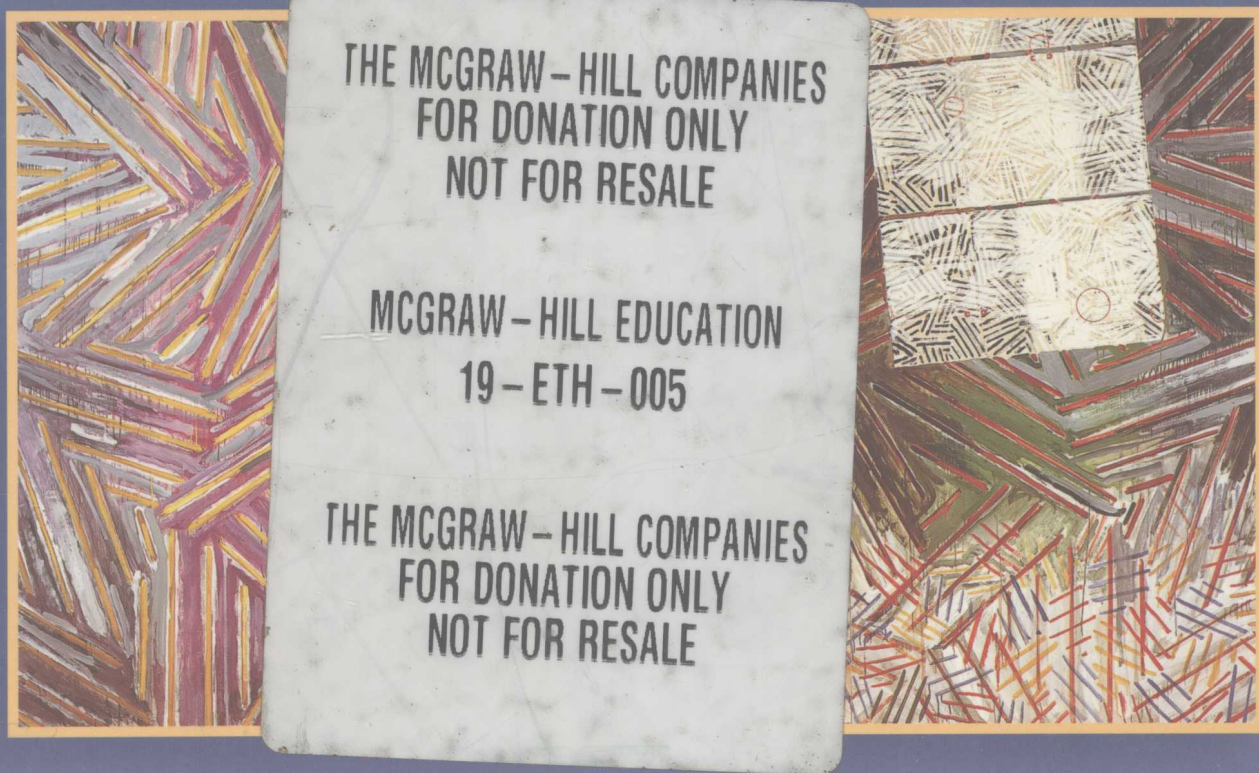


F O U R T H E D I T I O N

Discrete Mathematics and Its Applications



K E N N E T H H . R O S E N

Discrete Mathematics and Its Applications

Fourth Edition

Kenneth H. Rosen
American Telephone and Telegraph Company
AT&T Laboratories Division



**Mc
Graw
Hill** **WCB
McGraw-Hill**

Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis
Bangkok Bogotá Caracas Lisbon London Madrid Mexico City
Milan New Delhi Seoul Singapore Sydney Taipei Toronto

WCB/McGraw-Hill

A Division of The McGraw-Hill Companies

DISCRETE MATHEMATICS AND ITS APPLICATIONS

Copyright © 1999, 1995, 1991, 1988 by American Telephone and Telegraph Company. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

This book is printed on acid-free paper.

9 0 DOW DOW 9 4 3 2

ISBN 0-07-289905-0

Editorial director: Kevin Kane
Publisher: J. P. Lenney
Sponsoring editor: Maggie Rogers
Developmental editor: Nina Kreiden
Marketing manager: Mary Kittell
Production supervisor: Richard deVitto
Supplements coordinator: Louis Swaim
Cover designer: Amanda Kavanagh
Compositor: Publication Services, Inc.
Printer: R. R. Donnelley

Cover painting: *Between the Clock and the Bed*, 1981 by Jasper Johns. Collection of the artist. Photograph by Glenn Stieglman. © Jasper Johns/Licensed by VAGA, New York, NY.

Library of Congress Cataloging-in-Publication Data

Rosen, Kenneth H.
Discrete mathematics and its applications / Kenneth H. Rosen.—
4th ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-07-289905-0 (acid-free paper)
1. Mathematics. 2. Computer science—Mathematics. I. Title.
QA39.2.R654 1999
511—dc21 98-44231
CIP

ABOUT THE AUTHOR

Kenneth H. Rosen is a Distinguished Member of the Technical Staff in the Information Systems and Services Research Laboratory of AT&T Laboratories in Middletown, New Jersey.

Dr. Rosen received his B.S. in Mathematics from the University of Michigan, Ann Arbor (1972), and his Ph.D. in Mathematics from M.I.T. (1976), where he wrote his thesis in the area of number theory under the direction of Harold Stark. Before joining Bell Laboratories in 1982, he held positions at the University of Colorado, Boulder; the Ohio State University, Columbus; and the University of Maine, Orono, where he was an associate professor of mathematics. While working at AT&T Labs, Ken has taught in the evening program in computer science at Monmouth University, teaching courses in discrete mathematics, coding theory, and data security.

Dr. Rosen has published numerous articles in professional journals in the areas of number theory and mathematical modeling. He is the author of the textbooks *Elementary Number Theory and Its Applications*, currently in its third edition, published by Addison-Wesley, and *Discrete Mathematics and Its Applications*, in its fourth edition, published by McGraw-Hill. Both books have been used extensively at hundreds of universities. He is coauthor of *UNIX: The Complete Reference*, which has sold more than 100,000 copies and has been translated into Spanish and German, and *Best UNIX Tips Ever*, translated into Chinese, both published by Osborne McGraw-Hill. Ken is also the editor of the *Handbook of Discrete Mathematics*, a new publication to be published in 1999 by CRC Press, and he is the editor of the CRC series of books in discrete mathematics. Ken is also interested in integrating mathematical software into the educational and professional environments and is working on projects with Waterloo MAPLE software in both these areas.

At Bell Laboratories and now AT&T Laboratories, Dr. Rosen has worked on a wide range of projects, including operations research studies and product line planning for computers and data communications equipment. He has helped plan AT&T's future products and services in the area of multimedia, including video communications, speech recognition, and image networking. He has evaluated new technology for use by AT&T. He has also invented many new services and holds or has submitted many patents. One of his more interesting projects involved helping evaluate technology for the AT&T attraction at EPCOT Center.

PREFACE

In writing this book, I was guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all the mathematical foundations they need for their future studies; I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the fourth edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 400 schools where this book has been successfully used. There are many enhancements in this edition. The ancillary package has been enriched, and a companion Web site provides helpful material, making it easier for students and instructors to achieve their goals.

This text is designed for a one- or two-term introductory discrete mathematics course to be taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only explicit prerequisite.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; more importantly, such a course should teach students how to think mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, algorithmic thinking, and applications and modeling. A successful discrete mathematics course should carefully blend and balance all five themes.

1. *Mathematical Reasoning*: Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the

subsequent discussions of methods of proof. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.

2. *Combinatorial Analysis*: An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems, not on applying formulae.
3. *Discrete Structures*: A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Algorithmic Thinking*: Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.
5. *Applications and Modeling*: Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, botany, zoology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises in the book.

Why a Fourth Edition?

The third edition of this book has been used successfully at over 400 schools in the United States, at dozens of Canadian universities, and at universities in Europe, Asia, and Oceania. Many students and professors like the third edition as it is. Why then, do we need a fourth edition? This is a valid question deserving a careful answer.

First, although the third edition has been extremely effective, many instructors have asked for specific improvements. Many have wanted changes to the text, additional or clarified examples, more exercises of a certain type, or new topics covered. In this new edition I have improved the book by taking into account the numerous suggestions I have received. The changes I have made at the request of users make this a better text.

Second, discrete mathematics is an active subject. There are many new discoveries made every year, and some of these can be reflected in a text. So, I have included discoveries made after the publication of the third edition. (Subsequent discoveries will be included in later printings of this edition whenever possible and noted on the companion Web site.)

Third, since the publication of the third edition, the Internet has become extraordinarily important and useful. In this edition you will find examples and exercises relating applications of discrete mathematics to the structure of the Internet itself. And with this

web

edition there is an extensive Web site that supplements the text in meaningful ways, offering additional material for students and instructors and providing a gateway for learning more about discrete mathematics by providing links to relevant sites on the Web. However, since many people will choose not to use the Web in conjunction with this course, the text includes icons indicating the inclusion of Web links in the annotated Web Guide on the Web site for this book.

The following list highlights some of the changes in this edition that make the book more effective.

NEW TOPIC COVERAGE

- Big-Omega and big-Theta notation are now covered, in addition to big-O notation.
- New topics in probability theory include the variance of a random variable and Chebyshev's inequality. Also, the Monty Hall three-door problem is now discussed in the text.
- The halting problem is now treated, including a proof that it is unsolvable.
- The traveling salesman problem is discussed.

EXPANDED TOPIC COVERAGE

- Additional material on mathematical logic and mathematical reasoning has been added. New examples show how to translate between quantified statements and English. The discussion of rules of inference has been enhanced. In particular, rules of inference for quantified statements are now explicitly covered, and examples illustrating how rules of inference are used have been added.
- Coverage of the floor and ceiling functions has been enhanced.
- Generating functions are now treated in a separate section in the main body of the text, expanding the coverage previously found in the appendix. The focus of this section is to show how generating functions can be used to solve counting problems, solve recurrence relations, and prove combinatorial identities.
- Nonhomogeneous linear recurrence relations with constant coefficients are now discussed in the text, rather than in an exercise set.
- The topic of integer sequences has extended coverage; examples and exercises involving identifying possible formulas for the terms of a sequence from its initial terms have been added.
- New biographies have been added, including those for Peirce, Chebyshev, Knuth, Hardy, Ramanujan, Tukey, Sloane, and Mersenne.

UP-TO-DATE, MODERN EXAMPLES

- Examples have been added at some key points in the text to help explain important concepts that have proved troublesome to students and to make the book more interesting.
- Examples and exercises illustrating the application of discrete mathematics to the protocols and network architecture of the Internet have been added. These additions include counting problems involving Internet addresses and Internet Protocol packets; the topic of Boolean searching, used by Internet search engines; and an example about how spanning trees are used in IP multicasting.

- Material has been added to the text which demonstrates that discrete mathematics is an active subject with many open questions and with new discoveries. For example, Mersenne primes are now covered, including the discoveries of new primes in 1997 and 1998; the range for which the Goldbach conjecture has been verified is discussed; and the variation of the Tower of Hanoi puzzle with four pegs is described.

EXPANDED EXERCISE SETS

- More than 500 new exercises have been added, including both routine and challenging ones, as requested by instructors who used the third edition, as well as exercises based on logical and mathematical puzzles. New blocks of exercises develop key concepts in a series of steps. New exercises ensure that there are both odd- and even-numbered exercises of important exercise types. There are also more exercises that depend on the previous study of calculus; these are explicitly noted as usual and can be easily avoided if so desired.

WEB SUPPORT

web

- A Web site has been developed to supplement the text for both students and instructors. This Web site contains a wide range of features (see page xix), including an annotated Web Guide to relevant sites on the Internet, that is keyed to the text. This guide will be kept current and updated regularly during the life of this edition.
- An icon has been placed at points in the text whenever the Web Guide includes annotated links to Web sites pertinent to the material under discussion. (More than 200 different links are in the guide.) These sites include additional information about concepts and applications, biographies, the latest discoveries, downloadable source code, interactive applets, animated algorithms, and other interesting material.

Special Features

ACCESSIBILITY This text has proven to be easily read and understood by beginning students. There are no mathematical prerequisites beyond college algebra for almost all of this text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

WRITING STYLE The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Notation is introduced and used when appropriate. Care has been taken to balance the mix of notation and words in mathematical statements.

EXTENSIVE CLASSROOM USE This book has been used at over 400 schools, and more than 325 have used it more than once. The feedback from instructors and students at many of the schools has helped make the fourth edition an even more successful teaching tool than previous editions.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. Recursive definitions are explained and used extensively.

FIGURES AND TABLES This text contains more than 550 figures. The figures are designed to illustrate key concepts and steps of proofs. Color has been carefully used in figures to illustrate important points. Whenever possible, tables have been used to summarize key points and illuminate quantitative relationships.

WORKED EXAMPLES Over 650 examples are used to illustrate concepts, relate different topics, and introduce applications. In the examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, data networking, psychology, chemistry, engineering, linguistics, biology, business, and the Internet.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix A.2. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of more than 55 mathematicians and computer scientists are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics. In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text.

KEY TERMS AND RESULTS A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, not every term defined in the chapter.

EXERCISES There are over 3000 exercises in the text. There are many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and many challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded

for level of difficulty. Exercise sets contain special discussions, with exercises, that develop new concepts not covered in the text, permitting students to discover new ideas through their own work.

Exercises that are somewhat more difficult than average are marked with a single star; those that are much more challenging are marked with two stars. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the symbol \square . Answers or outlined solutions to all odd-numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS Each chapter is followed by a set of computer projects. The approximately 150 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises (approximately 100 in total) are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as MAPLE or Mathematica. Many of these exercises give students the opportunity to uncover new facts and ideas through computation. (Some of these exercises are discussed in the companion volume, *Exploring Discrete Mathematics with MAPLE*.)

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie together mathematical concepts and the writing process and help expose students to possible areas for future study. (Suggested references for these projects can be found in the *Student Solutions Guide*.)

APPENDICES There are two appendixes to the text. The first covers exponential and logarithmic functions, reviewing some basic material used heavily in the course; the second specifies the pseudocode used to describe algorithms in this text.

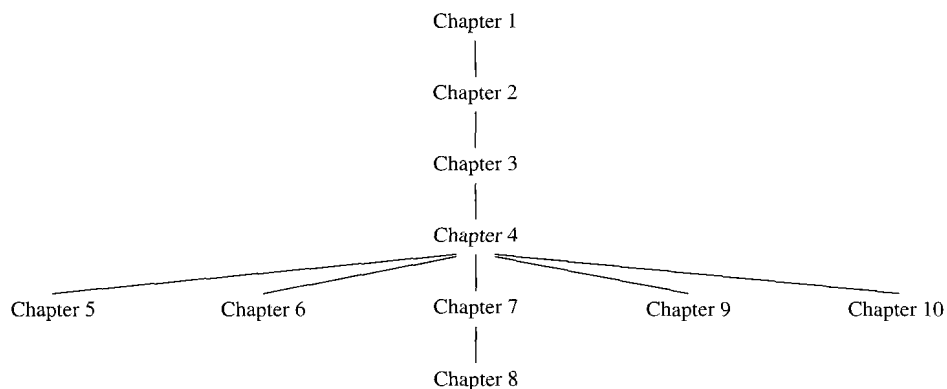
SUGGESTED READING A list of suggested readings for each chapter is provided in a section at the end of the text. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published.

How To Use This Book

This text has been carefully written and constructed to support discrete mathematics courses at several levels and with differing foci. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the instructor. A two-term introductory course could include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections.

<i>Chapter</i>	<i>Core Sections</i>	<i>Optional Computer Science Sections</i>	<i>Optional Mathematics Sections</i>
1	1.1–1.8 (as needed)		
2	2.1–2.3, 2.6 (as needed)	2.4	2.5
3	3.1–3.3	3.4, 3.5	
4	4.1–4.4	4.7	4.5, 4.6
5	5.1, 5.5	5.3	5.2, 5.4, 5.6
6	6.1, 6.3, 6.5	6.2	6.4, 6.6
7	7.1–7.5		7.6–7.8
8	8.1	8.2–8.4	8.5, 8.6
9		9.1–9.4	
10		10.1–10.5	

Instructors using this book can adjust the level of difficulty of their course by either choosing to cover or to omit the more challenging examples at the end of sections, as well as the more challenging exercises. The dependence of chapters on earlier chapters is shown in the following chart.



Ancillaries

STUDENT SOLUTIONS GUIDE This student manual, available separately, contains *full* solutions to all the odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem can be solved in several different ways. Suggested references for the writing projects found at the end of each chapter are also included in this volume. The guide contains a guide to writing proofs and a list of common mistakes students make in discrete mathematics. It also includes sample tests and a sample crib sheet for each chapter, both designed to help students prepare for exams. Students find this guide extremely useful.

INSTRUCTOR'S RESOURCE GUIDE This manual contains full solutions to even-numbered exercises in the text. It also provides suggestions on how to teach the material in each chapter of the book, including the points to stress in each section and how to put the material into perspective. Furthermore, the manual contains a test bank of sample examination questions for each chapter, including some sample tests as well as the solutions to the sample questions. Finally, sample syllabi are presented.

APPLICATIONS OF DISCRETE MATHEMATICS This ancillary is a separate text that can be used either in conjunction with the text or independently. It contains more than 20 chapters (each with its own set of exercises) written by instructors who have used the text. Following a common format similar to that of the text, the chapters in this book can be used as a text for a separate course, for a student seminar, or for a student doing independent study. Subsequent editions of this ancillary are planned that will broaden the range of applications covered. Instructors are invited to submit additional applications for possible inclusion in later versions.

TEST BANK An extensive test bank of more than 1300 questions is available for use on either Windows or Macintosh systems. Instructors can use this software to create their own tests by selecting questions of their choice or by random selection. Instructors can add their own headings and instructions, print scrambled versions of the same test, and edit the existing questions or add their own. A printed version of this test bank, including the questions and their answers, is included in the Instructor's Resource Guide.

EXPLORING DISCRETE MATHEMATICS AND ITS APPLICATIONS WITH MAPLE This ancillary is a separate book designed to help students use the MAPLE computer algebra system to do a wide range of computations in discrete mathematics. For each chapter of this text, this new ancillary includes the following: a description of relevant MAPLE functions and how they are used, MAPLE programs that carry out relevant computations, suggestions and examples showing how MAPLE can be used for the computations and explorations at the end of each chapter, and exercises that can be worked using MAPLE.

Acknowledgments

I would like to thank the many instructors and students at many different schools who have used this book and provided me with their valuable feedback and helpful

suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman and John Michaels for their technical reviews of the fourth edition and their “eagle eyes,” which have helped ensure the accuracy of this book.

I thank the many, many reviewers of the first, second, third, and fourth editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

- | | |
|--|--|
| Eric Allender,
<i>Rutgers University</i> | Laxmi N. Gupta,
<i>Rochester Institute of Technology</i> |
| Stephen Andrilli,
<i>La Salle University</i> | Daniel Gusfield,
<i>University of California at Davis</i> |
| Jack R. Barone,
<i>Baruch College</i> | Xin He,
<i>State University of New York
at Buffalo</i> |
| Alfred E. Borm,
<i>Southwest Texas State University</i> | Arthur M. Hobbs,
<i>Texas A&M University</i> |
| Ken W. Bosworth,
<i>University of Maryland</i> | Donald Hutchison,
<i>Clackamas Community College</i> |
| Lois Brady,
<i>California Polytechnic State
University, San Luis Obispo</i> | Kenneth Johnson,
<i>North Dakota State University</i> |
| Russell Campbell,
<i>University of Northern Iowa</i> | David Jonah,
<i>Wayne State University</i> |
| Kevin Carolan,
<i>Marist College</i> | W. Thomas Kiley,
<i>George Mason University</i> |
| Tim Carroll,
<i>Bloomsburg University</i> | Nancy Kinnersley,
<i>University of Kansas</i> |
| Kit C. Chan,
<i>Bowling Green State University</i> | Gary Klatt,
<i>University of Wisconsin</i> |
| Allan C. Cochran,
<i>University of Arkansas</i> | Nicholas Krier,
<i>Colorado State University</i> |
| Peter Collinge,
<i>Monroe Community College</i> | Lawrence S. Kroll,
<i>San Francisco State University</i> |
| Ron Davis,
<i>Millersville University</i> | Robert Lavelle,
<i>Iona College</i> |
| Nachum Dershowitz,
<i>University of Illinois,
Urbana–Champaign</i> | Yi-Hsin Liu,
<i>University of Nebraska
at Omaha</i> |
| Thomas Dowling,
<i>Ohio State University</i> | George Luger,
<i>University of New Mexico</i> |
| Patrick C. Fischer,
<i>Vanderbilt University</i> | David S. McAllister,
<i>North Carolina State University</i> |
| Jane Fritz,
<i>University of New Brunswick</i> | Robert McGuigan,
<i>Westfield State College</i> |
| Ladnor Geissinger,
<i>University of North Carolina</i> | Michael Maller,
<i>Queens College</i> |
| Paul Gormley,
<i>Villanova University</i> | Ernie Manes,
<i>University of Massachusetts</i> |
| Jerrold Grossman,
<i>Oakland University</i> | Francis Masat,
<i>Glassboro State College</i> |

- J. M. Metzger,
University of North Dakota
- John G. Michaels,
SUNY Brockport
- D. R. Morrison,
University of New Mexico
- Ho Kuen Ng,
San Jose State University
- Timothy S. Norfolk, Sr.,
University of Akron
- Jeffrey Nunemacher,
Ohio Wesleyan University
- Charles Parry,
*Virginia Polytechnic Institute
and State University*
- Thomas W. Parsons,
Hofstra University
- Mary K. Prisco,
*University of Wisconsin–
Green Bay*
- Harold Reiter,
University of North Carolina
- Adrian Riskin,
Northern Arizona State University
- Amy L. Rocha,
San Jose State University
- Janet Roll,
University of Findlay
- Alyssa Sankey,
Slippery Rock University
- Dinesh Sarvate,
College of Charleston
- Steven R. Seidel,
Michigan Technological University
- Douglas Shier,
Clemson University
- Hunter S. Snevily,
University of Idaho
- Daniel Somerville,
University of Massachusetts, Boston
- Bharti Temkin,
Texas Tech University
- Wallace Terwilligen,
Bowling Green State University
- Philip D. Tiu,
Oglethorpe University
- Lisa Townsley-Kulich,
Illinois Benedictine College
- George Trapp,
West Virginia University
- David S. Tucker,
Midwestern State University
- Thomas Upson,
Rochester Institute of Technology
- Roman Voronka,
New Jersey Institute of Technology
- James Walker,
University of South Carolina
- Anthony S. Wojcik,
Michigan State University

I would like to thank the staff of the McGraw-Hill Higher Education Group for their support of this project. In particular, thanks go to J. P. Lenney, Publisher, for his overall support; to Maggie Rogers, Sponsoring Editor, for her continued interest and enthusiasm; to Nina Kreiden, Developmental Editor, for her dedication and diligence; and to Amy Upgren, Editorial Assistant, for her valuable help. I would also like to thank the original editor, Wayne Yuhasz, whose insights and skills helped ensure the book's success, as well as Jack Shira, Tom Casson, and Mike Morales, who helped plan and initiate work on the fourth edition.

To the staff involved in the production of this fourth edition, I offer my thanks to Rich DeVitto, Production Supervisor; Suzanne Montazer, Designer; Ronald Tigges, Web Developer; Louis Swaim, Supplements Coordinator; and Mary Kittell, Marketing Manager.

As always, I am grateful for the support given to me by my management at AT&T Laboratories. They have provided an environment that has helped me develop professionally and generously given me the resources needed to make this book a success.

Kenneth H. Rosen

THE COMPANION WEB SITE

An extensive companion Web site has been developed and will be maintained and improved on a continuing basis. The URL for this site is <http://www.mhhe.com/rosen>. Following this URL takes you to a page that provides access to five different sections of the Web site:

- About the Book
- Instructor Resources
- Student Resources
- Web Guide for Discrete Mathematics
- Supplementary Resources

Each section will be in place with the publication of this new edition, although additional material will be added later.

The *About the Book* section includes basic information about the textbook and its ancillaries. It also contains an errata list and an e-mail address for the submission of errata and suggestions.

The *Instructor Resources* section is a secure portion of the Web site. It contains valuable tools and resources to supplement both the text and the discrete mathematics teaching experience.

The *Student Resources* section contains helpful reference and supplemental material to enhance students' learning experience.

The *Web Guide for Discrete Mathematics* section includes annotated links to relevant Web sites anchored to the Web icons in the text. (Links are included wherever the icon is found.) The links in this guide can be used to access sites that provide biographies, additional material on topics covered in the text, information on the latest discoveries, animated algorithms, downloadable source code, and so on.

The *Supplementary Resources* section, intended for use by both students and instructors, includes supplementary educational material, organized by chapter. This material is designed to clarify and expand on material in the text.

TO THE STUDENT

What is *discrete mathematics*? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here *discrete* means consisting of distinct or unconnected elements.) The kind of problems solved using discrete mathematics include:

- How many ways are there to choose a valid password on a computer system?
- What is the probability of winning a lottery?
- Is there a link between two computers in a network?
- What is the shortest path between two cities using a transportation system?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can a circuit that adds two integers be designed?
- How many valid Internet addresses are there?

You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite (or countable) sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity, that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills.

Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses, including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math. One student has sent me an electronic mail message to tell me that she used the contents of this book in every computer science course she took!

Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject).

Also, discrete mathematics contains the necessary mathematical background for solving problems in operations research (including many discrete optimization techniques), chemistry, engineering, biology, and so on. In the text, we will study applications to some of these areas.

I would like to offer some helpful advice to students about how best to learn discrete mathematics. You will learn the most by working exercises. I suggest you do as many as you possibly can, including both the exercises at the end of each section of the text and the supplementary exercises at the end of each chapter. Always try exercises yourself before consulting the answers at the end of the book or in the *Student Solutions Guide*. Only after you have put together a solution, or you find yourself at an impasse, should you look up the suggested solution. At that point you will find the discussions in the *Student Solutions Guide* most helpful. When doing exercises, remember that the more difficult ones are marked as described in the following table.

Key to the Exercises	
No marking	A routine exercise
*	A difficult exercise
**	An extremely challenging exercise
☞	An exercise containing a result used in the book
(Calculus required)	An exercise whose solution requires the use of limits or concepts from differential or integral calculus

Finally, I encourage you to explore discrete mathematics beyond what you see in the book. An excellent starting place is the Web Guide for Discrete Mathematics that can be found on the Web site for this book. The URL is <http://www.mhhe.com/rosen>.

Kenneth H. Rosen