

*editors*

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Series on Quality, Reliability and Engineering Statistics **Vol. 10**

**Modern Statistical and  
Mathematical Methods in Reliability**

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# **Modern Statistical and Mathematical Methods in Reliability**



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# **Modern Statistical and Mathematical Methods in Reliability**

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## PREFACE

This volume is published on the occasion of the fourth International Conference on Mathematical Methods in Reliability (MMR 2004). This bi-annual conference was hosted by Los Alamos National Laboratory (LANL) and the National Institute of Statistical Sciences (NISS), June 21-25, 2004, in Santa Fe, New Mexico. The MMR conferences serve as a forum for discussing fundamental issues on mathematical methods in reliability theory and its applications. They are a forum that bring together mathematicians, probabilists, statisticians, and computer scientists from within a central focus on reliability. This volume contains a careful selection of papers, that have been peer-reviewed, from MMR 2004.

A broad overview of current research activities in reliability theory and its applications is provided with coverage on reliability modeling, network and system reliability, Bayesian methods, survival analysis, degradation and maintenance modeling, and software reliability. The contributors are all leading experts in the field and include the plenary session speakers, Tim Bedford, Thierry Duchesne, Henry Wynn, Vicki Bier, Edsel Peña, Michael Hamada, and Todd Graves.

This volume follows Statistical and Probabilistic Models in Reliability: Proceedings of the International Conference on Mathematical Methods in Reliability, Bucharest, Romania (D. C. Ionescu and N. Limnios, eds.), Birkhauser, Series on Quality, Reliability and Engineering Statistics (1999); Recent Advances in Reliability Theory, Methodology, Practice, and Inference, Proceedings of the Second International Conference on Mathematical Methods in Reliability, Bordeaux, France (N. Limnios and M. Nikulin, eds.), Birkhauser, Series on Quality, Reliability and Engineering Statistics (2000); Mathematical and Statistical Methods in Reliability, Proceedings of the Third International Conference on Mathematical Methods in Reliability, Trondheim, Norway (B. Lindqvist and K. A. Doksum, eds.), World

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