CONTROL AND DYNAMIC SYSTEMS

ADVANCES IN THEORY AND APPLICATIONS

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Volume 17

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PREFACE

As noted in earlier volumes, this annual international series went to the format of theme volumes beginning with Volume 12. The theme for this volume is "differential game theory applications." Its purpose is to offer a unique presentation of the theory of differential games, including the development and presentation of powerfully effective algorithms for the solution of this complex class of problems, and to demonstrate these solutions through a comprehensive collection of applications. Users of this volume will have at their disposal a collection of the work on theory and proven applications for the analysis and synthesis of real time on-line computer control of dynamic systems, which may be categorized as differential games.

Modern techniques for the analysis and synthesis of optimal control systems were first established during the mid-1950s with the reintroduction of state space techniques. Thereafter, modern techniques were developed rapidly for control systems and continue to develop at a very substantial pace today. It was not until R. Isaacs's book, "Differential Games," was published in 1965 that much of consequence happened in the competitive or two-sided decision-makers dynamic system situation categorized as differential games, not be confused with earlier publications on the static two-sided decision-makers situation generally known as game theory. Because of the substantial theoretical and computational difficulties of differential games, progress has been slower than that for the development of modern optimal control techniques. This volume is one of the first attempts to present in a self-contained manner a comprehensive treatment of the subject of differential games and its applications.

In the first chapter, Narendra K. Gupta defines some of the fundamental concepts and terms and develops the structure of system problem formulation for differential game systems. A broad spectrum of application areas and examples is included. Once the problem is stated, effective methods for the solution must be developed. In the second chapter, James E. Rader presents and applies some computationally effective techniques. The contribution by Josef Shinar, one of the most active workers in the field, is devoted to pursuit–evasion games. Methods for solutions are presented and illustrated by examples.

In the continuing search for effective computational techniques for differential games, which could then result not only in effective algorithms for dynamic system design, but also in on-line real time computer control techniques, one of the most potentially effective techniques is the application of differential dynamic programming techniques in differential games. The chapter by Bernt A. S. Järmark, an

internationally recognized leader in developing these techniques, is devoted to differential dynamic programming techniques.

Using stochastic linear control systems with a graduate performance index, it is possible to develop effective control techniques by means of the separation principle, wherein the optimal control is given as a time variable gain matrix premultiplying a minimum variance estimate of the state vector. However, in differential games, this is not possible because of the "intertwining" nature of the vector matrix equations for the two players or opponents. As a result, early attempts at a solution for this problem in differential games produced a formal solution that was composed of an infinite number of integrals—not a very practical solution but one that followed from the nature of the problem. The chapter by B. Mons presents some powerfully effective means for the control of linear stochastic differential games and develops a separation principle. The techniques developed in this chapter are illustrated by examples.

Aran Chompaisal addresses problems of increasing complexity which, in fact, follow from "real-world" situations. In this chapter the author presents and illustrates powerful algorithms for the solution of stochastic differential games with bounds on the control and the state.

One of the most significant applications of differential games is to air combat systems analysis techniques. The contribution by Lynch, a prominent researcher in this area, is devoted to this subject. The following chapter, by Narendra K. Gupta, deals with reachable set methods for the solution of differential games. This is a potentially practical and effective method for the solution of differential games and was first motivated by air combat systems; Gupta's technique, however, is more widely applicable.

The contribution by P. R. Kumar and T. H. Shiau is devoted to a thorough exposition of zero-sum dynamic games, and it includes some significant illustrative applications. This volume closes with the contribution by Kamran Forouhar, an in-depth analysis of techniques developed for singular differential games and the development of closed-loop techniques.

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