MATHEMATICS
FOR
TECHNICAL STUDENTS

PART II
SECOND YEAR COURSE

FREDERICK G. W. BROWN

WITH ANSWERS

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BY

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MATHEMATICS FOR TECHNICAL STUDENTS

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MATHEMATICS FOR TECHNICAL STUDENTS.
PART I. (FIRST YEAR COURSE) SECOND EDITION.
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A SHORT PRACTICAL MATHEMATICS. SECOND EDITION.

COMMERCIAL ARITHMETIC.

ELEMENTARY MATHEMATICS FOR WORKSHOP STUDENTS.

A SHORT APPLIED WORKSHOP MATHEMATICS.

PREFACE

It is now generally recognised that little progress can be made by a technical student without some appropriate knowledge of mathematics. The more formal methods, fixed sequence and rigorous treatment of Pure Mathematics are, however, neither desirable nor interesting to students of applied science and, as a consequence, courses of Practical Mathematics were introduced some years ago. This subject in its early stages often consisted of little more than an enunciation of a number of pocket-book rules. In the progressive development of technical knowledge, it was soon found, however, that the intelligent student was compelled eventually to get back to those mathematical principles upon which depend so many of the applications he needs. Modern courses in technical mathematics have, therefore, become fuller and more logical than those of the earlier Practical Mathematics; for experience has also shewn that technical students cease to be intelligently interested when they are unable to understand the basic reasons for many of the mathematical processes they employ.

This volume is an attempt to provide such a course. Whilst not designed to cover any specific syllabus in detail, it is the outcome, not only of teaching experience, but also of a thorough survey of the requirements in technical mathematics prescribed by important examining authorities. It aims at weaving the fundamental principles of elementary applied arithmetic, algebra, trigonometry and simple deductive geometry into a coherent unity.

The book is divided into two parts, and the scope of these may be gauged by the fact that Parts I and II cover completely the First and Second Years' Courses, respectively, in Practical Mathematics laid down by the Union of Lancashire and Cheshire Institutes for the Preparatory Senior Technical Courses, whilst the whole book covers the First Year's Syllabus in Mathematics for the Senior Courses in Mechanical Engineering, Electrical Engineering and Naval Architecture, as well as the Third Year's Course in Electrical Installation of the same authority.

Parts I and II also cover the syllabuses in Mathematics for Stages I and II laid down for the classes conducted by the Scottish Educational Authorities affiliated with the Royal Technical College, Glasgow.

It may be pointed out that, whilst some courses may require an elementary introduction to graphs in Part I, it was not deemed advisable to deal piecemeal with the subject by including portions in each part. The whole of the work necessary has therefore been treated in Chap. XXIII, with the exception of the graphical representation of a linear equation considered on pages 264-266 in connection with simultaneous equations in two unknowns.

Mention may also be made of the *Frame* notation introduced on pages 163, 289, 290, 296-7 in connection with algebraic identities. This has proved to be a very effective device in teaching and was first published by the author in an article on *Algebraic Factorisation* which appeared in May 1910.

Considerable care has been taken to provide the student with a wide choice of numerous and varied exercises. Many of these are naturally of a formal type in illustration of fundamental principles, whilst a larger number are founded upon actual technical data. Some of the exercises have also been taken from recent examination papers, and acknowledgment is gladly accorded to the Union of Lancashire and Cheshire Institutes and to the Controller of H.M. Stationery Office for kindly permitting the use of these questions.

To Sir Richard Gregory, Bt., F.R.S., the author owes an increasing debt of gratitude for constant help and wise counsel at every stage in the production of the book.

The author also wishes to record his grateful thanks to Mr. A. J. V. Gale, M.A., and Mr. R. Holmes, M.Sc., for many valuable suggestions, as well as for their care and kindly interest in reading through the whole of the proof sheets. Mr. Holmes has indeed checked the answers to the whole of the exercises.

Thanks are also accorded to Mr. F. W. Dent for considerable help in the preparation of the manuscript; to the publishers for the use of the tables given at the end, which are taken from Mr. F. Castle's *Logarithmic and Other Tables for Schools*; and finally, to the printers for the excellence of their work.

It is quite probable that a few errors have escaped final detection, and notification of these will be welcomed.

F. G. W. BROWN.

April 1936.

LIST OF ABBREVIATIONS

I. IN THE TEXT

THE following symbols and abbreviations are used throughout the text of this volume :

$=$ *for* is, or are, equal to.

$\therefore$,, therefore.

$\angle$,, angle.

$>$,, is, or are, greater than.

$<$,, is, or are, less than.

parm. ,, parallelogram.

The solidus notation for fractions is also occasionally used ; thus

$$a/b \text{ denotes } \frac{a}{b}.$$

II. IN THE EXERCISES

The following abbreviations are used to indicate those exercises taken from examination papers :

C.S. Papers set by the Civil Service Commissioners.

U.L.C.I. Papers set by the Union of Lancashire and Cheshire Institutes.

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PART II

CHAPTER XI

THE LAWS OF INDICES

11·1. Products of Like Powers.

WE have already become familiar with the index notation, explained in Section 1·2, page 4, for writing down a product of equal factors. Let us study it a little further.

We know that

$$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3, \quad \text{and} \quad 3^4 = 3 \times 3 \times 3 \times 3,$$

$\therefore$ the product of 3^7 and 3^4 is

$$\begin{aligned} &(3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3) \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^{11}. \end{aligned}$$

Hence, $\quad 3^7 \times 3^4 = 3^{11}$, *i.e.* 3^{7+4}.

Thus the index of the product is the **sum** of the indices of the powers of 3 multiplied together.

Suppose, however, we required the product of 3^7 and 5^4, then

$$3^7 \times 5^4 = (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5) = 3^7 \times 5^4.$$

Here we cannot add the indices because the sets of equal factors are of *different* numbers.

Hence we can only add indices when powers of the **same number** are multiplied together.

EXERCISES 11A. MENTAL

Write down :

1. 9 as a power of 3.
2. 16 as a power of 2, and of 4.
3. 343 as a power of 7.
4. 625 as a power of 5.

State, as powers, the products of :

5. $2^5 \times 2^3$.
6. $5^9 \times 5$.
7. $3^3 \times 7^2$.
8. 8×2^4.
9. $9 \times 3^3 \times 2^2$.
10. 25×125.
11. $49 \times 7^3 \times 3$.
12. 81×32.

11·2. Quotients of Powers.

Suppose we had to divide 3^7 by 3^4, then since

$$3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \quad \text{and} \quad 3^4 = 3 \times 3 \times 3 \times 3,$$

$$3^7 \div 3^4 = \frac{3^7}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 3 \times 3 \times 3 = 3^3.$$

$$\therefore\ 3^7 \div 3^4 = 3^3 = 3^{7-4}.$$

Hence to find the index of the quotient, we **subtract** the index of the divisor from that of the dividend, provided we are dividing powers of the **same** number.

When the indices of both dividend and divisor are equal, we have a very important result, which must be remembered.

Thus $$7^6 \div 7^6 = 7^{6-6} = 7^0.$$

But $$7^6 \div 7^6 = \frac{7^6}{7^6} = 1.$$

$$\therefore\ 7^0 = 1.$$

Similarly, any number raised to the power of 0 is merely another way of writing 1, thus $3^0 = 1$, $10^0 = 1$, $879^0 = 1$, and so on.

EXERCISES 11A (*Continued*)

Express as powers :

13. $2^{10} \div 2^3$.
14. $5^7 \div 5$.
15. $243 \div 3^3$.
16. $343 \div 7^2$.
17. $5^7 \div 625$.
18. $6^3 \div 216$.
19. $3^4 \div 3^7$.
20. $11^5 \div 11^8$.
21. $3^2 \div 5^3$.
22. $729 \div 27$.

11·3. Powers of Powers.

The square of 9 is 81 ; but $9 = 3^2$, and $81 = 3^4$;

$$\therefore\ 9^2 = (3^2)^2 = 3^2 \times 3^2 = 3^4.$$

Similarly, $$(5^3)^4 = 5^3 \times 5^3 \times 5^3 \times 5^3 = 5^{12}.$$

Hence, in raising the power of a number to some other power, we **multiply** the indices together.

EXERCISES 11A (*Continued*)

State as powers :

23. $(3^8)^3$.
24. $(13^2)^4$.
25. $(7^5)^6$.
26. $(15^9)^5$.

Express :

27. 9^3 as a power of 3.
28. 16^4 as a power of 2.
29. 49^5 as a power of 7.
30. 125^3 as a power of 5.

11·4. Fractional Indices.

Can we express $\sqrt{5}$ as a power ? Let us see.

Suppose $\sqrt{5}$ can be written as 5^x, where x is some number.

Since $\sqrt{5} \times \sqrt{5} = 5$, therefore $5^x \times 5^x = 5^1$.

But, when we multiply powers of the same number, we add the indices to find the index of the product. Assuming this to be true in all cases,

$$5^x \times 5^x = 5^{x+x} = 5^{2x}.$$

Therefore $$5^{2x} = 5^1,$$

so that $2x$ must be the same number as 1.

Hence, $$2x = 1, \quad \text{or} \quad x = \tfrac{1}{2}.$$

$$\therefore\ \sqrt{5} = 5^{\frac{1}{2}}.$$

Thus any number raised to the power of $\frac{1}{2}$ is another way of indicating the square root of that number, hence

$$9^{\frac{1}{2}} = \sqrt{9} = 3\ ; \quad 169^{\frac{1}{2}} = \sqrt{169} = 13, \text{ and so on.}$$

Similarly, $$5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^1 = 5\,;$$
$$\therefore\ 5^{\frac{1}{3}} = \sqrt[3]{5},$$

i.e. $5^{\frac{1}{3}}$ is only another way of writing down the cube root of 5.

In the same way, we may shew that

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3\,;\quad 125^{\frac{1}{3}} = \sqrt[3]{125} = 5, \text{ and so on.}$$

Again, $\sqrt[3]{4} = 4^{\frac{1}{3}}$, but $4 = 2^2$, therefore $4^{\frac{1}{3}} = (2^2)^{\frac{1}{3}} = 2^{\frac{2}{3}}$.

Hence, $$2^{\frac{2}{3}} = \sqrt[3]{4} = \sqrt[3]{2^2},$$

so that, when an index is expressed by a fraction, the numerator indicates a **power** and the denominator indicates a **root**; this is true in all cases.

Ex. 1. *Find the simplest values of* (i) $16^{\frac{3}{4}}$ *and* (ii) $343^{\frac{2}{5}}$.

(i) By the above rule, $16^{\frac{3}{4}} = \sqrt[4]{16^3}$.

But $16 = 2^4$, so that $16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2$,

and $16^{\frac{3}{4}} = (16^{\frac{1}{4}})^3 = 2^3 = \mathbf{8}$.

(ii) Similarly, $343 = 3^5$;

$\therefore\ 343^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3$.

Hence, $343^{\frac{2}{5}} = (343^{\frac{1}{5}})^2 = 3^2 = \mathbf{9}$.

EXERCISES 11B

Find the values of :

1. $6241^{\frac{1}{2}}$.
2. $9261^{\frac{1}{3}}$. (Use factors)
3. $64^{\frac{2}{3}}$.
4. $324^{\frac{1}{2}}$.
5. $27^{\frac{2}{3}}$.
6. $256^{\frac{3}{4}}$.
7. $(37^2 - 35^2)^{\frac{1}{2}}$.
8. $(28^2 - 21^2)^{\frac{1}{3}}$.
9. $(72^2 + 65^2)^{\frac{1}{2}}$.
10. $(153^2 - 136^2)^{\frac{2}{3}}$.

11·5. Negative Indices.

Sometimes in practical problems we have to deal with a power which is expressed by a negative index. It will therefore be necessary to find out what this really means.

Now we know that $7^4 \times 7^4 = 7^{4+4} = 7^8$.

In the same way, $7^{-4} \times 7^4 = 7^{-4+4} = 7^0 = 1$, by Sect. 11·2.

Hence, on dividing both sides by 7^4, we have

$$7^{-4} = \frac{1}{7^4}.$$

Similarly it may be shewn that

$$3^{-5} = \frac{1}{3^5}; \quad 11^{-3} = \frac{1}{11^3}; \quad 13^{-\frac{1}{2}} = \frac{1}{13^{\frac{1}{2}}} = \frac{1}{\sqrt{13}};$$

and so on.

EXERCISES 11B (*Continued*)

Find the values of:

11. 5^{-3}, as a decimal.
12. $1369^{-\frac{1}{2}}$.
13. $16^{-1} + 2^{-5}$.
14. $729^{-\frac{1}{3}}$. (Use factors)
15. $4^{-\frac{1}{2}} + 27^{-\frac{1}{3}} + 6^{-1}$.
16. $9^{-2} \times 3^4$.
17. $(\frac{1}{3})^{-2} \times 5^{-3}$, as a decimal.
18. $64^{-\frac{2}{3}} \div 32^{-\frac{3}{5}}$.
19. $(\frac{1}{1728})^{-\frac{1}{3}} \div 625^{\frac{1}{2}}$, as a decimal.
20. $194481^{-\frac{1}{4}} \times 343^{\frac{1}{3}}$.
21. $(\frac{4}{9})^{-\frac{1}{2}} \div (\frac{16}{27})^{-\frac{1}{3}} \times (\frac{1}{2})^{-\frac{2}{3}}$.

11·6. Revision Summary.

The laws of indices which we have been considering may now be summarised in the following general forms.

Let N, m, and n be any three positive whole numbers or fractions, then

I. $N^m \times N^n = N^{m+n}$. (Section 11·1.)

II. $N^m \div N^n = N^{m-n}$. (,, 11·2.)

III. $(N^m)^n = N^{mn}$. (,, 11·3.)

IV. $N^{\frac{m}{n}} = \sqrt[n]{N^m}$. (,, 11·4.)

V. $N^{-m} = \frac{1}{N^m}$. (,, 11·5.)

VI. $N^0 = 1$. (,, 11·2.)

11·7. Miscellaneous Applications.

Ex. 2. *Simplify* $625^{\frac{1}{4}} \times \sqrt[3]{0{\cdot}125} \div (\frac{2}{5})^{-2}$.

Now
$$625 = 25^2 = (5^2)^2 = 5^4;$$
$$\therefore\ 625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5.$$

Also
$$0{\cdot}125 = \frac{1}{8} = \frac{1}{2^3};$$
$$\therefore\ \sqrt[3]{0{\cdot}125} = \sqrt[3]{\frac{1}{8}} = \sqrt[3]{\frac{1}{2^3}} = \frac{1}{2}.$$

And
$$(\tfrac{2}{5})^{-2} = \frac{1}{(\frac{2}{5})^2} = (\tfrac{5}{2})^2 = \tfrac{25}{4};$$
$$\therefore\ 625^{\frac{1}{4}} \times \sqrt[3]{0{\cdot}125} \div (\tfrac{2}{5})^{-2} = 5 \times \tfrac{1}{2} \div \tfrac{25}{4}$$
$$= \frac{5 \times 4}{2 \times 25}$$
$$= \tfrac{2}{5} = \mathbf{0{\cdot}4}.$$

Ex. 3. *Calculate, by a short method, the value of:*
$$\frac{16{\cdot}98 \times 5{\cdot}754 \times 3{\cdot}311}{43{\cdot}65 \times 74{\cdot}13},$$
given that
$$3{\cdot}311 = 10^{0\cdot52},\quad 5{\cdot}754 = 10^{0\cdot76},\quad 16{\cdot}98 = 10^{1\cdot23},$$
$$43{\cdot}65 = 10^{1\cdot64} \quad\text{and}\quad 74{\cdot}13 = 10^{1\cdot87}.$$

Expressing each of the given numbers as powers of 10 from the values given, we have,
$$\frac{16{\cdot}98 \times 5{\cdot}754 \times 3{\cdot}311}{43{\cdot}65 \times 74{\cdot}13} = \frac{10^{1\cdot23} \times 10^{0\cdot76} \times 10^{0\cdot52}}{10^{1\cdot64} \times 10^{1\cdot87}}$$
$$= \frac{10^{1\cdot23+0\cdot76+0\cdot52}}{10^{1\cdot64+1\cdot87}} = \frac{10^{2\cdot51}}{10^{3\cdot51}} = 10^{2\cdot51-3\cdot51} = 10^{-1} = \tfrac{1}{10} = \mathbf{0{\cdot}1}.$$

EXERCISES 11c

1. Express as powers of 2 and simplify, $(2^7)^2 \times 16^{-5} \times 256^{\frac{3}{4}}$.

2. Simplify $(16^{\frac{5}{2}} - 10^3) \times 2^{-3}$.

3. Calculate the value of $(3^2 + 3^{-2}) \div (3^2 - 3^{-2})$.