

CONTROL AND DYNAMIC SYSTEMS

ADVANCES IN THEORY
AND APPLICATIONS

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**CONTROL AND
DYNAMIC SYSTEMS**

*Advances in Theory
and Applications*

Volume 17

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PREFACE

As noted in earlier volumes, this annual international series went to the format of theme volumes beginning with Volume 12. The theme for this volume is "differential game theory applications." Its purpose is to offer a unique presentation of the theory of differential games, including the development and presentation of powerfully effective algorithms for the solution of this complex class of problems, and to demonstrate these solutions through a comprehensive collection of applications. Users of this volume will have at their disposal a collection of the work on theory and proven applications for the analysis and synthesis of real time on-line computer control of dynamic systems, which may be categorized as differential games.

Modern techniques for the analysis and synthesis of optimal control systems were first established during the mid-1950s with the reintroduction of state space techniques. Thereafter, modern techniques were developed rapidly for control systems and continue to develop at a very substantial pace today. It was not until R. Isaacs's book, "Differential Games," was published in 1965 that much of consequence happened in the competitive or two-sided decision-makers dynamic system situation categorized as differential games, not be confused with earlier publications on the static two-sided decision-makers situation generally known as game theory. Because of the substantial theoretical and computational difficulties of differential games, progress has been slower than that for the development of modern optimal control techniques. This volume is one of the first attempts to present in a self-contained manner a comprehensive treatment of the subject of differential games and its applications.

In the first chapter, Narendra K. Gupta defines some of the fundamental concepts and terms and develops the structure of system problem formulation for differential game systems. A broad spectrum of application areas and examples is included. Once the problem is stated, effective methods for the solution must be developed. In the second chapter, James E. Rader presents and applies some computationally effective techniques. The contribution by Josef Shinar, one of the most active workers in the field, is devoted to pursuit-evasion games. Methods for solutions are presented and illustrated by examples.

In the continuing search for effective computational techniques for differential games, which could then result not only in effective algorithms for dynamic system design, but also in on-line real time computer control techniques, one of the most potentially effective techniques is the application of differential dynamic programming techniques in differential games. The chapter by Bernt A. S. Järmark, an

internationally recognized leader in developing these techniques, is devoted to differential dynamic programming techniques.

Using stochastic linear control systems with a graduate performance index, it is possible to develop effective control techniques by means of the separation principle, wherein the optimal control is given as a time variable gain matrix premultiplying a minimum variance estimate of the state vector. However, in differential games, this is not possible because of the “intertwining” nature of the vector matrix equations for the two players or opponents. As a result, early attempts at a solution for this problem in differential games produced a formal solution that was composed of an infinite number of integrals—not a very practical solution but one that followed from the nature of the problem. The chapter by B. Mons presents some powerfully effective means for the control of linear stochastic differential games and develops a separation principle. The techniques developed in this chapter are illustrated by examples.

Aran Chompaisal addresses problems of increasing complexity which, in fact, follow from “real-world” situations. In this chapter the author presents and illustrates powerful algorithms for the solution of stochastic differential games with bounds on the control and the state.

One of the most significant applications of differential games is to air combat systems analysis techniques. The contribution by Lynch, a prominent researcher in this area, is devoted to this subject. The following chapter, by Narendra K. Gupta, deals with reachable set methods for the solution of differential games. This is a potentially practical and effective method for the solution of differential games and was first motivated by air combat systems; Gupta’s technique, however, is more widely applicable.

The contribution by P. R. Kumar and T. H. Shiau is devoted to a thorough exposition of zero-sum dynamic games, and it includes some significant illustrative applications. This volume closes with the contribution by Kamran Forouhar, an in-depth analysis of techniques developed for singular differential games and the development of closed-loop techniques.

CONTENTS OF PREVIOUS VOLUMES

Volume 1

On Optimal and Suboptimal Policies in Control Systems, Masanao Aoki

The Pontryagin Maximum Principle and Some of Its Applications, James J. Meditch

Control of Distributed Parameter Systems, P. K. C. Wang

Optimal Control for Systems Described by Difference Equations, Hubert Halkin

An Optimal Control Problem with State Vector Measurement Errors, Peter R. Schultz

On Line Computer Control Techniques and Their Application to Reentry Aerospace Vehicle Control, Francis H. Kishi

Author Index—Subject Index

Volume 2

The Generation of Liapunov Functions, D. G. Schultz

The Application of Dynamic Programming to Satellite Intercept and Rendezvous Problems, F. T. Smith

Synthesis of Adaptive Control Systems by Function Space Methods, H. C. Hsieh

Singular Solutions in Problems of Optimal Control, C. D. Johnson

Several Applications of the Direct Method of Liapunov, Richard Allison Nesbit

Author Index—Subject Index

Volume 3

Guidance and Control of Reentry and Aerospace Vehicles, Thomas L. Gunckel, II

Two-Point Boundary-Value-Problem Techniques, P. Kenneth and R. McGill

The Existence Theory of Optimal Control Systems, W. W. Schmaedeke

Applications of the Theory of Minimum-Normed Operators to Optimum-Control-System Problems, James M. Swiger

Kalman Filtering Techniques, H. W. Sorenson

Application of State-Space Methods to Navigation Problems, Stanley F. Schmidt

Author Index—Subject Index

Volume 4

Algorithms for Sequential Optimization of Control Systems, David Isaacs

Stability of Stochastic Dynamical Systems, Harold J. Kushner

Trajectory Optimization Techniques, Richard E. Kopp and H. Gardner Moyer

Optimum Control of Multidimensional and Multilevel Systems, R. Kulikowski

Optimal Control of Linear Stochastic Systems with Complexity Constraints, Donald E. Johansen

Convergence Properties of the Method of Gradients, Donald E. Johansen

Author Index—Subject Index

Volume 5

Adaptive Optimal Steady State Control of Nonlinear Systems, Allan E. Pearson

An Initial Value Method for Trajectory Optimization Problems, D. K. Scharmack

Determining Reachable Regions and Optimal Controls, Donald R. Snow

Optimal Nonlinear Filtering, J. R. Fischer

Optimal Control of Nuclear Reactor Systems, D. M. Wiberg

On Optimal Control with Bounded State Variables, John McIntyre and Bernard Paiewonsky

Author Index—Subject Index

Volume 6

The Application of Techniques of Artificial Intelligence to Control System Design, Jerry M. Mendel and James J. Zapalac

Controllability and Observability of Linear, Stochastic, Time-Discrete Control Systems, H. W. Sorenson

Multilevel Optimization Techniques with Application to Trajectory Decomposition, Edward James Bauman

Optimal Control Theory Applied to Systems Described by Partial Differential Equations, William L. Brogan

Author Index—Subject Index

Volume 7

Computational Problems in Random and Deterministic Dynamical Systems, Michael M. Connors

Approximate Continuous Nonlinear Minimal-Variance Filtering, Lawrence Schwartz

Computational Methods in Optimal Control Problems, J. A. Payne

The Optimal Control of Systems with Transport Lag, Roger R. Bate

Entropy Analysis of Feedback Control Systems, Henry L. Weidemann

Optimal Control of Linear Distributed Parameter Systems, Elliot I. Axelband

Author Index—Subject Index

Volume 8

Method of Conjugate Gradients for Optimal Control Problems with State Variable Constraint, Thomas S. Fong and C. T. Leondes

Final Value Control System, C. E. Seal and Allen Stubberud

Final Value Control System, Kurt Simon and Allen Stubberud

Discrete Stochastic Differential Games, Kenneth B. Bley and Edwin B. Stear

Optimal Control Applications in Economic Systems, L. F. Buchanan and F. E. Norton

Numerical Solution of Nonlinear Equations and Nonlinear, Two-Point Boundary-Value Problems, A. Miele, S. Naqvi, A. V. Levy, and R. R. Iyer

Advances in Process Control Applications, C. H. Wells and D. A. Wismer

Author Index—Subject Index

Volume 9

Optimal Observer Techniques for Linear Discrete Time Systems, Leslie M. Novak

Application of Sensitivity Constrained Optimal Control to National Economic Policy Formulation, D. L. Erickson and F. E. Norton

Modified Quasilinearization Method for Mathematical Programming Problems and Optimal Control Problems, A. Miele, A. V. Levy, R. R. Iyer, and K. H. Well

Dynamic Decision Theory and Techniques, William R. Osgood and C. T. Leondes

Closed Loop Formulations of Optimal Control Problems for Minimum Sensitivity, Robert N. Crane and Allen R. Stubberud

Author Index—Subject Index

Volume 10

The Evaluation of Suboptimal Strategies Using Quasilinearization, R. G. Graham and C. T. Leondes

Aircraft Symmetric Flight Optimization, Michael Falco and Henry J. Kelley

Aircraft Maneuver Optimization by Reduced-Order Approximation, Henry J. Kelley

Differential Dynamic Programming—A Unified Approach to the Optimization of Dynamic Systems, David Q. Mayne

Estimation of Uncertain Systems, Jack O. Pearson

Application of Modern Control and Optimization Techniques to Transport Systems, Daniel Tabak

Integrated System Identification and Optimization, Yacov Y. Haimes

Author—Subject Index

Volume 11

Fitting Multistage Models to Input/Output Data, Patrick L. Smith

Computer Aided Control Systems Design Techniques, J. A. Page and E. B. Stear

Multilevel Optimization of Multiple Arc Trajectories, Ronald D. Sugar

Nonlinear Smoothing Techniques, John B. Peller

Toward the Synthesis of Solutions of Dynamic Games, L. C. Westphal

A Survey of Soviet Contributions to Control Theory, Alexander Ya. Learner

Volume 12

An Overview of Filtering and Stochastic Control in Dynamic Systems, Harold W. Sorenson

Linear and Nonlinear Filtering Techniques, George T. Schmidt

Concepts and Methods in Stochastic Control, Yaakov Bar-Shalom and Edison Tse

The Innovations Process with Applications to Identification, W. C. Martin and A. R. Stubberud

Discrete-Time Optimal Stochastic Observers, Leslie M. Novak

Discrete Riccati Equations: Alternative Algorithms, Asymptotic Properties, and System Theory Interpretations, Leonard M. Silverman

Theory of Disturbance-Accommodating Controllers, C. D. Johnson

Identification of the Noise Characteristics in a Kalman Filter, H. Whitin Brewer

Adaptive Minimum Variance Estimation in Discrete-Time Linear Systems, R. F. Oshay and A. R. Stubberud

Volume 13

Optimal Operation of Large Scale Power Systems, M. E. El-Hawary and G. S. Christensen

A New Approach to High-Speed Tracked Vehicle Suspension Synthesis, Christian Guenther

Economic Systems, Michael D. Intriligator

Modern Aerospace Systems, Randall V. Gressang and Demetrius Zonars

Optimization of Distributed Parameter Structures under Dynamic Loads, E. J. Haug and T. T. Feng

Optimal Observers for Continuous Time Linear Stochastic Systems, John F. Yocum, Jr.

Optimal Estimation and Control of Elastic Spacecraft, Victor Larson and Peter W. Likins

Stability Analysis of Stochastic Interconnected Systems, A. N. Michel and R. D. Rasmussen

Index

Volume 14

Techniques of Modeling and Model Error Compensation In Linear Regulator Problems, Robert E. Skelton and Peter W. Likins

Dynamic Analysis and Control of Pressurized Water Reactors, T. W. Kerlin

Models for Jet Engine Systems

Part I. Techniques for Jet Engine Systems Modeling, John R. Szuch

Jet Engine Systems Models

Part II. State Space Techniques and Modeling for Control, R. L. DeHoff and W. E. Hall, Jr.

Power Systems Modeling: Estimation and Control Applications, Samir A. Arafah

Index

Volume 15

Optimal Control Policies for the Prescription of Clinical Drugs: Dynamics of Hormone Replacement for Endocrine Deficiency Disorders, Patrick H. Mak and Joseph J. DiStefano III

Method in the Modeling of Chemical Engineering Systems, Rutherford Aris

Modeling, Prediction, and Control of Fish Behavior, Jens G. Balchen

Modeling for Process Control, Morton M. Denn

Water Resource Systems Models, William W-G. Yeh and Leonard Becker

Sensitivity Analysis and Optimization of Large Scale Structures, J. S. Arora and E. J. Haug, Jr.

Advances in Adaptive Filtering, Leonard Chin

Index

Volume 16

Gradient Algorithms for the Optimization of Dynamic Systems, A. Miele

Modal Methods in Optimal Control Syntheses, Arthur E. Bryson, Jr., and W. Earl Hall, Jr.

Linear Time-Invariant Robust Servomechanism Problem: A Self-Contained Exposition,
C. A. Desoer and Y. T. Wang

Parameter Identification of Linear Discrete Stochastic Systems with Time Delays, Edward
C. Wong

UDU^T Covariance Factorization for Kalman Filtering, Catherine L. Thornton and Gerald J.
Bierman

Direct Recursive Estimation of Noise Statistics, Katsumi Ohnishi

Modern Techniques for the Simulation of Large Closed-Loop Systems, William D. Wade

Optimal Low-Order Feedback Controllers for Linear Discrete-Time Systems, John O'Reilly

Index

CONTENTS

CONTRIBUTORS	ix
PREFACE	xi
CONTENTS OF PREVIOUS VOLUMES	xiii

An Overview of Differential Games

Narendra K. Gupta

I. Games	2
II. General Formulation	5
III. Two-Person Games	12
IV. Solution Techniques	14
V. Applications	17
VI. Future of Differential Games	23
References	24

Use of Parameter Optimization Methods to Determine the Existence of Game Theoretic Saddle Points

James E. Rader

I. Introduction	27
II. Solutions to Differential Game Problems	29
III. Polynomial Approximations and Saddle Points	34
IV. Vertical Plane Pursuit-Evasion with Range Payoff	39
V. Conclusions	58
References	60

Solution Techniques for Realistic Pursuit-Evasion Games

Josef Shinar

I. Introduction	64
II. Problem Statement	67
III. Modeling Considerations	71

IV.	Linear Differential Games with Bounded Controls	78
V.	Singular Perturbation Technique for Nonlinear Pursuit–Evasion Games	94
VI.	Concluding Remarks	120
	References	121

Differential Dynamic Programming Techniques in Differential Games

Bernt A. S. Järmark

I.	Introduction	126
II.	The Optimal Control Problem	127
III.	Solving the Optimal Control Problem	152
IV.	Optimal Miss Distance for a Variable Speed Model	138
V.	Sequence of Events	145
VI.	Conclusions	159
	References	160

Stochastic Differential Game Techniques

B. Mons

I.	Introduction	162
II.	General Problem Formulation	165
III.	Solution Concepts and Difficulties	166
IV.	Perfect Information/Noise-Corrupted Game	173
V.	Payoff Relationships	189
VI.	Noise-Corrupted/Noise-Corrupted Games	192
VII.	A Pursuit–Evasion Example	195
VIII.	Conclusions and Remarks	215
	References	216

Algorithms for Differential Games with Bounded Control and State

Aran Chompaissal

I.	Introduction	220
II.	Problem Statement	222
III.	Algorithm for Security-Level Strategies	223
IV.	Linear–Quadratic Differential Game	226
V.	Nonlinear Stochastic Missile Antimissile Problem	233
VI.	Conclusions	241
	References	242

Air Combat Systems Analysis Techniques

Urban H. D. Lynch

I. Differential Games and Military Systems Decisions	244
II. Differential Game/Barrier Theory	249
III. The Simplest Model	263
IV. Limited Pursuer Model	271
V. A Relative Evaluation of Fighter A/C Capability: An Application ..	297
VI. The Barrier and Its Use in Air-to-Air Combat Role Decision	305
VII. Recommendations	318
Appendix	319
References	320

Reachable Set Methods

Narendra K. Gupta

I. Introduction	324
II. Methodology Development	325
III. Applications to Missile Guidance	328
IV. Conclusions	343
References	343

Zero-Sum Dynamic Games

P. R. Kumar and T. H. Shiau

I. Introduction	346
II. Zero-Sum Static Games	346
III. Formulation of Zero-Sum Games	351
IV. The Theory of Zero-Sum Games	353
V. The Tank versus Gun Problem	367
VI. Continuous Models, Discounted Costs, and Other Extensions ...	373
References	376

Singular Differential Game Techniques and Closed-Loop Strategies

Kamran Forouhar

I. Introduction	380
II. Linear-Quadratic Singular Differential Games with Bounds on Control	382
III. Numerical Techniques	389
IV. Closed-Loop Solutions	402
V. Conclusion and Remarks	413
Appendix	415
References	418

INDEX	421
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