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ADVANCES IN DESIGN AUTOMATION — 1988

edited by
S. S. RAO

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FOREWORD

This volume contains 68 papers presented at the fourteenth Annual Design Automation Conference held September 24-28, 1988 in Orlando,Florida. The conference, sponsored by the ASME Design Engineering Division, has been organized by the Division's Design Automation Committee. It is the goal of the conference to provide a collection of papers which, along with the session presentations and discussions and the luncheon program, will give mechanical design engineers not only some added knowledge in the areas of design automation, but also accompanying motivation which will result in them being to some degree better equipped to meet the current and future challenges in the area of mechanical design. It is hoped that this Conference Proceedings will provide an efficient dissemination of information to all the design automation community.

The future growth in industrial productivity and technical advancement of products depend upon research and development in the area of mechanical systems design and design methodology. The importance of the research in mechanical design has been acknowledged in recent years by the National Science Foundation through its programs in mechanical systems and design methodologies. Industrial research in the field is also increasing.

The design automation conferences are held annually each fall where sessions are devoted to theoretical and applied synthesis and analysis of mechanical systems. Topics of interest include expert systems in design, optimal design, computer aided design and engineering, design methodologies, hardware/software systems evaluation, simulation and automated design of mechanical systems, and related areas. The areas that are receiving increased attention in the design automation community are expert systems, artificial intelligence, design for manufacturability and assembly, and computational geometry- specifically the underlying theories and principles as these relate to design automation. The papers included in this volume on these subjects is a reflection of this trend.

As Papers Review Chairman, I would like to gratefully acknowledge the cooperation and help of all reviewers in reviewing the papers on time. I am thankful to Professor Glen Johnson, Chairman of the Design Automation Committee, for his valuable suggestions and help in handling my job. I wish to thank the Chairman of the Design Automation Conference, Professor Bahram Ravani, for his cooperation and help in bringing out this proceedings volume. Thanks are also to the exceptional ASME staff members in New York for getting this volume printed. My special thanks are due to Dr. Winfred M. Phillips, Head, School of Mechanical Engineering, Purdue University for his support and help in carrying out the task of Papers Review Chairmanship. My final thanks are to Ms. Patricia Booth who cheerfully helped me in the secretarial work on numerous occassions.

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Fourteenth Design Automation Conference

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A COMPUTER AIDED GEOMETRIC METHOD FOR DEVELOPMENT OF THICK SURFACES

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ABSTRACT

Uniformly thick surfaces are considered for development. A thick surface is considered to be a set of thin surfaces lying one over the other. A mean surface is defined. Other surfaces in the set are defined in terms of the mean surface as well as the distance between them and the mean surface. Once the mean surface is developable, other surfaces are shown to be developable. These surfaces are developed individually and then a co-ordinate transformation is applied to these developments to take into account the fact that the origin and axes of the development are different for different surfaces. This completes the geometrical development of the thick surface. Plastic deformations taking place during the manufacturing process are not taken into account here.

INTRODUCTION

Surfaces can be classified into developable and non-developable surfaces. Planes and single-curved ruled surfaces are developable. Geometric methods like parallel line method, radial line method and triangulation are available for developing surfaces [1,2]. These are manual, time consuming and are subjected to drafting inaccuracies and instability. To overcome these defects, suitable mathematical modelling has to be developed and the development process has to be computerised. Taking into account the fact that the geodesic curvature of a space curve is bending invariant and that the isometric as well as the isogonal and isoareal mapping of a surface [3] onto a tangent plane of it is the development of the surface, suitable mathematical modelling and algorithms have been proposed by Gurunathan and Dhande for the development of ruled surfaces with single or double directrices [4,5]. Based on this, conical and helical convolute surfaces have been analysed and mathematical modelling and algorithm for their development have been proposed [5,6,7] . Also an approximate method for the development of ducts - double-curves surfaces - has been proposed by treating the duct to be a series of conical convolutes [5].

So far the development of thin surfaces only have been considered. Many of the surfaces that are used in industry, such as volute casing of turbines, various ducts under pressure, have considerable thickness. Large amount of plastic deformation takes plaee during their manufacture. So the development process has to take into account not only the geometrical aspects of development but also the plastic deformations taking place during manufacture. It is proposed to consider the geometrical aspects first and obtain the development of the thick surface and then to take into account the plastic deformations and get the final development of the surface. In this paper the first part of the development process - development taking into account the geometrical aspects only - is considered [5].

Surfaces of uniform thickness are considered here. A mean surface that divides the thick surface into two halves of equal thickness can be considered such that, at any point on it, the top and bottom surfaces of the thick surface are at equal distance. These two halves can further be divided into a number of slices of uniform thickness. The surfaces that separate these slices from one another, together with the top and bottom surfaces of the thick surface form the set of surfaces representing the thick surface. These are parallel to one another (Figure 1). The end surfaces of the thick surface bound the set of surfaces. The edges of the various surfaces in the set are the lines of intersection of these surfaces with the end surfaces of the thick surface.

THE SET OF SURFACES

Let h be the thickness of the thick surface and s_h be the number of slices into which each half of the thick surface is to be divided. Then the total number of thin surfaces in the set of surfaces representing the thick surface is $2s_h + 1$. Let S_{kt}, $kt = 1,2,\ldots(2s_h+1)$, represent a surface in the set, Let S_1 represent the mean surface. The direction of the inward normal or that of the outward normal to the mean surface can be taken as positive. Then S_{kt}, $kt = 2,3,\ldots,(s_h+1)$ and S_{kt}, $kt = (s_h + 2), (s_h + 3),\ldots,(2s_h+1)$ represent respectively the surfaces in the positive and negative directions of the normal in the increasing order of distance from the mean surface. Let c_{kt} be the distance of the surface S_{kt} from the mean surface. The distance is positive if it is along the positive direction of the normal to the mean surface; otherwise it is negative. If the slices are of equal thickness, then

$$c_{kt} = (h/2s_h)\,(kt - 1) \quad kt = 2,3,\ldots,(s_h+1)$$

and

$$c_{kt} = -(h/2s_h)\,(kt - s_h - 1)$$

$$kt = (s_h+2),(s_h+3),\ldots.(2s_h+1) \quad \cdots (1)$$

THE MEAN SURFACE

The mean surface is considered to be a developable ruled surface. Let the lines $D_{i,1}$ and $D_{j,1}$, the lines of intersection of the mean surface with the end surfaces E_1 and E_2 be respectively the primary and secondary directrices (refer to Figure 2). Let $\underline{r}_{i,1}$ and $\underline{r}_{j,1}$ be the position vectors of the generic points on the primary and secondary directrices respectively. The subscript 1 stands for the mean surface and the subscripts i and j stand for the primary and secondary directrices respectively. If θ_i and θ_j are the parameters of the directrices, then the condition for the developability of the mean surface is given by [4,5]

$$\left(\frac{d\underline{r}_{i,1}}{d\theta_i} \times \frac{d\underline{r}_{j,1}}{d\theta_j}\right) \cdot (\underline{r}_{j,1} - \underline{r}_{j,1}) = 0 \quad \cdots (2)$$

A generatrix $P_1\,Q_1$ of the mean surface the tangent vectors $d\underline{r}_{i,1}/d\theta_i$ and $d\underline{r}_{j,1}/d\theta_j$, the unit normal to the mean surface, $\underline{n}_s$ and the unit vector $\underline{g}$ along the generatrix are shown in Figure 2. Also shown in Figure 2 is a unit vector $\underline{e}$ which together with the vectors $\underline{g}$ and $\underline{n}_s$ form a right-handed system of mutually perpendicular unit vectors; $\underline{e} = \underline{g} \times \underline{n}_s$. With the point P_1 and Q_1 as origins, local co-ordinate frames $P_1 - \underline{g}\,\underline{n}_s\,\underline{e}$ and $Q_1 - \underline{g}\,\underline{n}_s\,\underline{e}$ can be considered.

OTHER SURFACES

$P_{kt}\,Q_{kt}$ is the line of intersection of the surface S_{kt} with the $\underline{g} - \underline{n}_s$ plane of the local co-ordinate frame $P_1 - \underline{g}\,\underline{n}_s\,\underline{e}$ (Figure 3). It is at a distance c_{kt} from $P_1\,Q_1$. P_{kt} and Q_{kt} are the points of intersection of the line $P_{kt}\,Q_{kt}$ with the end surfaces E_1 and E_2 respectively. Let $O_{L1} - X_{L1}\,Y_{L1}\,Z_{L1}$ be a local co-ordinate frame such that $X_{L1}\,Y_{L1}$ is the plane of the end surface E_1 and Z_{L1} axis is normal to the plane E_1. $O - XYZ$ is the global co-ordinate frame.

Let the position vector of the point P_{kt} be $[x,y,z,1]$, $[x_1, y_1, 0, 1]$ and $[x^x, c_{kt}, 0, 1]$ with respect to three co-ordinate frames $O - XYZ$, $O_{L1} - X_{L1}\,Y_{L1}\,Z_{L1}$ and $P_1 - \underline{g}\,\underline{n}_s\,\underline{e}$ respectively. Then

$$\begin{bmatrix} x_1 \\ y_1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \underline{g}.X_{L1} & \underline{n}_s.X_{L1} & \underline{e}.X_{L1} & (\underline{r}_{i,1}^{(P_1)} - \underline{r}^{(O_{L1})}).X_{L1} \\ \underline{g}.Y_{L1} & \underline{n}_s.Y_{L1} & \underline{e}.Y_{L1} & (\underline{r}_{i,1}^{(P_1)} - \underline{r}^{(O_{L1})}).Y_{L1} \\ \underline{g}Z_{L1} & \underline{n}_s.Z_{L1} & \underline{e}.Z_{L1} & (\underline{r}_{i,1}^{(P_1)} - \underline{r}^{(O_{L1})}).Z_{L1} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x^x \\ c_{kt} \\ 0 \\ 1 \end{bmatrix} \quad \cdots (3)$$

where $\underline{r}_{i,1}^{(P_1)}$ and $\underline{r}^{(O_{L1})}$ are the position vectors of the points P_1 and O_{L1} respectively expressed in global co-ordinates. In the above equation, to avoid confusion, vector sign is not used with X_{L1}, Y_{L1} and Z_{L1} although these are unit direction vectors along the axes X_{L1}, Y_{L1} and Z_{L1} respectively. Since the point P_1 is on the $X_{L1}\,Y_{L1}$ plane,

$$(\underline{r}_{i,1}^{(P_1)} - \underline{r}^{(O_{L1})}).Z_{L1} = 0 \; .$$

So, from the third row of the Eqn. (3),

$$x^x = -\frac{\underline{n}_s \cdot Z_{L1}}{\underline{g} \cdot Z_{L1}}\, c_{kt}.$$

Hence the position vector of the point P_{kt} is

$$\underline{r}_{i,kt}^{(P_{kt})} = \underline{r}_{i,1}^{(P_1)} - \frac{\underline{n}_s.Z_{L1}}{\underline{g}.Z_{L1}}\, c_{kt}\,\underline{g} + c_{kt}\underline{n}_s. \quad \cdots (4)$$

Similarly a local co-ordinate frame $O_{L2} - X_{L2}\,Y_{L2}\,Z_{L2}$ where $X_{L2}\,Y_{L2}$ is the plane of the end surface E_2 and Z_{L2} axis is normal to the end surface E_2, can be considered. Then the position vector of the point Q_{kt} is given by

$$\underline{r}_{j,kt}^{(Q_{kt})} = \underline{r}_{j,1}^{(Q_1)} - \frac{\underline{n}_s.Z_{L2}}{\underline{g}\,.Z_{L2}}\, c_{kt}\,\underline{g} + c_{kt}\,\underline{n}_s. \quad \cdots (5)$$

The conditions to be satisfied by Eqns.(4) and (5) are that

$$\underline{g} \cdot Z_{L1} \neq 0$$

and

$$\underline{g} \cdot Z_{L2} \neq 0. \quad \cdots (6)$$

These conditions imply that the generatrix does not lie on the end surfaces E_1 and E_2 respectively.

If the generatrix is perpendicular to the end surface E_1, then $\underline{g} = Z_{L1}$ and Eqn.(4) reduces to

$$\underline{r}_{i,kt}^{(P_{kt})} = \underline{r}_{i,1}^{(P_1)} + c_{kt}\,\underline{n}_s . \qquad (7)$$

Similarly when the generatrix is perpendicular to the end surface E_2, $\underline{g} = Z_{L2}$ and

$$\underline{r}_{j,kt}^{(Q_{kt})} = \underline{r}_{j,1}^{(Q_1)} + c_{kt}\,\underline{n}_s . \qquad (8)$$

The vectors $\underline{r}_{i,1}^{(P_1)}$, $\underline{g}$ and $\underline{n}_s$ have θ_i as their parameter. The end surface E_1 is a planar one and hence the vector Z_{L1} is a constant vector. Hence it can be seen from Eqn. (4) that the vector $\underline{r}_{i,kt}^{(P_{kt})}$ has θ_i as its parameter. Similarly the vector $\underline{r}_{j,kt}^{(Q_{kt})}$ has θ_j as its parameter.

Corresponding to each generatrix of the mean surface the vectors $\underline{g}$, $\underline{n}_s$ and $\underline{e}$ can be obtained and then the line of intersection of the $\underline{g}$ - $\underline{n}_s$ plane with the surface S_{kt} can be obtained. The points of intersection of this line with the end surfaces E_1 and E_2 can be obtained from Eqns. (4) and (5). Thus the lines $D_{i,kt}$ and $D_{j,kt}$, the lines of intersection of the surface S_{kt} with the end surfaces E_1 and E_2, can be obtained. The points P_{kt} and Q_{kt} are the generic points on these lines respectively. The lines $P_1\,P_{kt}$ and $Q_1\,Q_{kt}$ are the lines of intersection of the $\underline{g}$ - $\underline{n}_s$ plane with the end surfaces E_1 and E_2 respectively.

DEVELOPABILITY OF THE SET OF SURFACES

Differentiating the vector $\underline{r}_{i,kt}^{(P_{kt})}$ with respect to θ_i,

$$\frac{d\underline{r}_{i,kt}^{(P_{kt})}}{d\theta_i} = \frac{d\underline{r}_{i,1}^{(P_1)}}{d\theta_i} - \frac{\underline{n}_s . Z_{L1}}{\underline{g} . Z_{L1}}\, c_{kt}\frac{d\underline{g}}{d\theta_i} + c_{kt}\frac{d\underline{n}_s}{d\theta_i}$$
$$- \frac{(\underline{g}.Z_{L1})\left(\frac{d\underline{n}_s}{d\theta_i}.Z_{L1} + \underline{n}_s.\frac{dZ_{L1}}{d\theta_i}\right) - (\underline{n}_s.Z_{L1})\left(\frac{d\underline{g}}{d\theta_i}.Z_{L1} + \underline{g}.\frac{dZ_{L1}}{d\theta_i}\right)}{(\underline{g}.Z_{L1})^2}\, c_{kt}\underline{g} \qquad \text{.... (9)}$$

The end surface E_1 is planar and hence $\frac{dZ_{L1}}{d\theta_i}$. $\frac{d\underline{r}_{i,1}^{(P_1)}}{d\theta_i}$, $\frac{d\underline{g}}{d\theta_i}$ and $\frac{d\underline{n}_s}{d\theta_i}$ are in the $\underline{g}$-$\underline{e}$ plane[5].

Hence the vector $\frac{d\underline{r}_{i,kt}^{(P_{kt})}}{d\theta_i}$ is acting in a plane parallel to the $\underline{g}$ - $\underline{e}$ plane. Similarly the derivative of the vector $\underline{r}_{j,kt}^{(Q_{kt})}$ with respect to the parameter θ_j, $\frac{d\underline{r}_{j,kt}^{(Q_{kt})}}{d\theta_i}$, is also acting in the plane parallel to the $\underline{g}$ - $\underline{e}$ plane.

Thus the three vectors $(\underline{r}_{j,kt}^{(Qkt)} - \underline{r}_{i,kt}^{(P_{kt})})$, $d\underline{r}_{i,kt}^{(P_{kt})}/d\theta_i$ and $d\underline{r}_{j,kt}^{(Q_{kt})}/d\theta_j$ are co-planar, acting in a plane parallel to the $\underline{g}$ - $\underline{e}$ plane. The vectors $d\underline{r}_{i,kt}^{(P_{kt})}/d\theta_i$ and $d\underline{r}_{j,kt}^{(Q_{kt})}/d\theta_j$ are the tangents to the lines $D_{i,kt}$ and $D_{j,kt}$ respectively. So the line $P_{kt}\,Q_{kt}$ can be considered as the generatrix of the surface S_{kt} and the condition for developability of the surface S_{kt} is satisfied. Thus the surface S_{kt} is a developable ruled surface and the lines $D_{i,kt}$ and $D_{j,kt}$ are the primary and secondary directrices of the surface S_{kt}.

Hence it can be seen that

(i) If the mean surface is a developable ruled surface, all other surfaces are also developable ruled surface,

(ii) if the lines of intersection of the mean surface with the end surfaces E_1 and E_2 are respectively the primary and secondary directrices of the mean surface, then the lines of intersection of the other surfaces with the end surfaces E_1 and E_2 are respectively the primary and secondary directrices of the surface concerned,

(iii) for a given position of the generic points of the primary and secondary directrices of the mean surface, the positions of the generic points of the primary and secondary directrices of other surfaces are given by the lines of intersection of the $\underline{g}$ - $\underline{n}_s$ plane with the end surfaces E_1 and E_2 respectively.

(iv) the parameters for the primary and secondary directrices of all surfaces are the same θ_i and θ_j respectively as for the primary and secondary directrices of the mean surface. Corresponding to a particular generatrix of the mean surface defined by a pair of (θ_i, θ_j) values, the generatrices of all other surfaces are also defined by the same pair of (θ_i, θ_j) values.

DEVELOPMENT OF THE SET OF SURFACES

The set of surfaces are developable ruled surfaces with two directrices and the development of these surfaces are carried out individually by isometrically mapping the primary directrix of surface concerned and then isogonally as well as isometrically mapping its generatrices [4] . The geodesic curvature of the primary directrix is equal to the curvature of the curve of the development of the primary directrix. The Serret-Frenet equations are used for isometrically mapping the primary directrix.

ARC LENGTH OF THE PRIMARY DIRECTRIX OF THE SURFACE S_{kt}

Equations (4) and (5) can be rewritten as

$$\underline{r}_{i,kt} = \underline{r}_{i,1} + c_{kt}\,\underline{v}_i \qquad \text{...(10)}$$

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and

$$\underline{r}_{j,kt} = \underline{r}_{i,1} + c_{kt}\,\underline{v}_j \qquad \cdots (11)$$

where

$$\underline{v}_i = -\frac{\underline{n}_s \cdot Z_{L1}}{\underline{g} \cdot Z_{L1}}\,\underline{g} + \underline{n}_s \qquad \cdots (12)$$

and

$$\underline{v}_j = -\frac{\underline{n}_a \cdot Z_{L2}}{\underline{g} \cdot Z_{L2}}\,\underline{g} + \underline{n}_s\,. \qquad \cdots (13)$$

The vectors $\underline{v}_i$ and $\underline{v}_j$ are along the lines of intersection of the $\underline{g} - \underline{n}_s$ plane with the end surfaces E_1 and E_2 respectively.

The derivative of $\underline{r}_{i,kt}$ with respect to θ_i is

$$\dot{\underline{r}}_{i,kt} = \dot{\underline{r}}_{i,1} + c_{kt}\,\dot{\underline{v}}_i \qquad \cdots (14)$$

where the dot above the vectors indicate their derivative with respect to θ_i. The rate of variation of the arc length of the primary directrix with respect to θ_i is given by

$$\dot{s}_{i,kt} = (\dot{\underline{r}}_{i,1}\cdot\dot{\underline{r}}_{i,1} + 2c_{kt}\,\dot{\underline{r}}_{i,1}\cdot\dot{\underline{v}}_i + c_{kt}^2\,\dot{\underline{v}}_i\cdot\dot{\underline{v}}_i)^{1/2} \qquad \cdots (15)$$

The arc length of the primary directrix is obtained by integrating the above expression. For a given value of θ_i, the values of $\dot{\underline{r}}_{i,1}$ and $\dot{\underline{v}}_i$ are constants and hence $\dot{s}_{i,kt}$ is a function of c_{kt}.

MAGNITUDE OF GEODESIC CURVATURE OF THE PRIMARY DIRECTRIX OF THE SURFACE S_{kt}

The second derivative of $\underline{r}_{i,kt}$ with respect to θ_i is

$$\ddot{\underline{r}}_{i,kt} = \ddot{\underline{r}}_{i,1} + c_{kt}\,\ddot{\underline{v}}_i \qquad \cdots (16)$$

where the two dots above the vectors indicate their second derivative with respect to θ_i. The magnitude of the geodesic curvature for the primary directrix is

$$k_{g_{i,kt}} = \frac{\dot{\underline{r}}_{i,kt} \times \ddot{\underline{r}}_{i,kt}}{\dot{s}_{i,kt}^3}\cdot\underline{n}_s$$

and substituting for $\dot{\underline{r}}_{i,kt}$, $\ddot{\underline{r}}_{i,kt}$ and $\dot{s}_{i,kt}$,

$$k_{g_{i,kt}} = \frac{\dot{\underline{r}}_{i,1} \times \ddot{\underline{r}}_{i,1} + c_{kt}(\dot{\underline{r}}_{i,1} \times \ddot{v}_i + \dot{v}_i \times \ddot{\underline{r}}_{i,1}) + c_{kt}^2\,\dot{\underline{v}}_i \times \ddot{\underline{v}}_i}{(\dot{\underline{r}}_{i,1}\cdot\dot{\underline{r}}_{i,1} + 2c_{kt}\,\dot{\underline{r}}_{i,1}\cdot\dot{\underline{v}}_i + c_{kt}^2\,\dot{\underline{v}}_i\cdot\dot{\underline{v}}_i)^{3/2}}\cdot\underline{n}_s \qquad \cdots (17)$$

For a given value of θ_i the magnitude of geodesic curvature $k_{g_{i,kt}}$ is a function of c_{kt}.

DEVELOPMENT OF THE THICK SURFACE

The origin and axes of the development are different for different surfaces of the set of surfaces representing the thick surface. Hence a co-ordinate transformation is to be carried out and the co-ordinates of the generic points on the development of the primary and secondary directrices of the surfaces in the set are expressed in terms of the origin and the co-ordinate axes of the development of the mean surface.

Consider the co-ordinate frame $O_{d_{kt}}-X_{d_{kt}}\,Y_{d_{kt}}\,Z_{d_{kt}}$ where $O_{d_{kt}}$, $X_{d_{kt}}$ and $Y_{d_{kt}}$ are respectively the origin and the x and y axes of the development of the surface S_{kt}. The $Z_{d_{kt}}$ axis is perpendicular to the plane of the development. Let $\underline{t}_{0_{kt}}$, $\underline{e}_{0_{kt}}$ and $\underline{n}_{0_{kt}}$ be the unit vectors along the $X_{d_{kt}}$, $Y_{d_{kt}}$ and $Z_{d_{kt}}$ axes. Here $\underline{t}_{0_{kt}}$ is the unit tangent vector to the primary directrix of the surface S_{kt} at the starting point and $\underline{n}_{0_{kt}}$ is the unit normal to the surface at that point. The vector $\underline{e}_{0_{kt}}$ is given by

$$\underline{e}_{0_{kt}} = \underline{n}_{0_{kt}} \times \underline{t}_{0_{kt}}\,.$$

Let the co-ordinate frame for the development of the mean surface be $O_{d_1} - X_{d_1}\,Y_{d_1}\,Z_{d_1}$ and the corresponding unit vectors be $\underline{t}_{0_1}$, $\underline{e}_{0_1}$ and $\underline{n}_{0_1}$. Then the co-ordinates of the development of the surface S_{kt}, $x_{d_{kt}}$, $y_{d_{kt}}$, 0, 1 can be expressed with respect to the co-ordinate frame $O_{d_1}-X_{d_1}\,Y_{d_1}\,Z_{d_1}$ as

$$\begin{bmatrix} x_{dm_{kt}} \\ y_{dm_{kt}} \\ z_{dm_{kt}} \\ 1 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} \begin{bmatrix} x_{d_{kt}} \\ y_{d_{kt}} \\ 0 \\ 1 \end{bmatrix}$$

where

$$q_{11} = \underline{t}_{0_{kt}}\cdot\underline{t}_{0_1},\; q_{12} = \underline{e}_{0_{kt}}\cdot\underline{t}_{0_1},\; q_{13} = \underline{n}_{0_{kt}}\cdot\underline{t}_{0_1}$$

$$q_{14} = (\underline{r}^{(0_{d_{kt}})} - \underline{r}^{(0_{d_1})})\cdot\underline{t}_{0_1},\; q_{21} = \underline{t}_{0_{kt}}\cdot\underline{e}_{0_1},$$

$$q_{22} = \underline{e}_{0_{kt}}\cdot\underline{e}_{0_1},\; q_{23} = \underline{n}_{0_{kt}}\cdot\underline{e}_{0_1},$$

$$q_{24} = (\underline{r}^{(0_{d_{kt}})} - \underline{r}^{(0_{d_1})})\cdot\underline{e}_{0_1},\; q_{31} = \underline{t}_{0_{kt}}\cdot\underline{n}_{0_1},$$

$$q_{32} = \underline{e}_{0_{kt}}\cdot\underline{n}_{0_1},\; q_{33} = \underline{n}_{0_{kt}}\cdot\underline{n}_{0_1},$$

$$q_{34} = (\underline{r}^{(0_{d_{kt}})} - \underline{r}^{(0_{d_1})})\cdot\underline{n}_{0_1},\; q_{41} = 0,\; q_{42} = 0,$$

$$q_{43} = 0,\; q_{44} = 1 \qquad \cdots (18)$$

where $\underline{r}^{(0_{d_{kt}})}$ and $\underline{r}^{(0_{d_1})}$ are the positive vectors of the points $O_{d_{kt}}$ and O_1, which are nothing but the generic points on the primary directrices of the surface S_{kt} and the mean surface at the starting point. From Eqn.(4),

$$\underline{r}^{(0_{d_{kt}})} - \underline{r}^{(0_{d_1})} = -\frac{\underline{n}_{0_1}\cdot Z_{L1}}{\underline{g}_{0_1}\cdot Z_{L1}} c_{kt}\,\underline{g}_{0_1} + c_{kt}\,\underline{n}_{0_1} \qquad \cdots (19)$$

where $\underline{g}_{0_1}$ is the unit vector along the generatrix of the mean surface at the starting point. Since all surfaces are parallel, $\underline{n}_{0_{kt}} = \underline{n}_{0_1}$. Then Eqns. (18) reduce to

$$\begin{bmatrix} x_{dm_{kt}} \\ y_{dm_{kt}} \\ z_{dm_{kt}} \\ 1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} x_{d_{kt}} \\ y_{d_{kt}} \\ 0 \\ 1 \end{bmatrix} \qquad \cdots (20)$$

where

$k_{11} = \underline{t}_{0_{kt}}\cdot\underline{t}_{0_1}$, $k_{12} = \underline{e}_{0_{kt}}\cdot\underline{t}_{0_1}$, $k_{13} = 0$,

$k_{14} = -\dfrac{\underline{n}_{0_1}\cdot Z_{L1}}{\underline{g}_{0_1}\cdot Z_{L1}} c_{kt}\,\underline{g}_{0_1}\cdot\underline{t}_{0_1}$, $k_{21} = \underline{t}_{0_{kt}}\cdot\underline{e}_{0_1}$,

$k_{22} = \underline{e}_{0_{kt}}\cdot\underline{e}_{0_1}$, $k_{23} = 0$, $k_{24} = -\dfrac{\underline{n}_{0_1}\cdot Z_{L1}}{\underline{g}_{0_1}\cdot Z_{L1}}$ $c_{kt}\,\underline{g}_{0_1}\cdot\underline{e}_{0_1}$, $k_{31} = 0$, $k_{32} = 0$, $k_{33} = 1$,

$k_{34} = c_{kt}$, $k_{41} = 0$, $k_{42} = 0$, $k_{43} = 0$, $k_{44} = 1$.

From the above equations it can be seen that $z_{dm_{kt}} = c_{kt}$ always. This is because the surface S_{kt} and the mean surface are parallel and are at a distance c_{kt} from one another. So not only the two surfaces but also their developments are parallel and are at a distance c_{kt} from one another.

ALGORITHM FOR THE DEVELOPMENT OF THICK SURFACE

Step 1(a). The end surfaces E_1 and E_2 are specified i.e. the local co-ordinate frames $O_{L1} - X_{L1}\,Y_{L1}\,Z_{L1}$ and $O_{L2} - X_{L2}\,Y_{L2}\,Z_{L2}$ are specified.

(b). The primary and secondary directrices of the mean surface and the thickness of the thick surface are specified.

Step 2 . The number of surfaces in the set of thin surfaces representing the thick surface is fixed and their distances from the mean surface is calculated.

Step 3 . The development of the mean surface is carried out,

(a) The value of the parameter θ_i of the primary directrix of the mean surface is fixed and from the condition for developability the value of the parameter θ_j of the secondary directrix of the mean surface is fixed.

(b) The vectors $\underline{r}_{i,1}$, $\underline{r}_{j,1}$, $\underline{g}$, $\underline{n}_s$, $\underline{v}_i$ and $\underline{v}_j$ are calculated.

(c) The arc length, the magnitude of geodesic curvature and the arc-tangent angle of the primary directrix and the angle between the arc-tangent and the generatrix are calculated.

(d) The co-ordinates $(x_{d_{i,1}}, y_{d_{i,1}})$ and $(x_{d_{k,1}}, y_{d_{j,1}})$ of the ends of the generatrix in the development are calculated.

(e) For the initial position of the generatrix, the vectors $\underline{n}_{01}$, $\underline{g}_{01}$ and $\underline{t}_{01}$ are noted. The vector $\underline{e}_{01}$ is calculated. For other position this step 3(e) is omitted.

(f) The parameter θ_i is incremented and the above steps 3(a) to 3(d) are repeated till the entire mean surface is developed.

Step 4. Corresponding to the various positions generatrix of the mean surface, the first and second derivatives of the vector v_i with respect to the parameter θ_i are calculated.

Step 5. The initial position of the generatrix of the mean surface S_1 is considered.

Step 6. The surface S_{kt}, kt = 2, is considered.

Step 7. The position vectors $\underline{r}_{i,kt}$ and $\underline{r}_{j,kt}$ of the ends of the corresponding generatrix of the surface S_{kt} are calculated using Eqns.(10) and (11). Also the vectors $\dot{\underline{r}}_{i,kt}$ and $\ddot{\underline{r}}_{i,kt}$ are calculated using Eqns.(14) and (16). The unit tangent vector to the primary directrix of the surface S_{kt} is also calculated.

Step 8. The arc length, the magnitude of geodesic curvature, the arc-tangent angle of the primary directrix and the angle between the arc-tangent and generatrix of the surface S_{kt} are calculated.

Step 9. The co-ordinates $(x_{d_{i,kt}}, y_{d_{i,kt}})$ and $(x_{d_{j,kt}}, y_{d_{j,kt}})$ of the ends of the generatrix in the development of the surface S_{kt} are calculated.

Step 10. For the initial position of the generatrix of the surface S_{kt}, the vector $\underline{t}_{0_{kt}}$ (Eqn.20) is noted. Also the vector $\underline{e}_{0_{kt}}$ is calculated. For other positions of the generatrix this step is omitted.

Step 11. The other surfaces in the set of surfaces are considered one by one and Steps 7 through 10 are carried out.

Step 12. The other positions of the generatrix of the mean surface S_1 are considered one by one and Steps 6 through 11 are carried out till the development of all the surfaces in the set are obtained individually.

Step 13. The development of the individual surfaces are stacked properly using Eqns.(20) and the development of the thick surface is obtained.

CASE STUDY

The development of a thick super-conical surface whose mean surface is a super-conical convolute is presented here. If atleast one of the directrices of a conical convolute is a super-ellipse, then the surface is called a super-conical convolute. A generic point on a super-ellipse, with respect to a local co-ordinate frame $O_L - X_L\ Y_L\ Z_L$, is given by

$$\underline{r}_L = \begin{bmatrix} a \cos^{2/n} \theta \\ b \sin^{2/n} \theta \\ 0 \\ 1 \end{bmatrix} \quad \cdots (21)$$

where a,b,n and θ are respectively the semi-major diameter, the semi-minor diameter, the power index and the parameter of the super-ellipse. The origin O_L and the X_L and Y_L axes of the local co-ordinate frame coincide respectively with the centre and the major and minor diameters of the super-ellipse. With respect to the global co-ordinate frame O-XYZ, the generic point on the super-ellipse is given by

$$\underline{r} = [T]\ \underline{r}_L \quad \cdots (22)$$

where [T] is the 4 x4 transformation matrix connecting the local co-ordinate frame $O_L - X_L\ Y_L\ Z_L$ with the global co-ordinate frame O-XYZ. Mathematical models and suitable algorithm for the development of super-conical convolutes are given by Gurunathan and Dhande [5,6].

The geometric details of the super-conical surface are given in Table 1. The orthographic views and the development of the thick surface are given in Figures 4 and 5 respectively.

CONCLUSIONS

A geometrical method for the development of thick surfaces is given. In this geometrical method the large amount of plastic deformations that take place during the manufacturing of the thick surface from thick plates are not considered. To facilitate the development process, the plastic deformations and the geometrical aspects of development are considered separately. Obtaining the development considering only the geometrical aspects is the first stage of the process. Suitable mathematical models are to be evolved to apply the effects of plastic deformations to the development obtained in the first stage and get the final development. The thick plate cut as per this final development and subjected to the required manufacturing processes should give the required thick surface. Work in this direction is being pursued by Gurunathan and Dhande.

Sometimes it may be necessary to develope a thick surface whose mean surface is, as a whole, not developable, but is piecewise developable. Then the mean surface can be divided into a number of surface patches, each one of which is individually developable, The corresponding portions of the thick surface constitute a set of thick surfaces in series. Adjacent thick surfaces meet along a common edge. Mathematical expressions to define the common edge, suitable mathematical models and necessary algorithm to geometrically develop these multiple thick surfaces in series are given by Gurunathan [5,8].

Volute casings of turbines and ducts under pressure are fabricated out of thick plates. Such thick ducts, which are double-curved surfaces, can be approximated to a number of thick super-conical surfaces which are in series and each one of these super-conical surfaces can be developed individually.

Table 1. Details of the Thick Surface

Thickness of the surface	= 0.4
Number of slices into which each half of the thick surface is to be divided	= 2

Details of the Mean Surface

<u>Primary Directrix</u>

Semi-major diameter of the super-ellipse	0.9			
Semi-minor diameter of the super-ellipse	0.58			
Power index of the super-ellipse	3.1			
Transformation matrix	-0.463	-0.835	0.293	4.72
	0.605	-0.052	0.794	3.52
	-0.648	0.548	0.530	4.48
	0.0	0.0	0.0	1

<u>Secondary Directrix</u>

Semi-major diameter of the super-ellipse	1.0			
Semi-minor diameter of the super-ellipse	0.6			
Power index of the super-ellipse	3.2			
Transformation matrix	-0.447	-0.894	0.0	5.0
	0.0	0.0	1.0	5.0
	-0.894	0.447	0.0	5.0
	0.0	0.0	0.0	1

Normal to the end surface E_1, Z_{L1} = $\begin{bmatrix} 0.293 \\ 0.794 \\ 0.530 \end{bmatrix}$

Normal to the end surface E_2, Z_{L2} = $\begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$

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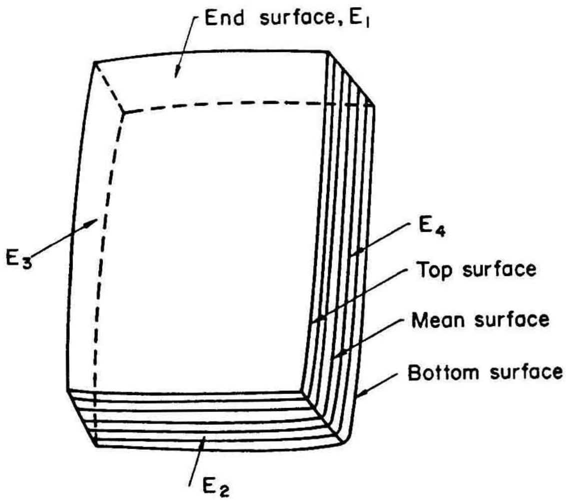

Fig.1 A Schematic Diagram of a Set of Surfaces

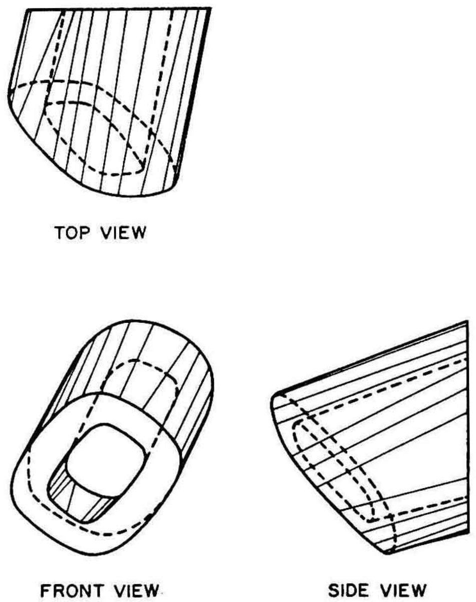

Fig. 4. Orthographic Views of a Thick Surfaces

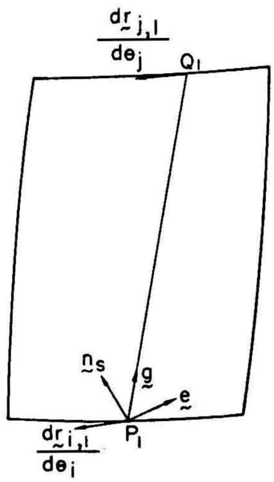

Fig.2. A View of the Mean Surface

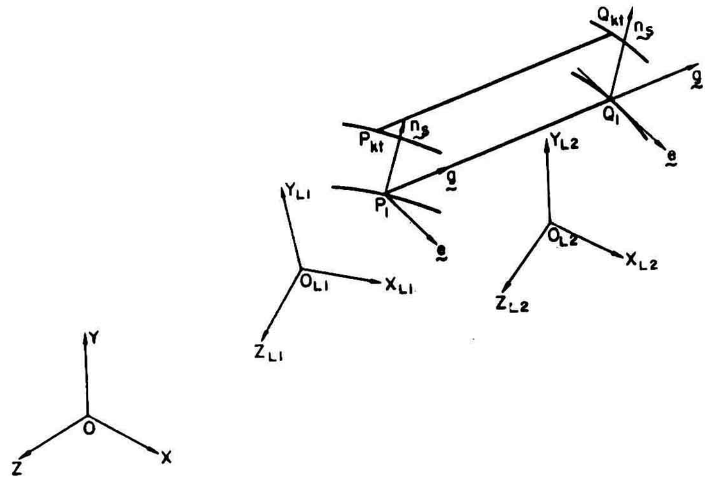

Fig.3. The Set Up of Coordinate Systems

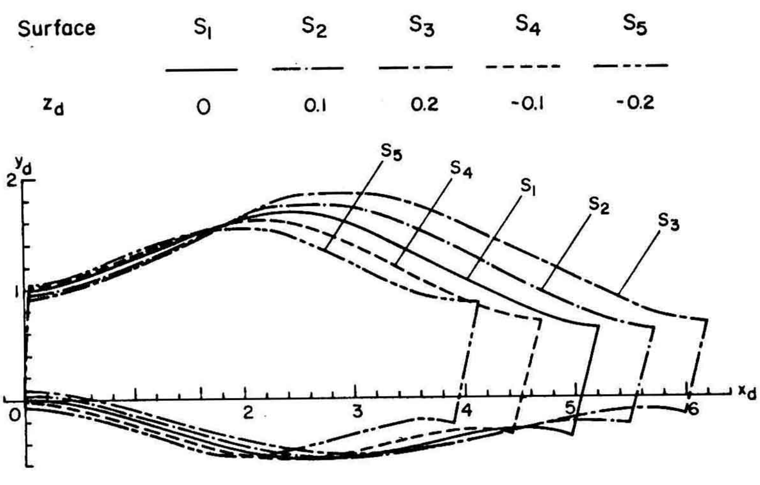

Fig. 5. Development of the Thick Surfaces given in Fig. 4.

CONSTRUCTIVE SOLID GEOMETRY OF THE TRIHEDRON

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Abstract

As a first step toward a Constructive Solid Geometry for designing general polyhedra, this paper develops the set theory of the trihedron, loosely speaking any set combination of three planar halfspaces (monohedra). The trihedron can be decomposed precisely into its primitive monohedra and its CSG-tree of union or intersection operations with no designer topological input other than the convexity or concavity of each edge, giving a human-computer interface simpler than those for existing right-hand rule boundary representation methods. The somewhat visual trigonometric concepts of classical solid geometry are formulated in terms of vectors and matrices appropriate for numerical computation. This reorganization may be useful not only for designers of CAD systems, but also for educators seeking to strengthen and modernize the geometric education of engineering students wanting to make full use of CAD/CAM technology.

Introduction

Computer Aided Design (CAD) has in the last decade evolved from computerized two dimensional drafting and documentation through 3D wireframe programs to the contemporary solid modeling systems needed for computer integration of design and manufacturing (2,4,6,8). But in the words of pioneers Voelcker and Wesley (11), "One hard-learned lesson of computer geometry is that implementing the simple cases of geometric problems is rather easy, but the ability to handle all possible cases may require an order of magnitude more work".

Most solid modelers employ either boundary representation (b-rep) or constructive solid geometry (CSG) (6,10). CSG systems assume much of the burden for insuring geometric and topological validity that B-reps place on the designer, but in CSG it is difficult or impossible to design and detail certain objects with non-rectangular corners. Although such arbitrary polyhedra can in principle be designed with B-rep systems, the level of detail required to specify all those vertices, edges, and faces limits the complexity the designer can tolerate. The present article begins a theoretical study intended eventually to produce a CSG system appropriate for designing arbitrary polyhedra of interest to designers not only of complicated multifunctional machine parts, but also of structures beyond, if not the imagination, then at least the detailing capability of 20th century architects.

CSG based on primitive solids (9) creates set unions, intersections, or differences of such primitive solids as rectangular prisms, cylinders, cones, and spheres -- all bounded point sets. The range of describable objects is limited by the primitives available, which is why such existing CSG has difficulty making, even with skew wedge primitives (10), polyhedra even as simple as a star tetrahedron. But by adding an unbounded general trihedron primitive, one might with proper precaution make a CSG system for polyhedra. Thus as a first step toward polyhedral CSG this paper develops the set theory of the trihedron, loosely speaking any set combination of three planar halfspaces (monohedra).

Because the trihedron is the simplest possible object with a solid interior, it has received much study in classical solid geometry (1) and can be generated on a drafting board with the techniques of descriptive geometry (5). But existing theory neither distinguishes between interior and exterior nor lends itself to digital calculation without visual control by the designer. The

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