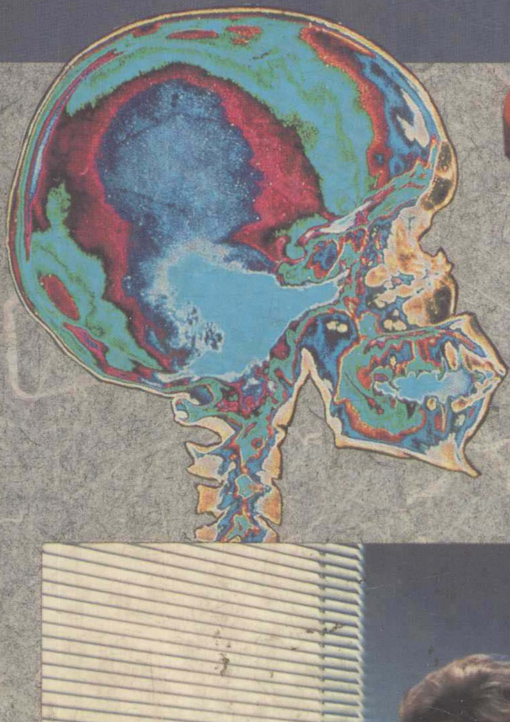
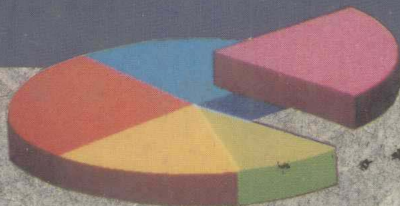
CALCULUS

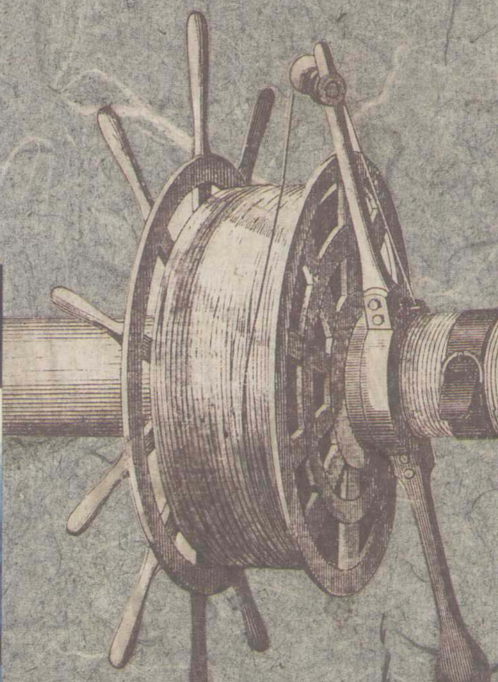
For Business, Economics, and the Social and Life Sciences

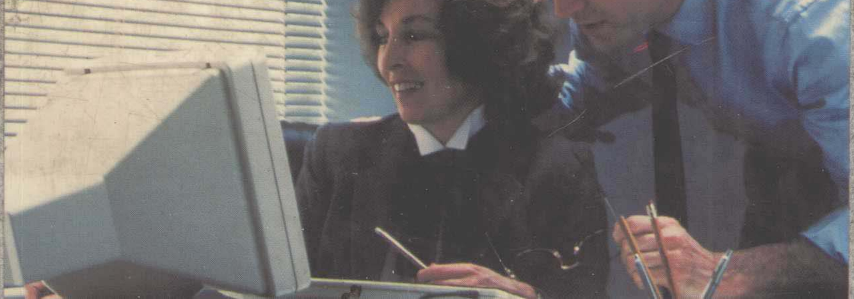
Laurence D. Hoffmann
Gerald L. Bradley

FOURTH EDITION









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About the Authors

Laurence D. Hoffmann came to Claremont McKenna College in 1970 and became Professor of Mathematics in 1983. He is a three-time recipient of the College's Huntoon Award for Excellence in Teaching, an award for "best teacher" voted by the students. He received his B.A. from Brown University and his Ph.D. from the University of Wisconsin. He is also the author of "Applied Calculus" (McGraw-Hill, 1983) and coauthor of "Mathematics with Applications" and "Finite Mathematics with Applications" (both McGraw-Hill, 1979).

Gerald L. Bradley has taught at Claremont McKenna College since 1966. He received his B.S. degree from Harvey Mudd College and his Ph.D. from the California Institute of Technology. His major field of interest is matrix theory, and he is the author of "A Primer of Linear Algebra" (Prentice-Hall, 1975).

Preface

Overview

Objectives

If you are preparing for a career in business, economics, psychology, sociology, architecture, or biology, and if you have taken high school algebra, then this book was written for you. Its primary goal is to teach you the techniques of differential and integral calculus that you are likely to encounter in undergraduate courses in your major and in your subsequent professional activities.

Applications

The text is applications-oriented. Each new concept you learn is applied to a variety of practical situations. The techniques and strategies you will need to solve applied problems are stressed. The applications are drawn from the social, managerial, and life sciences, with special emphasis on business and economics.

Level of Rigor

The exposition is designed to give you a sound, intuitive understanding of the basic concepts without sacrificing mathematical accuracy. Thus, the main results are stated carefully and completely, and whenever possible, explanations are intuitive or geometric.

Problems

You learn mathematics by doing it. Each section in this text is followed by an extensive set of problems. Many involve routine computation and are designed to help you master new techniques. Others ask you to apply the new techniques to practical situations. There is a set of review problems at the end of each chapter. At the back of the book, you will find the answers to the odd-numbered problems and to all the review problems.

Algebra Review

If you need to brush up on your high school algebra, there is an extensive algebra review in the appendix that includes worked examples and practical problems for you to do. You will be advised throughout the text when it might be appropriate to consult this material.

Major Features of the New Edition

This edition retains the straightforward style, intuitive approach, and applications orientation of its predecessors, but contains the following additions and revisions:

1. The treatment of limits and continuity has been expanded and moved from the Appendix to Chapter 1. The intention is to make the limit concept and its notation more readily accessible to students.
2. All topics on definite integration have been combined into a single chapter (Chapter 6). This provides a more coherent treatment of integration and its applications.
3. A new chapter (Chapter 8) on limits at infinity and improper integrals has been included. This chapter includes new material on L'Hôpital's rule and a discussion of indeterminate forms, as well as the application of integration to the study of probability density functions.
4. Numerous new routine exercises have been added throughout the text.

Supplementary Materials

A Student's Solutions Manual, prepared by Stanley Lukawecki at Clemson University, provides detailed solutions to all odd-numbered exercises. Students may purchase this manual through their local bookstore.

An Instructor's Manual provides solutions to the even-numbered exercises, along with sample tests and transparency masters. This complimentary manual is available to adopters.

In addition to the sample tests provided in the Instructor's Manual, further testing is provided in both computerized and print form. The computerized testing provides the instructor with over 1800 test questions from throughout the text. Several test question types are used, including multiple choice, open-ended, matching, true-false, and vocabulary. The testing system enables the teacher to find these questions by section, topic, question type, difficulty level, and other criteria. Instructors may add their own criteria and edit their own questions. The Print Test Bank is a hard copy listing of the questions found in the computerized version.

A computerized study guide is also available. This tutorial provides additional coverage and support for all sections of the text. Students can work additional problems of many different question types, receiving constructive feedback based on their answers. Virtually no computer training is needed for the student to work with this supplement.

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Economist Susan Feigenbaum offered valuable advice during the development of the new material on business and economics that appeared for the first time in the third edition.

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Laurence D. Hoffmann
Gerald L. Bradley

Contents

CHAPTER

1

FUNCTIONS, GRAPHS, AND LIMITS

1 Functions

In many practical situations, the value of one quantity may depend on the value of a second. For example, the consumer demand for beef may depend on its current market price; the amount of air pollution in a metropolitan area may depend on the number of cars on the road; the value of a bottle of wine may depend on its age. Such relationships can often be represented mathematically as **functions.**

Function

> A function is a rule that assigns to each object in a set A, one and only one object in a set B.

This definition is illustrated in Figure 1.1.

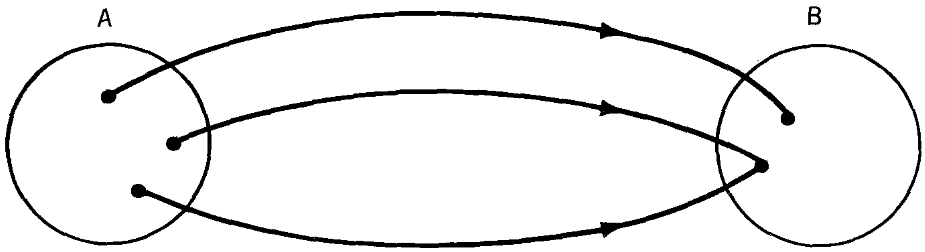

Figure 1.1 A visual representation of a function.

For most of the functions in this book, the sets A and B will be collections of real numbers. You can think of such a function as a rule that assigns "new" numbers to "old" numbers. To be called a function, the rule must have the property that it assigns one and only one "new" number to each "old" number. Here is an example.

EXAMPLE 1.1 According to a certain function, the "new" number is obtained by adding 4 to the square of the "old" number. What number does this function assign to 3?

SOLUTION
The number assigned to 3 is $3^2 + 4$, or 13.

Variables

Often you can write a function compactly by using a mathematical formula. It is traditional to let x denote the old number and y the new number, and write an equation relating x and y. For instance, you can express the function in Example 1.1 by the equation

$$y = x^2 + 4$$

The letters x and y that appear in such an equation are called **variables.** The numerical value of the variable y is determined by that of the variable x. For this reason, y is sometimes referred to as the **dependent variable** and x as the **independent variable.**

Functional Notation

There is an alternative notation for functions that is widely used and somewhat more versatile. A letter such as f is chosen to stand for the function itself, and the value that the function assigns to x is denoted by $f(x)$ instead of y. The symbol $f(x)$ is read "f of x." Using this **functional notation,** you can rewrite Example 1.1 as follows.

EXAMPLE 1.2 Find $f(3)$ if $f(x) = x^2 + 4$.

SOLUTION

$$f(3) = 3^2 + 4 = 13$$

Observe the convenience and simplicity of this notation. In Example 1.2, the compact formula $f(x) = x^2 + 4$ completely defines the function, and the simple equation $f(3) = 13$ indicates that 13 is the number that the function assigns to 3.

The use of functional notation is illustrated further in the following examples. Notice that in Example 1.3, letters other than f and x are used to denote the function and its independent variable.

EXAMPLE 1.3 If $g(t) = (t - 2)^{1/2}$, find (if possible) $g(27)$, $g(5)$, $g(2)$, and $g(1)$.

SOLUTION

Rewrite the function as $g(t) = \sqrt{t - 2}$. (If you need to brush up on fractional powers, you can consult the discussion of exponential notation in the Algebra Review at the back of the book.) Then,

$$g(27) = \sqrt{27 - 2} = \sqrt{25} = 5$$

$$g(5) = \sqrt{5 - 2} = \sqrt{3} \simeq 1.7321$$

and

$$g(2) = \sqrt{2 - 2} = \sqrt{0} = 0$$

However, $g(1)$ is undefined since

$$g(1) = \sqrt{1 - 2} = \sqrt{-1}$$

and negative numbers do not have real square roots.

In the next example, two formulas are needed to define the function.

EXAMPLE 1.4 Find $f\left(-\frac{1}{2}\right)$, $f(1)$, and $f(2)$ if

$$f(x) = \begin{cases} \dfrac{1}{x - 1} & \text{if } x < 1 \\ 3x^2 + 1 & \text{if } x \geq 1 \end{cases}$$

SOLUTION

From the first formula,

$$f\left(-\frac{1}{2}\right) = \frac{1}{-1/2 - 1} = \frac{1}{-3/2} = -\frac{2}{3}$$

and from the second formula,

$$f(1) = 3(1)^2 + 1 = 4 \quad \text{and} \quad f(2) = 3(2)^2 + 1 = 13$$

The next example illustrates how functional notation is used in a practical situation. Notice that to make the algebraic formula easier to interpret, letters suggesting the relevant practical quantities are used for the function and its independent variable. (In this example, the letter C stands for "cost" and q for "quantity" manufactured.)

EXAMPLE 1.5 Suppose the total cost in dollars of manufacturing q units of a certain commodity is given by the function $C(q) = q^3 - 30q^2 + 500q + 200$.

(a) Compute the cost of manufacturing 10 units of the commodity.
(b) Compute the cost of manufacturing the 10th unit of the commodity.

SOLUTION

(a) The cost of manufacturing 10 units is the value of the total cost function when $q = 10$. That is,

$$\begin{aligned}\text{Cost of 10 units} &= C(10)\\ &= (10)^3 - 30(10)^2 + 500(10) + 200\\ &= \$3{,}200\end{aligned}$$

(b) The cost of manufacturing the 10th unit is the difference between the cost of manufacturing 10 units and the cost of manufacturing 9 units. That is,

$$\text{Cost of 10th unit} = C(10) - C(9) = 3{,}200 - 2{,}999 = \$201$$

The Domain of a Function

The set of values of the independent variable for which a function can be evaluated is called the **domain** of the function. For instance, the function $f(x) = x^2 + 4$ in Example 1.2 can be evaluated for any real number x. Thus, the domain of this function is the set of all real numbers. The domain of the function $C(q) = q^3 - 30q^2 + 500q + 200$ in Example 1.5 is also the set of all real numbers [although $C(q)$ represents total cost only for nonnegative values of q]. In the next example are two functions whose domains are restricted for algebraic reasons.

EXAMPLE 1.6 Find the domain of each of the following functions:

(a) $f(x) = \dfrac{1}{x - 3}$

(b) $g(x) = \sqrt{x - 2}$

SOLUTION

(a) Since division by any real number except zero is possible, the only value of x for which $f(x) = \dfrac{1}{x - 3}$ cannot be evaluated is $x = 3$, the value that makes the denominator of f equal to zero. Hence the domain of f consists of all real numbers except 3.

(b) Since negative numbers do not have real square roots, the only values of x for which $g(x) = \sqrt{x - 2}$ can be evaluated are those for which $x - 2$ is nonnegative, that is, for which

$$x - 2 \geq 0 \quad \text{or} \quad x \geq 2$$

That is, the domain of g consists of all real numbers that are greater than or equal to 2.

Composition of Functions

There are many situations in which a quantity is given as a function of one variable which, in turn, can be written as a function of a second variable. By combining the functions in an appropriate way, you can express the original quantity as a function of the second variable. This process is known as the **composition of functions.**

Composition of Functions

The composite function $g[h(x)]$ is the function formed from the two functions $g(u)$ and $h(x)$ by substituting $h(x)$ for u in the formula for $g(u)$.

The situation is illustrated in Figure 1.2.

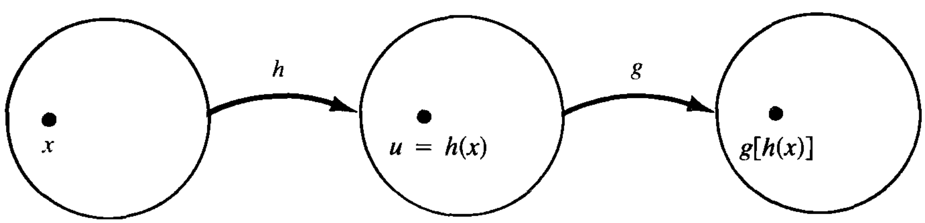

Figure 1.2 The composition of functions.