INTRODUCTION TO

CLASSICAL MECHANICS

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Preface

This text is written to present a reasonably complete account of classical mechanics at an intermediate undergraduate level. The text affords maximum flexibility in the selection and arrangement of topics for a two-semester, 3 credit-hour course at a sophomore or junior level. But with proper selection and omission of material, it may be used for a one-semester course. The first chapter is a review of the basic concepts of mechanics, which includes Newton's laws of motion and gravitation and their application to a few selected examples. Chapters 2 through 10 may be covered in the first semester, while the remaining six chapters may be covered in the second semester. For a one-semester course most of the first twelve chapters (with material deleted equivalent to two chapters) may covered.

Students with adequate preparation in general physics and calculus are ready to start this course. Mathematical topics are presented as needed, such as differential equations (Chapter 3), Fourier series (Chapter 4), vector algebra and matrix transformations (Chapter 5), and tensor analysis (Chapter 13). Most of Chapter 5 includes a review of vector analysis. Average students need not go through most of this material, but they may use it as a convenient reference for other chapters.

Mechanics is the foundation of pure and applied sciences. Its principles apply to a vast range and variety of physical systems. I have presented this text to steadily take students who have had introductory mechanics in general physics to an intermediate level mechanics, which will give them a strong basis for their future work in applied and pure sciences, especially advanced physics. Attention has been paid to the following topics of modern interest: (a) nonlinear oscillators (Chapter 4); (b) central force motion (Chapter 7), which includes the (i) capture of comets, (ii) satellite orbits and maneuvers, (iii) stability of circular orbits, and (iv) interplanetary transfer orbits; (c) collisions in CMCS, which are discussed in detail (Chapter 8); (d) horizontal wind circulation (weather systems) (Chapter 11); and the relations between conservation laws and symmetry princples (Chapter 12).

If one is to fully appreciate mechanics (or physics in general), one must learn to solve problems. It is not necessary to solve the most difficult problems; solving even simple problems increases understanding of basic concepts. One difficulty most students face in mechanics is that, after reading a given chapter, they find it hard to attempt the problems at the end of the chapter. Even an average student, if exposed to solved examples, with solutions that explain the basic principles and mathematical techniques, can find problem solving both interesting and rewarding. To overcome this difficulty, I have included about 60 worked out examples, which are presented throughout the text. Furthermore, the presence of solved examples saves a great deal of class time and allows the class to progress at a good pace. Each example is followed by an exercise, and the student should do these before attempting the problems at the end of the chapter. I have included a generous sampling of problems of varying degrees of difficulty.

At the end of each chapter there is a list of Suggestions for Further Reading. Most of the references are for the material discussed in the chapter. A few references are for the prerequisite preparation material, while those references marked with asterisks are of an advanced nature. Furthermore, this list contains most of the references used in writing this textbook, and hence it serves as an acknowledgment of my debt to these authors.

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1

Introduction to Newtonian Mechanics

1.1. INTRODUCTION

Mechanics is one of the oldest and most familiar branches of physics. It deals with bodies at rest and in motion and the conditions of rest and motion when bodies are under the influence of internal and external forces. The laws of mechanics apply to a whole range of objects, both microscopic to macroscopic, such as the motion of electrons in atoms and that of planets in space or even to the galaxies in distant parts of the universe.

Mechanics does not explain why bodies move; it simply shows how a body will move in a given situation and how to describe such motion. The study of mechanics may be divided into two parts: kinematics and dynamics. *Kinematics* is concerned with a purely geometrical description of the motion (or trajectories) of objects, disregarding the forces producing the motion. It deals with concepts and the interrelation between position, velocity, acceleration, and time. *Dynamics* is concerned with the forces that produce changes in motion or changes in other properties, such as the shape and size of objects. This leads us to the concepts of force and mass and the laws that govern the motion of objects. A special case is *statics*, which deals with bodies at rest under the influence of external forces.

Although mechanics had its beginning in antiquity, a significant impetus was given to the thought process involved in mechanics during Aristotle's time. However, it was not until the seventeenth century A.D. that the science of mechanics was truly founded by Galileo, Huygens, and Newton. They showed that objects move according to certain rules, and these rules were stated in the form of laws of motion. Classical or Newtonian mechanics essentially is the study of the consequences of the laws of motion as formulated by Newton in his *Philosophiae Naturalis Principia Mathematica* (the *Principia*) published in 1686.

Although Newton's laws provide a direct and simple approach to the subject of classical mechanics, there are a number of other ways of formulating the principles of classical mechanics. Among these, the two most significant approaches are the formulations of Lagrange and Hamilton. These two approaches take *energy* rather than force as the fundamental concept. In more than half of this text, we will use the classical approach of Newton, while in the later part of the text we will introduce Lagrange and Hamilton formulations.

Until the beginning of the present century, Newton's laws were completely applicable to all well-known situations. The difficulties arose when these laws were applied to certain definite situations: (a) to very fast moving objects (objects moving with speeds approaching the speed of light) and (b) to objects of microscopic size such as electrons in atoms. These difficulties led to modifications in the laws of Newtonian mechanics: (a) to the formulation of the *special theory of relativity* for objects moving with high speeds, and (b) to the formulation of *quantum mechanics* for objects of microscopic size. The failure of classical mechanics in these situations is the result of inadequacies in classical concepts of space and time as discussed briefly in Chapter 16, Special Theory of Relativity.

Before we start an in-depth study of mechanics, we devote this chapter to summarizing briefly a few essential concepts of interest from introductory mechanics. We especially emphasize the importance of the role of Newton's laws of motion.

1.2. UNITS AND DIMENSIONS

Measurements in physics involve such quantities as velocity, force, energy, temperature, electric current, magnetic field, and many others. The most surprising aspect is that all these quantities can be expressed in terms of a few basic quantities, such as length, mass, and time. These three quantities are called *fundamental* or *basic quantities* (*base units*); all others that are expressed in terms of these are called *derived quantities*.

Three Basic Standards: Length, Mass, and Time

Three different sets of units are in use. The most prevalent is that in which length is measured in *meters*, mass in *kilograms*, and time in *seconds*, hence the name *MKS system* (or *metric system*).

Standard of Length: The Meter. The meter has been defined as the distance between the two marks on the ends of a platinum–iridium alloy metal bar kept in a temperature-controlled vault at the International Bureau of Weights and Measures in Sèvres, near Paris, France. In 1960, by international agreement, the General Conference on Weights and Measures changed the standard of length to an atomic constant by the following procedure. A glass tube is filled with krypton gas in which an electrical discharge is maintained. The standard *meter* is defined to be equal to exactly 1,650,763.73 wavelengths of orange-red light emitted in a vacuum from krypton-86 atoms. To improve the accuracy still further, a meter was redefined in 1983 as equal to a distance traveled by light in vacuum in a time interval of 1/299,792,458 of a second.

Standard of Time: The Second. In the past, the spinning motion of the Earth about its axis, as well as its orbital motion about the Sun, have been used to define a second. Thus, a second is defined to be 1/86,400 of a mean solar day. In October 1967, the time standard was redefined in terms of an atomic clock, which makes use of the periodic atomic vibrations of certain atoms. According to the cesium clock, a *second* is defined to be exactly equal to the time interval of 9,192,631,770 vibrations of radiation from cesium-133. This method has an accuracy of 1 part in 10^{11}. It is possible that two cesium clocks running over a period of 5000 years will differ by only 1 second.

Standard of Mass: The Kilogram. A platinum–iridium cylinder is carefully stored in a repository at the International Bureau of Weights and Measures. The mass of the cylinder is defined to be exactly equal to a *kilogram*. This is the only base unit still defined by an artifact. The basic aim of scientists has been to define the three basic standards in such a way that they are accurately and easily reproducible in any laboratory.

Different Systems of Units

Systeme International. The International System of Units, abbreviated SI after the French *Systeme International*, is the modern version of the metric system established by international agreement. For convenience it uses seven base units:

1. Length, in meters (m)
2. Mass, in kilograms (kg)
3. Time, in seconds (s)
4. Electric current, in amperes (A)
5. Temperature, in kelvins (K)
6. Amount of substance, in moles (mol)
7. Luminous intensity, in candelas (cd)

The SI also uses two supplementary units:

1. Plane angle, in radians (rad)
2. Solid angle, in steradians (sr)

The CGS or Gaussian System. In this system the unit of length is the *centimeter* ($=10^{-2}$ m), the unit of mass is the *gram* ($=10^{-3}$ kg), and the unit of time is the *second*.

The British System. In this system the unit of length is the *foot* and the unit of time is the *second*. This system does not use mass as a basic unit; instead, *force* is used, the unit of which is the *pound* (lb). The unit of mass derived from the pound is called the *slug* ($=32.17$ lb mass). The unit of temperature in the British system is the *degree Fahrenheit*.

Dimensions

Most physical quantities may be expressed in terms of length L, mass M, and time T, where L, M, and T are called dimensions. A quantity expressed as $L^a M^b T^c$

means that its length dimension is raised to the power a, its mass dimension is raised to the power b, and its time dimension is raised to the power c. Thus the dimensions of volume are L^3, that of acceleration are LT^{-2}, and that of force are MLT^{-2}.

To add or subtract two quantities in physics, they must have the same dimensions. Similarly, no matter what system of units is used, all mathematical relations and equations must be dimensionally correct. That is, the quantities on both sides of the equations must have the same dimensions. For example, in the equation $x = v_0 t + \frac{1}{2}at^2$, x has dimensions of L, $v_0 t$ has dimensions of $(L/T)T = L$, and $\frac{1}{2}at^2$ has dimensions of $\frac{1}{2}(L/T^2)(T^2) = L$. Thus dimensional analysis may be used to (1) check the correctness of the form of the equation, that is, every term in the equation must have the same dimensions, (2) to check an answer computed from an equation for plausibility in a given situation, and (3) to arrive at a formula if we know the dependence of a certain quantity on other physical quantities.

EXAMPLE 1.1: The magnitude of the radial acceleration a_R is a function of the magnitude of the velocity of the object and the radius R of the curve. By the method of dimensional analysis, find an expression for a_R.

We are given

$$a_R = f(v, R) \tag{i}$$

that is,

$$a_R = v^a R^b \tag{ii}$$

Substituting the dimensions

$$LT^{-2} = \left(\frac{L}{T}\right)^a (L)^b = L^{a+b}T^{-a} \tag{iii}$$

and comparing the two sides,

$$a + b = 1 \qquad \text{and} \qquad -a = -2$$

which gives $a = 2$ and $b = -1$, lead to the following expression for radial acceleration:

$$a_R = \frac{v^2}{R} \tag{iv}$$

EXERCISE 1.1: The time period T of a simple pendulum depends only on its length l and the acceleration due to gravity g. Find the expression for the time period by the method of dimensional analysis.

1.3. NEWTON'S LAWS AND INERTIAL SYSTEMS

Newton's laws may be stated in a brief and concise form as below:

Newton's first law: *Every object continues in its state of rest or uniform motion in a straight line unless a net external force acts on it to change that state.*

Newton's second law: *The rate of change of momentum of an object is directly proportional to the force applied and takes place in the direction of the force.*

Newton's third law: *To every action there is always an equal and opposite reaction; that is, whenever a body exerts a certain force on a second body, the second body exerts an equal and opposite force on the first.*

These statements do look simple; but that is deceptive. Newton's laws are the results of a combination of definitions, experimental observations from nature, and many intuitive concepts. We cannot do justice to these concepts in a short space here, but we will try to expand our thinking horizon by discussing these statements further in some detail.

The motion of objects in our immediate surroundings is complicated by ever present frictional and gravitational forces. Let us consider an isolated object that is moving with a constant (or uniform) velocity in space. Being an isolated object implies that it is far away from any surrounding objects so that it does not interact with them; hence no net force (gravitational or otherwise) acts on it. To describe the motion of the object, we must draw a coordinate system with respect to which the object moves with uniform velocity. Such a coordinate system is called an *inertial system*. The essence of Newton's first law is that it is always possible to find a coordinate system with respect to which an isolated body moves with uniform velocity, that is, *Newton's first law asserts the existence of inertial systems.*

Newton's second law deals with such matters as what happens when there is an interaction between objects? How do you represent interaction? And still further, what is inertia and how do we measure this property of an object? As we know, *inertia* is a property of a body that determines its resistance to motion when that body interacts with another body. The quantitative measure of *inertia* is called *mass*, as we explore now.

Consider two bodies that are completely isolated from the surroundings but interact with one another. The interaction between these objects may result from being connected by means of a rubber band or a spring. The interaction results in acceleration of the bodies. Such accelerations may be measured by stretching the bodies apart by the same amount and then measuring the resultant accelerations. All possible measurements show that the accelerations of these two bodies are always in opposite directions and that the ratio of the accelerations is constant. That is,

$$\frac{a_A}{a_B} = -K_{BA} \tag{1.1}$$

where K_{BA} is the measure of the relative inertia of body B with respect to body A. Equation (1.1) also implies that

$$K_{BA} = -\frac{a_A}{a_B} = \frac{1}{-(a_B/a_A)} = \frac{1}{K_{AB}} \tag{1.2}$$

where K_{AB} is the measure of the relative inertia of body A with respect to body B. That is,

$$K_{BA} = \frac{1}{K_{AB}} \tag{1.3}$$

Since K_{BA} is a measure of a ratio, we may define

$$K_{BA} = \frac{m_B}{m_A} \tag{1.4}$$

where m_A and m_B are called the masses (or the inertial masses) of body A and body B, respectively. The ratio m_B/m_A must be independent of units. The two objects always have a unique mass ratio, m_B/m_A, no matter how the interaction is applied. This definition of mass is an operational definition of mass. By combining Eqs. (1.1) and (1.4), we obtain

$$\frac{a_A}{a_B} = -\frac{m_B}{m_A}$$

or

$$m_A a_A = -m_B a_B \tag{1.6}$$

Thus the effect of interaction is that the product of mass and acceleration is constant and denotes the *change in motion*. This product is called *force* and it represents interaction. Thus we may say that the force F_A acting on A due to interaction with B is

$$F_A = m_A a_A \tag{1.7}$$

while the force F_B acting on B due to interaction with A is

$$F_B = m_B a_B \tag{1.8}$$

Thus, in general, using vector notation, we may write

$$\mathbf{F} = m\mathbf{a} \tag{1.9}$$

This equation is the definition of force and holds good only in inertial systems. It is important to keep in mind that the force $\mathbf{F}$ arises because of an interaction or simply stands for an interaction. No acceleration could ever be produced without an interaction.

Let us now proceed to obtain the definition of force starting directly with the statement of Newton's second law given previously. Suppose an object of mass m is moving with velocity $\mathbf{v}$ so that the linear momentum $\mathbf{p}$ is defined as

$$\mathbf{p} = m\mathbf{v} \tag{1.10}$$

According to Newton's second law, the rate of change of momentum is defined as force $\mathbf{F}$; that is,

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \tag{1.11}$$

This equation takes a much simpler form if mass m remains constant at all speeds. If $\mathbf{v}$ is very small as compared to the speed of light c ($=3 \times 10^8$ m/s), the variation in mass m is negligible. Hence, we may write