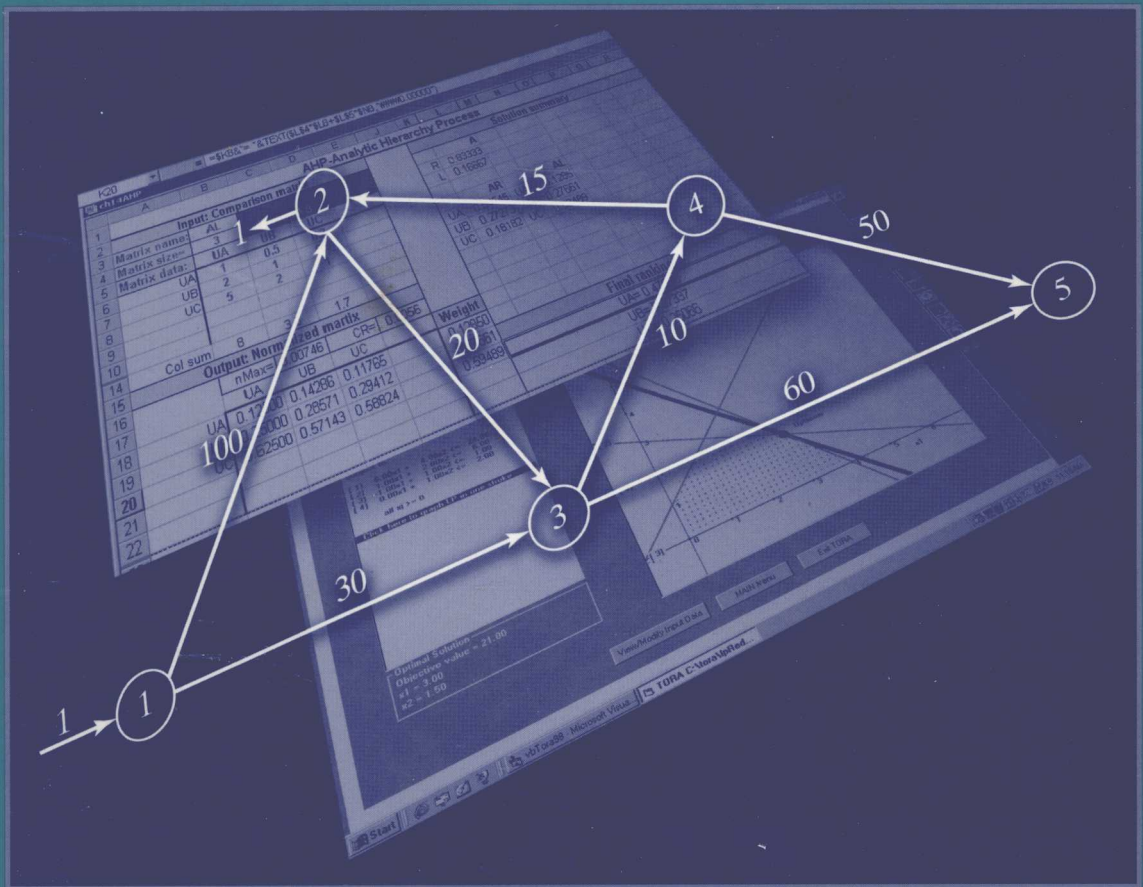


OPERATIONS RESEARCH

AN INTRODUCTION
SEVENTH EDITION



HAMDY A. TAHA



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Seventh Edition

Hamdy A. Taha

University of Arkansas, Fayetteville



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To Karen

Los ríos no llevan agua,
el sol las fuentes secó...
¡Yo sé donde hay una fuente
que no ha de secar el sol!
La fuente que no se agota
es mi propio corazón...

—*V. Ruiz Aguilera (1862)*

Preface

It is gratifying that, for over 30 years, hundreds of thousands of students worldwide have been introduced to operations research through the various editions of this book. This success carries with it the responsibility of meeting the needs of future generations of students. The seventh edition is the result of a dedicated effort to live up to this responsibility.

The main thrust of the seventh edition is the extensive software support used throughout the book:

1. Windows-based TORA.
2. Excel spreadsheet templates.
3. Examples of LINGO and AMPL applications.

The TORA software offers modules for matrix inversion, solution of simultaneous linear equations, linear programming, transportation models, network models, integer programming, queuing models, project planning with CPM and PERT, and game theory. TORA can be executed in automated or tutorial mode. The automated mode reports the final solution of the problem, usually in the standard format followed in commercial packages. The tutorial mode is a unique feature that provides immediate feedback to test the reader's understanding of the computational details of each algorithm. As with its DOS predecessor, the different screens in TORA are accessed in a logical and unambiguous manner, essentially eliminating the need for a user's manual.

Excel spreadsheet templates complement TORA's modules. These templates include linear programming, dynamic programming, analytical hierarchy process (AHP), inventory models, histogramming of raw data, decision theory, Poisson queues, P-K formula, simulation, and nonlinear models. Some of the templates are direct spreadsheets. Others use Excel Solver or VBA macros. Regardless of the design, all templates offer the unique feature of being equipped with an input data section that allows solving different problems without the need to modify the formulas or the layout of the spreadsheet. In this manner, the user can experiment with, test, and compare different sets of input data in a convenient manner. Where possible, the formulas and the layout of the spreadsheets have been protected to minimize the chance of inadvertently corrupting them.

The book includes examples of the commercial packages LINGO and AMPL for solving linear programming problems. The objective is to familiarize the reader with how very large mathematical programming models are solved in practice.

TORA software and the Excel spreadsheets are integrated into the text in a manner that facilitates introducing and testing concepts that otherwise could not be presented effectively. From my personal experience, I have found TORA's tutorial module and Excel spreadsheets to be highly effective in classroom presentations. Many concepts can be demonstrated instantly, simply by changing the data of the problem. To

cite a few examples, TORA can be used to demonstrate the bizarre behavior of the branch-and-bound algorithm by applying it to a (small) integer programming problem, where the solution is found in nine iterations but its optimality verified in more than 25,000 iterations. Without the software and the special design of TORA, it would be impossible to demonstrate this situation in an effective manner. Another example is the unique design of the dynamic programming and the AHP spreadsheets, where the user interactive input is designed to enhance effective understanding of the details of these two topics. A third example deals with explaining the congruential method for generating 0-1 pseudo-random numbers. With the spreadsheet, one can instantly demonstrate the effect of selecting the seed (and the parameters) on the “quality” of the generator, particularly with regard to the cycle length of the random number sequence and, hence, warn the student about the danger of a “causal” implementation of the congruential method within a simulation model.

In addition to the software support in the book, all chapters have been streamlined (many rewritten) to present the material in a concise manner. New material includes a new introduction to operations research (Chapter 1); the generalized simplex method (Chapter 4); representation of all network models, including CPM, as linear programs (Chapter 6); PERT networks (Chapter 6); solution of the traveling sales Person problem (Chapter 9); and the golden section method (Chapter 21).

As in the sixth edition, the book is organized into three parts: deterministic models, probabilistic models, and nonlinear models. Appendices A through D include a review of matrix algebra, a TORA primer (though TORA’s design makes a user’s manual unnecessary), basic statistical tables, and partial answers to selected problems.

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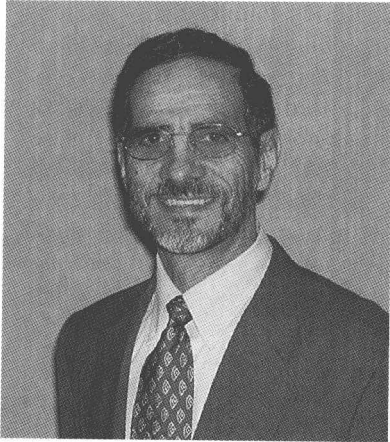
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Hamdy A. Taha is a University Professor of Industrial Engineering with the University of Arkansas, where he teaches and conducts research in operations research and simulation. He is the author of three other books on integer programming and simulation, and his works have been translated into Chinese, Korean, Spanish, Japanese, Russian, Turkish, and Indonesian. He is also the author of several book chapters. His articles have appeared in *Management Science*, *Operations Research*, and *Interfaces* [Institute for Operations Research and Management Science], *Naval Research Logistics* [John Wiley & Sons], the *European Journal of Operations Research* [International Federation of Operations Research Societies] and the *AIIE Transactions*.

Professor Taha was named a Senior Fulbright Scholar to Carlos III University, Madrid, Spain. He received an Alumni Award for excellence in research and The Nadine Baum Faculty Teaching Award, both from the University of Arkansas, and numerous other research and teaching awards from the College of Engineering, University of Arkansas. He is fluent in three languages and has held positions in Mexico and the Middle East.

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CHAPTER 1

What Is Operations Research?

The first formal activities of operations research (OR) were initiated in England during World War II when a team of British scientists set out to make decisions regarding the best utilization of war material. Following the end of the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sector. Today, OR is a dominant and indispensable decision-making tool.

A cornerstone of OR is mathematical modeling. Though the solution of the mathematical model provides a basis for making a decision, intangible (unquantifiable) factors (such as human behavior) must be accounted for before a final decision can be reached.

1.1 OPERATIONS RESEARCH MODELS

Imagine that you have a 5-week business commitment between Fayetteville (FYV) and Denver (DEN). You fly out of Fayetteville on Monday and return on Wednesday. A regular round-trip ticket costs \$400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

We can look at the situation as a decision-making problem whose solution requires identifying three components.

1. What are the decision **alternatives**?
2. Under what **restrictions** is the decision made?
3. What is an appropriate **objective criterion** for evaluating the alternatives?

Three alternatives are considered:

1. Buy five regular FYV-DEN-FYV.
2. Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.