# industrial calculating devices

wilcox — butler

# INDUSTRIAL CALCULATING DEVICES

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## **PREFACE**

THE processing of information is as fundamental as life itself. It forms the basis of every rational decision and of every purposeful activity. It occurs, to a greater or lesser degree, in the brains of all living, rational creatures. The human mind is a complex and wonderful machine for storing and processing information, but, in spite of its complexity, it has its limitations. To help overcome these limitations, man has always had to devise artificial aids to assist him in gathering, processing, summarizing, and interpreting information. In our fast-moving technical age this need has become much more acute than it ever was before.

Computation with numerical data is an essential and central part of data processing. In particular, the need for saving time and promoting efficiency has made mechanical computing devices almost indispensable to the engineer and to the technician.

In preparing this book the authors have attempted to present certain information, rules of procedure, and types of problems basic to the understanding and operation of certain simple computing devices commonly used by engineers and technicians in industry as a part of the processing of numerical information. This is not a theoretical treatise on computing. It is intended, rather, as a book which will be of practical use to students who find it important in their work to become proficient in the use of these aids to calculation.

Industrial Calculating Devices has a unique organization; this is intentional. Chapters 1, 2, and 3 deal with the basic calculating aids found in book and pamphlet form. This section relates the background of the student to the major content of the text.

Chapters 4, 5, 6, and 7 explain the operations that may be performed on a Mannheim slide rule. Other types of slide rules are incorporated in the discussion, but all the operations may be performed on the Mannheim rule.

Chapters 8, 9, and 10 complete a section of the book that covers

the content given in many schools. Thus, Chapters 1 through 10 provide material for a 12-week course. In such a course, Chapter 7 serves as a mid-term review and Chapter 10 as a final review. This permits additional time to be spent on the operation of mechanical calculators.

Chapters 11, 12, and 13 utilize problems applicable to the log-log type of slide rule. With the addition of Chapters 14 and 15, the book is suitable for an 18-week course. It will be noted that Chapters 14 and 15 are each self-contained and independent of the rest of the book. They can be omitted, if necessary, without impairing the primary objective of the book. Some teachers may not wish to cover the content in the last two chapters as a formal part of a course. Appendix I, "Errors in Computation with Approximate Data," has been added to supplement such courses. This content was not incorporated into the text as it applies to other than computing devices.

Glade Wilcox Charles H. Butler

Kalamazoo, Michigan November, 1961

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# INFORMATION

ANY rational discussion or plan of action must be based on relevant information, but, unless the information is properly represented and properly processed, it is of little use. The kinds of information which are relevant, and the ways in which information can be most helpfully systematized, depend upon the purposes for which it is to be used. Certain information about plants and animals is systematized under such headings as genus, species, family, etc. Information about chemical elements is presented in the form of a periodic table or an atomic chart. Automobiles are classified according to manufacturer, year of marketing, body style, and various other characteristics. Regardless of the kind of information with which one is concerned, it is more meaningful and helpful if it is organized than if it is not.

QUANTITATIVE INFORMATION. Some information is purely descriptive, but a great deal of the information needed in technical work is quantitative in nature. Original or primary quantitative information can be obtained at first hand only by the processes of counting and measuring. If it is obtained by counting, it is exact, because the only numbers used in counting are the positive integers or natural numbers—1, 2, 3, etc. If it is obtained by measuring, it is only approximate. Even though the measurement may be very precise, it cannot be known to be exact. Thus, if a measurement of the diameter of a No. 20 wire is recorded as 0.0319 in., this means only that this measurement is taken to be correct to the nearest tenthousandth of an inch. The true diameter, however, might have had any value between 0.03185 and 0.03195 in. Numerical information known to be exact is sometimes called digital information, and the name analog information is sometimes applied to approximate

numerical data. A great deal of the numerical information used in science, engineering, and technology is of this latter kind.

1.2 STANDARD UNITS. In order to obtain numerical information by measuring, one must have *units* of measure, and in order to be generally useful these units must be standardized.

Our whole system of physical measurement is based upon three primary and independent kinds of physical magnitude—length, mass, and time. In the *English system*, in common use in this country, the basic standard units for these three kinds of magnitude are, respectively, the foot, the pound, and the second. In the metric or *cgs system*, the basic standard units are, respectively, the centimeter, the gram, and the second. Conversions from one system to the other are sometimes necessary and can be made with the help of suitable conversion tables.

Standard units for the measurement of all other kinds of physical properties and phenomena (work, power, momentum, velocity, tensile strength, acceleration, electrical potential, heat, sound, and hundreds of others) are derived from and defined, directly or indirectly, in terms of the basic units in either the English system, the cgs system, or the mks system (see Appendix V).

1.3 SYMBOLS. In order to communicate any sort of information, one must use symbols of one kind or another to represent the concepts or relationships or ideas he wishes to convey. In a sense, even the words one uses in speaking are symbols. They are sounds to which meanings have been attached. Most people, however, think of symbols in a narrower sense—as written or printed marks to which meanings have been attached. In this sense, written words are symbols through which ideas can be communicated. In a still narrower sense, symbols are thought of as a sort of shorthand for representing words and ideas more briefly and economically than would be possible through the use of words themselves. Thus the symbols @, ", 73, =,  $\pi$ , \$, &, %, and many others represent words or phrases. Their meanings are widely understood, and they are in common use.

The symbols with which we shall be most directly concerned in this book are those used in representing numbers. In the course of history, a variety of different systems of numeration symbols have been invented at different times and by different peoples. Most of the civilized world now uses what is called the *Hindi-Arabic* system of numeration, and this is the system of number symbols to which we are accustomed. It consists of a set of nine individual symbols, or *digits*, for the numbers 1 through 9, and a symbol for zero: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. It is called a *decimal* system of numeration because it employs a base of 10. The principle of place value enables us to combine any two or more of these individual symbols in various sequences, and thus to represent (either exactly or approximately) any real number. It lends itself well to longhand computation and to simple mechanical calculating devices. Simpler systems of numeration are possible and, indeed, are actually used in high-speed electronic digital computers, but these need not be considered here. In this book all numerical data will be represented in the common decimal system of numeration with which we are all familiar.

1.4 EXPONENTS. If a number is multiplied by itself, as  $9 \times 9$  or  $5 \times 5$ , the products are often written in the notation  $9^2$  or  $5^2$ . The small number 2, written to the right of and slightly above the base number (9 or 5) is called an *exponent*. The exponent in such cases tells how many 9's or 5's are multiplied together. The symbol  $5^3$  indicates the product of 3 fives, or  $5 \times 5 \times 5$ . Similarly,  $3^4$  indicates the product of 4 threes, or  $3 \times 3 \times 3 \times 3$ . This is often a useful way of writing such products. The number represented by  $5^3$  is called the *third power* of 5, and the number represented by the symbol  $3^4$  is called the *fourth power* of 3. The exponent is sometimes called the *index of the power*, and the number upon which it is applied is called the *base*.

In the cases shown above the exponents are positive integers; that is, the counting numbers. The meaning of such exponents gives rise to certain extremely useful laws for multiplying and dividing numbers which can be expressed as powers of some common base. For example,  $7 \times 7 \times 7 = 7^3$  and  $7 \times 7 = 7^2$ . If we wish to multiply  $7^3$  by  $7^2$ , we wish, in effect, to find the product of  $(7 \times 7 \times 7) \times (7 \times 7)$ , and this is evidently  $7^5$ . Now the exponent  $^5$  is the *sum* of the exponents 3 and 2 of the respective factors; that is,  $7^3 \times 7^2 = 7^{3+2} = 7^5$ . Similarly,  $10^3 \times 10^2 = 10^5$ , and  $5^4 \times 5^3 = 5^7$ . These examples illustrate the following law which is valid for all positive integral exponents: If b is a number and M and N are positive

integers, then  $b^M \times b^N = b^{M+N}$ . This law is valid also when more than two such factors are involved. Thus  $b^M \times b^N \times b^P = b^{M+N+P}$ . As an extension of this law, it can be easily shown that  $(b^M)^T = b^{MT}$ .

This law is a consequence of the meaning that was given to exponents which are positive integers. It is often helpful to use exponents which are not positive integers. Such exponents may be in the form of negative integers, zero, or fractions (rational numbers). It is possible to assign meanings to exponents having these forms, in such a way that they can be used under the rule given above. These meanings will be made clear by the following defining equations and illustrations.

If b is a number other than zero, the symbol  $b^0$  will always represent the number 1.

Thus 
$$7^0 = 1$$
,  $\left(\frac{1}{2}\right)^0 = 1$ ,  $10^0 = 1$ , etc.

The symbol  $b^{-M}$  will always mean  $\frac{1}{b^M}$ . Thus  $7^{-2}$  means  $\frac{1}{7^2}=\frac{1}{49}$ , and  $10^{-3}$  means  $\frac{1}{10^3}=\frac{1}{1,000}$ .

In like manner, 
$$\frac{1}{7^{-2}}$$
 means  $7^2 = 49$ , and  $\frac{1}{10^{-3}}$  means  $10^3 = 1,000$ .

If p and r are integers, the symbol  $b^{\frac{p}{r}}$  will always mean  $(\sqrt[r]{b})^p$  or the pth power of the rth root of b.\* Thus  $(8)^{\frac{2}{3}}$  means  $(\sqrt[3]{8})^2 = 2^2 = 4$ , and  $(5)^{\frac{3}{2}}$  means  $(\sqrt[2]{5})^3 = \sqrt{5^3} = \sqrt{125}$ .

By assigning these meanings to negative exponents, the exponent zero, and fractional (rational) exponents, it is possible to operate with these exponents under the foregoing rule in the same manner as with positive integral exponents. No new rules are necessary, but the field of possible operations is extended now to include division and roots as well as multiplication and powers of numbers. The following examples present further illustrations of the use of exponents.

$$\frac{7^{6}}{7^{4}} = 7^{6} \times \left(\frac{1}{7^{4}}\right) = 7^{6} \times 7^{-4} = 7^{6+(-4)} = 7^{2} = 49$$

$$\frac{10^{5}}{10^{-2}} = 10^{5} \times \left(\frac{1}{10^{-2}}\right) = 10^{5} \times 10^{2} = 10^{5+2} = 10^{7} = 10,000,000$$

<sup>\*</sup> If b is a positive number, or if r is an odd integer, this can be written also as  $\sqrt[r]{b^p}$ . It is not necessary for both of these conditions to exist. Either one is sufficient.

$$\frac{5^3}{5^3} = 5^3 \times \left(\frac{1}{5^3}\right) = 5^3 \times 5^{-3} = 5^{3+(-3)} = 5^0 = 1$$

$$[(4)^{\frac{1}{2}}]^3 = (\sqrt{4})^3 = 2^3 = 8$$

$$(2)^{\frac{5}{3}} = (\sqrt[3]{2})^5 = \sqrt[3]{2^5} = \sqrt[3]{32}$$

$$(5^2)^4 = (5)^{2\times 4} = (5)^8 = 391,125$$

1.5 POWERS OF 10 AND SCIENTIFIC NOTATION. Engineers and technicians often have to deal with complicated expressions involving very large or very small numbers. Problems such as  $36,000 \times 1.1 \times 0.06$  are not uncommon. Evaluating such expressions

sions by ordinary arithmetic is laborious and time-consuming.

Moreover it increases the likelihood of operational errors and un-

Moreover, it increases the likelihood of operational errors and uncertainty as to the placement of the decimal point in the final result.

These difficulties can be reduced materially by a device called *scientific notation*. This is based on the fact that any number can be written as the product of two factors, one of which is a number between 1 and 10, and the other an integral power of 10. For example, 47,322 can be written as  $4.7322 \times 10,000$ , or  $4.7322 \times 10^4$ . The following partial table of integral powers of 10 will be helpful for reference in rewriting numbers in scientific notation.

$$10,000 = 10^{4}$$

$$1,000 = 10^{3}$$

$$100 = 10^{2}$$

$$10 = 10^{1}$$

$$1 = 10^{0}$$

$$0.1 = 10^{-1}$$

$$0.01 = 10^{-2}$$

$$0.001 = 10^{-3}$$

$$0.0001 = 10^{-4}$$

The examples on page 6 give further illustrations of how numbers can be rewritten in scientific notation.

$$60,000 = 6 \times 10,000 = 6 \times 10^{4}$$

$$538 = 5.38 \times 100 = 5.38 \times 10^{2}$$

$$543,000,000 = 5.43 \times 100,000,000 = 5.43 \times 10^{8}$$

$$0.00675 = 6.75 \times 0.001 = 6.75 \times 10^{-3}$$

$$0.000023 = 2.3 \times 0.00001 = 2.3 \times 10^{-5}$$

Now let us see how this scientific notation and the law of exponents can help in multiplying or dividing numbers that are unusually large or unusually small. In particular, let us note how the use of scientific notation takes the uncertainty out of the placement of the decimal point in the product or quotient.

Consider the following examples:

$$2,560,000 \times 41,000 = 2.56 \times 10^{6} \times 4.1 \times 10^{4}$$

$$= (2.56 \times 4.1) \times (10^{6} \times 10^{4})$$

$$= 10.496 \times 10^{6+4} = 10.496 \times 10^{10}$$

$$= 104,960,000,000$$

$$\frac{72,000}{0.0012} = \frac{7.2 \times 10^{4}}{1.2 \times 10^{-3}} = \frac{7.2}{1.2} \times \frac{10^{4}}{10^{-3}}$$

$$= \frac{7.2}{1.2} \times 10^{4} \times \frac{1}{10^{-3}}$$

$$= \frac{7.2}{1.2} \times 10^{4} \times 10^{3} = 6 \times 10^{7} = 60,000,000$$

$$\frac{0.0044}{0.00011} = \frac{4.4 \times 10^{-3}}{1.1 \times 10^{-4}} = \frac{4.4}{1.1} \times 10^{-3} \times 10^{4}$$

$$= 4 \times 10^{1} = 40$$

### PRACTICE EXERCISES

1. Write the following values, using scientific notation.

6,725	=	0.0827	=
508	=	0.0009	=
6,065	=	58.28	=
28,400	=	7.68	=

Use scientific notation to test problems 2 through 11 to determine whether the indicated operations give the results shown on the right sides of the equations. Check those that are correct. If there are any which are not correct, find the correct answers.

$$2. 10,000 \times 0.0001 \times 100 = 100$$

3. 
$$140,000 \times 0.000057 \times 600 = 4,788$$

4. 
$$0.00350 \times 5,000,000 \times 34,000 \times 0.0004 = 23,800$$

5. 
$$3,875 \times 0.000032 \times 3,000,000 = 372,000$$

6. 
$$7,000 \times 0.015 \times 1.78 = 186.9$$

7. 
$$\frac{0.000063 \times 50.4 \times 0.0072}{780 \times 0.682 \times 0.018} = 2.38 \times 10^{-6}$$

$$8. \frac{0.015 \times 216 \times 1.78}{72 \times 0.0624 \times 0.0353} = 36.4$$

9. 
$$\frac{0.000079 \times 0.00036}{29 \times 10^{-8}} = 0.098$$

10. 
$$\frac{1}{2 \times 3.14 \times 458,000 \times 6,100} = 0.539 \times 10^{-12}$$

11. 
$$\frac{14.1 \times 156,000}{85 \times 10^{-3} \times 350 \times 10^{-6}} = 7.4 \times 10^{10}$$

12. Verify that 
$$(2^3)^2 = 2^{3 \times 2} = 2^6 = 64$$
.

13. Verify that 
$$(3^2)^4 = 3^8 = 6,561$$
.

14. Verify that 
$$(16)^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$$
.

15. To find the horsepower of an 8-cylinder gas engine with a piston of 2-in. diameter, substitute in the SAE horsepower-rating formula to get:  $2^2 \times 8$ 

horsepower = 
$$\frac{2^2 \times 8}{2.5}$$
. What is the horsepower?

16. A 5-ton truck and load are traveling 44 feet per second (fps). To find the average force required to stop the truck within 20 ft,  $f = \left(\frac{5 \times 2,000 \times 44 \times 44}{2 \times 32.2}\right) \div 20$ . What is the force?

- 17. To index a machine for  $34^{\circ}$  12', turns + remainder in 20ths =  $34 \times 3,600 + 12 \times 60$ . How many full turns and spaces on a 20-hole circle are needed?
- 18. The cutting speed of a material is  $s = \frac{\text{revolutions per minute} \times \text{diameter}}{4}$ . If a turret lathe cuts at 150 revolutions per minute (rpm) on a 4-in-diameter piece, what is its cutting speed in feet per minute?
- 19. Substituting in the formula for stress developed by compressing a beam,  $S \left[ \text{in pounds per square inch (psi)} \right] = \frac{0.0000065 \times 120 \times 60 \times 30 \times 10^6}{120} \,.$  What is the stress?
- 20. To find the torque in a certain coupling from the tension formula, T in inch-pounds (in.-lb) =  $\frac{10,000 \times 3.14 \times 5^4}{2.5 \times 32}$ . What is the torque?
- 21. To determine unit labor costs on a premium plan, an equation is used which results in the following form for John Doe:

Unit labor cost = 
$$\frac{6 \times \$1.00 + (6.5 - 6) \times \frac{\$1.00}{2}}{65}$$

What is John Doe's unit labor cost?

22. The core losses of a transformer in ohms are

$$R_c = \frac{2\pi^2 10^{-16} (60)^2 440^2 (100)}{(50) \, \frac{150}{15,000}}$$

What is the core loss in ohms?

- 23. The ratio of N/E in turns per volt for a certain transformer is  $N/E = \frac{10^8}{60,000(1)60(4.44)}$ . With E=110, what does N equal?
- 24. The core area of a transformer is  $A = \frac{5,280 \times 60 \times 60 \times 6.28 \times 10^{-18}}{2\pi (264 \times 3.6)}$

What is the core area?