understanding of probability did not appear until the mid-1500s, and the subject was not seriously discussed until almost one hundred years later. Historians have wondered why conceptual progress in this field was so slow, given that humans have encountered chance repeatedly from earliest times.

The key seems to be the difficulty of understanding randomness. Probability is based on the concept of a random event, and statistical inference is based on the distribution of random samples. Often we assume that the concept of randomness is obvious, but in fact, even today, the experts hold distinctly different views of it.

This book will examine randomness and several other notions that were critical to the historical development of probabilistic thinking—and that also play an important role in any individual's understanding (or misunderstanding) of the laws of chance. We will investigate a series of ideas over the course of the following chapters: ➤Why, from ancient times to today, have people resorted to chance in making decisions? ➤Is a decision made by random choice a fair decision? ➤What role has gambling played in our understanding of chance? ➤Are extremely rare events likely in the long run? ➤Why do some societies and individuals reject randomness? ➤Does true randomness exist? ➤What contribution have computers made to modern probabilistic thinking? ➤Why do even the experts disagree about the many meanings of randomness? ➤Why is probability so counter-intuitive?

We all have some notion about the "chances" of an event occurring. We come to the subject of probability with some intuition about the topic. Yet, as the eminent eighteenth-century mathematician Abraham De Moivre pointed out long ago, problems having to do with chance generally appear simple and amenable to solution with natural good sense, only to be proven otherwise.[9]

Why Resort to Chance?

2

"It isn't fair, it isn't right," Mrs. Hutchinson screamed. And then they were upon her.

SHIRLEY JACKSON
The Lottery

Everyday randomizers are not very sophisticated. To settle a dispute over which child gets to ride in the front seat of the car, for example, a parent may resort to a game called "drawing straws." One child holds two thin sticks or broom straws in her hand, with the ends concealed, and the other child chooses. The child with the shorter straw wins. Many adult entertainments—from old-fashioned cake walks and Friday Night Bingo to school raffles and million-dollar jackpots—are driven by simple lotteries. Spinners turn up on children's board games, among teenagers playing spin-the-bottle, and in Las Vegas gambling casinos. Dice, which are among the oldest randomizers known, are still popular today among a range of ages and ethnic groups.

Although hand games, lots, spinners, dice, coins, and cards are not very complicated devices, our attitudes about using them are a great deal more complex. When primitive societies needed to make a selection of some sort, they often resorted to randomizers for three basic reasons: to ensure fairness, to prevent dissension, and to acquire divine direction. Modern ideas about the use of chance in decision-making also invoke issues of fairness, dispute resolution, and even supernatural intervention, though we usually think of these concepts somewhat differently today.

Interestingly, these three reasons are exactly the ones given by children when asked by psychologists why they used counting-out games, such as one potato/two potato. Ninety percent of the time children responded that counting out gave them an equal chance of being selected. Other reasons given were to avoid friction and to allow some kind of magical or supernatural intervention.[1] Clearly, the idea of fairness is an important intuitive element in children's notions of randomness. Of course children eventually learn that counting-out games are not really fair: the choosing can be manipulated by speeding up or slowing down the verse, or by changing the starting point for counting out. Once they figure this out, children generally move on to better methods of randomization.

When chance determines the outcome, no amount of intelligence, skill, strength, knowledge, or experience can give one player an advantage, and "luck" emerges as an equalizing force. Chance is a fair way to determine moves in some games and in