CONTEMPORARY MATHEMATICS

Random Matrices and Their Applications

Proceedings of a Summer Research Conference held June 17–23, 1984

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Random Matrices and Their Applications

Proceedings of the AMS-IMS-SIAM Joint Summer Research Conference held June 17-23, 1984, with support from the National Science Foundation

Joel E. Cohen, Harry Kesten and Charles M. Newman, Editors

AMERICAN MATHEMATICAL SOCIETY
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Random Matrices and Their Applications

In memory of Mark Kac

1914-1984

Mathematician, teacher, raconteur, friend

par excellence

PREFACE

The purpose of this volume is to describe some recent and not so recent discoveries about random matrices and their applications to a wide audience of scientists and pure and applied mathematicians. The authors were requested to provide exposition, not detailed reports of new research; to give simple, concrete and vivid examples of significant phenomena, rather than abstract but inaccessible theorems that might cover all possibilities. The papers provide heuristic interpretations of results, sketch the ideas of proofs, and point to or comment on the original reports of research. The papers draw on or describe applications in the fields of probability theory, functional analysis, group theory, mathematical physics, statistics, computer science, population biology, and number theory. All papers were refereed for expository and technical quality. The editors are grateful for the co-operative efforts of the authors and the referees.

The papers in this volume fall into two broad groups: those concerned with products of random matrices and those in which the multiplication of random matrices plays no role. Interest in random matrices per se has a fairly long tradition in statistics and in physics (e.g. Mehta, 1967) [citations are given in the bibliography at the end of the book]. Interest in products of random matrices can be dated from the first published paper on the topic by Bellman (1956). The results about the asymptotic behavior of products of random matrices discovered by Furstenberg and Kesten (1960) led to much of the interest in products of random matrices over the last twenty-five years, as many of the papers on products of random matrices in this volume demonstrate.

Most of these papers were presented at a meeting on random matrices and their applications, organized as an AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences at Bowdoin College, Brunswick, Maine, 17-23 June 1984. This may have been the first, but was certainly not the only, conference devoted to random matrices and their applications. Simultaneously with the Bowdoin meeting, a meeting on products of random matrices was held in Toulouse, France. In November 1984 another meeting on Lyapounov exponents was held in Bremen, West Germany.

The papers in this volume, like the participants at the Bowdoin meeting, represent well the world community of mathematics and mathematical sciences. At the meeting were contributors and participants whose national origins or nationalities included China and Taiwan, France, Germany, Greece, India, Israel, Japan, Korea, The Netherlands, Poland, Sweden, and the United States. In recognition of the outstanding contributions of Soviet mathematicians to the field, the organizers extended invitations to several through official channels but unfortunately received no response.

The organizers are grateful to the National Science Foundation for financial support of the meeting, to Bowdoin College for a most congenial setting, to the sponsoring organizations (Institute of Mathematical Statistics, Society for Industrial and Applied Mathematics, American Mathematical Society), and to the AMS for administrative support in the friendly and effective person of Carole Kohanski. The organizers of this conference are grateful to the Organizing Committee of the Joint Summer Research Conferences, R. O. Wells, Jr., Chairman, and in particular to member James Glimm, for the invitation to organize this conference and prepare a volume of papers based on it. Anne Marie Welna provided administrative and secretarial assistance in producing this book.

Joel E. Cohen
Harry Kesten
Charles M. Newman

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Basic theory of products of random matrices

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LIMIT THEOREMS FOR PRODUCTS OF RANDOM MATRICES: A COMPARISON OF TWO POINTS OF VIEW

Joseph C. Watkins[1]

As we set out to prove limit theorems for sums of random variables, we may choose to reason along either one of two separate lines. Let us begin by considering a sequence $X, X_1, X_2, \ldots$ of independent identically distributed random variables.

1. If we view the sequence of partial sums as an outcome for a random walk, then we may interpret the law of large numbers as saying that despite the fact that the sums are random, their growth is roughly linear,

$$\sum_{k=1}^{n} X_k \sim n\ EX.$$

The error in this statement is made more precise via the central limit theorem. The difference between the actual random growth and the approximated linear growth is, upon division by $\sqrt{n}$, nearly normally distributed,

$$\frac{1}{\sqrt{n}}\left(\sum_{k=1}^{n} X_k - nEX\right) \sim N(0, \text{Var } X).$$

Thus the first line is specified by having each contribution to the sum remain large, but rescaling the sum in order to analyze the asymptotic behavior.

2. The second line, generalized by the language of infinitesimal triangular arrays, begins by considering the outcomes of a sum of many small independent perturbations. Guided by such considerations, we may now interpret the law of large numbers as saying

$$\sum_{k=1}^{n} \frac{1}{n} X_k \sim EX.$$

The difference between the sum and its limit is converging to zero, and so,

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