Mathematics for Management and Finance

Stephen P. Shao

3d Edition

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PREFACE

The third edition of *Mathematics for Management and Finance* has updated and reorganized many illustrations and problems which appeared in the previous edition. New material, such as an explanation of interest compounded continuously, also has been added to this new edition.

This third edition, like the two previous editions, is designed for the first course in mathematics for students of business administration. Students who have had little or perhaps no algebra in high school, but who have determination in learning, will find that this text is suitable to their capacities. Those who have a strong background in algebra will find challenging material throughout the text. After the completion of this course, the student should be prepared to continue with more advanced work in subjects involving quantitative analysis, such as accounting, statistics, investments, and insurance. This leads the author to believe that, for a student of business administration, this text is more adequate and practical than a one-year course in mathematical analysis or college algebra. The latter courses traditionally emphasize theoretical mathematics, which is less pertinent to the foreseeable needs of a business student.

This edition is divided into four parts. Part I discusses basic and modern mathematics. It presents fundamental arithmetic and algebraic operations, beginning with very elementary concepts. Students who have not had sufficient mathematics in high school will thus have an opportunity to strengthen their background. Furthermore, sufficient material has been provided for those students who may have had mathematics some time ago but now need a review in beginning a study of the subject of mathematics for management and finance.

Part II discusses mathematics in business management. The topics included in this part represent problem areas found in almost every type of business enterprise. These topics are fundamental statistical methods, common percentage problems, simple interest, and bank discount. Part III discusses the basic topics of the mathematics of long-term investment—compound interest and annuities. Part IV discusses the applications of the mathematics of compound interest and annuity to debt extinction, bonds, depreciation, depletion, perpetuity, capitalization, life annuities, and life insurance.

Experience shows that when students readily know the principles involved in each type of calculation, they usually become better prepared for problem solving and for more advanced work. Throughout this text, the basic principle for each topic in financial problems is illustrated in detail. Every special term is clearly defined and explained before it is applied. Diagrams are frequently employed as aids to illustrate the more complex examples. After the principles have been illustrated, formulas are often used to facilitate computation. However, the number of formulas has been kept to a minimum and they are presented in simple manner. Proofs for the more complicated formulas are given in the footnotes.

Enough number problems (drills) are placed at the beginning of each exercise so that students can quickly learn the mechanics of the new symbols and terms in each new process. In the statement problems that follow, the student has the opportunity to exercise his reasoning ability. In addition, ample review problems are provided at the end of certain chapters to give the student an opportunity to solve problems independently without referring to the illustrations in the individual sections of the text.

The problems in each exercise are carefully arranged so that either all odd-numbered or all even-numbered problems can be assigned by the instructor without fear of omitting the material that has been illustrated in the examples. Answers to the odd-numbered problems are placed at the end of the book. Detailed solutions to the odd-numbered and even-numbered problems are given in the instructor's manual.

The material presented in this text is sufficient for a one-year course, offered either in two semesters or in three quarters. However, if there is not sufficient time to cover all the material, those sections and problems which have been starred ★ may be omitted without interrupting the continuity of the text organization. The text may also be used as a one-semester course under either one of the following two suggestions:

I. To emphasize general management and financial problems—Chapters 6 through 13 and Chapters 15 and 16.
II. To emphasize investment problems—Chapters 9 through 13 and Chapters 15 through 19.

Each of the two suggestions may easily be adjusted for either a one-quarter or a two-quarter course. More detailed assignment suggestions are given in the instructor's manual.

The tables included in the Appendix have been designed primarily for this textbook. However, coverage in the tables is complete enough for most practical business problems involving logarithms, compound interest, annuities, and life insurance. The tables of logarithms include six-place and seven-place mantissas. The compound interest and annuity tables include the 38 most commonly used periodic interest rates on the investment market today. The insurance mortality table is the latest, the Commissioners 1958 Standard Ordinary Table of Mortality. Other special features of the tables are summarized in the Preface to the tables.

I am indebted to my colleagues of the School of Business Administration of Old Dominion University for their encouragement and for their suggestions made from reading and teaching the first two editions of this book. Above all, I am deeply grateful to my late wife, Betty Outen Shao, for her expert services in editing the manuscript of the first edition of this book.

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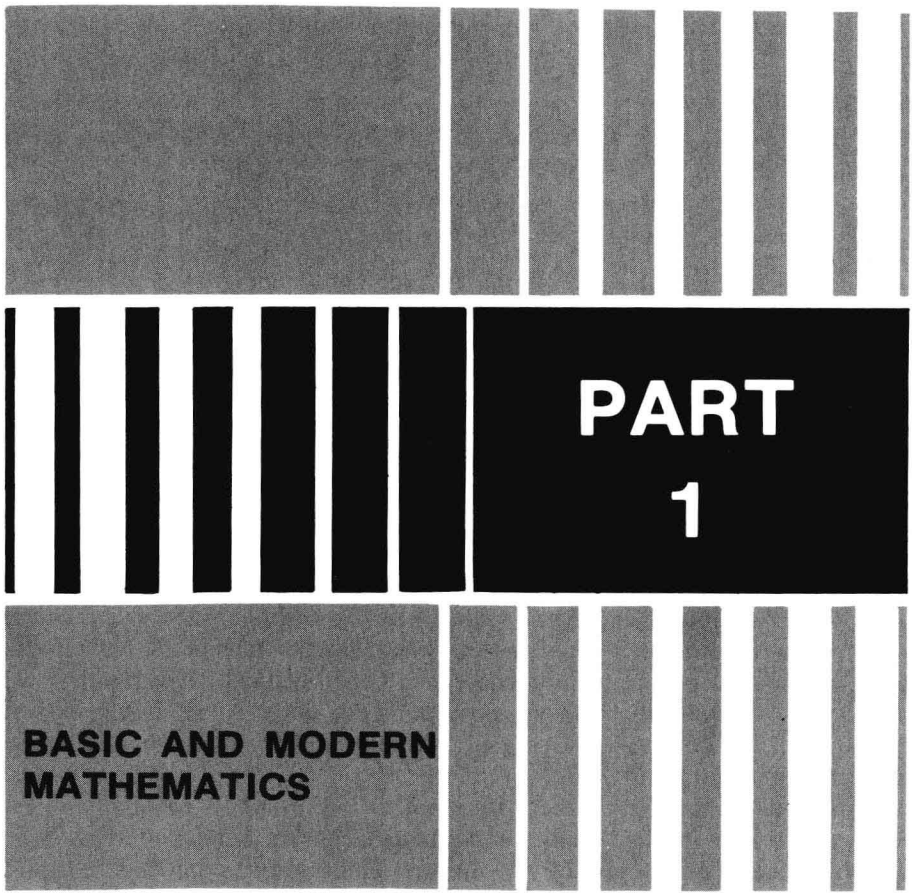
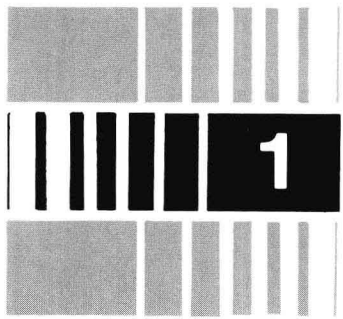
Part 1 Basic and Modern Mathematics





1 REVIEW OF ARITHMETIC

This chapter reviews basic arithmetic topics which are fundamental to further study in mathematical operations. However, emphasis is placed on the topic of fractions since fractional operations are relatively more complicated in arithmetic areas and are frequently used in solving common business problems.

1.1 FUNDAMENTAL OPERATIONS

The fundamental operations in arithmetic are *addition*, *subtraction*, *multiplication*, and *division*. Students not only should know how to do these operations but also should learn how to perform the basic operations rapidly and accurately. Of course, an accurate result is the final goal of mathematical operations. To accomplish this goal, commonly used methods of checking answers are introduced in this section.

Numbers containing decimal places are treated as whole numbers in performing the four fundamental operations and checking answers. However, care must be taken in placing the decimal points during an operation and rounding the decimal places in an answer. A method of rounding decimal places is also included in this section.

A. ADDITION. A common way to check an answer in addition is to use the *reverse order adding method*. In using this method, addition is performed by adding each column from the *top to the bottom*. The *checking operation* is then performed from the *bottom to the top*.

EXAMPLE 1 Add 5,674, 3,922, 6,137, and 2,308.

Add:			Check:	
Addend	5,674	↓	5,674	↑
Addend	3,922		3,922	
Addend	6,137		6,137	
Addend	2,308		2,308	
	21	$= 4 + 2 + 7 + 8$	21	$= 8 + 7 + 2 + 4$
	12	$= 7 + 2 + 3 + 0$	12	$= 0 + 3 + 2 + 7$
	19	$= 6 + 9 + 1 + 3$	19	$= 3 + 1 + 9 + 6$
	16	$= 5 + 3 + 6 + 2$	16	$= 2 + 6 + 3 + 5$
Total	18,041		18,041	

When addends include decimal fractions, it is important to place the decimal points in one column so that each column represents a definite unit. When this arrangement is used, it is easy to add tenths to tenths, hundredths to hundredths, and so on.

EXAMPLE 2 Find the sum of 2.54, .123, and 579.

2.54			2.540
.123		or	0.123
+579			+579.000
581.663			581.663

Because zeros do not change the value of a number when placed on the left side of a whole number or on the right side of a decimal fraction, they may be supplied for ease in addition. In the example above, the addition at the right is an illustration of this rule. However, if a zero is *annexed* to the right of an *integer* (a whole number), the value of the number is changed. For example, 579 is not equal to 579,000, but 579 is equal to 579.000.

B. SUBTRACTION. To check an answer in subtraction, a simple and effective method is to add the remainder to the subtrahend; the sum should be equal to the minuend.

EXAMPLE 3

Subtract:	Check:
2,761 (Minuend)	2,215 (Remainder)
− 546 (Subtrahend)	+ 546 (Subtrahend)
2,215 (Remainder)	2,761 (Minuend)

From the example above, it is easy to see that *subtraction is the reverse operation of addition.*

When subtraction involves decimals, the rule used in addition also applies; that is, place the decimal points of the minuend and the subtrahend in the same vertical column (if the subtraction is computed in a columnar form).

EXAMPLE 4 Subtract 169.564 from 295.4562.

$$\begin{array}{r} 295.4562 \\ -169.5640 \\ \hline 125.8922 \end{array}$$

C. MULTIPLICATION. When an answer in multiplication must be checked precisely, either one of the following two methods may be used.

Interchanging Multiplicand and Multiplier

EXAMPLE 5

Multiply:	Check:
463 (Multiplicand)	72 (Multiplier)
×72 (Multiplier)	×463 (Multiplicand)
926	216
32 41	4 32
33,336 (Product)	28 8
	33,336 (Product)

Dividing the Product by One of the Factors. The multiplicand and the multiplier are also called *factors* of the product. By dividing the product by one of the factors, the quotient thus obtained must be equal to the other factor. The product in Example 5 may be checked as follows:

```
        72 (Quotient, which is the other factor)
(Factor) 463)33,336 (Product)
           32 41
             926
             926
```

or,

```
          463 (Factor)
(Factor) 72)33,336 (Product)
           28 8
            4 53
            4 32
              216
              216
```

An answer in multiplication can be checked mentally by estimating the product after the multiplicand and the multiplier have been rounded to simple numbers. For instance, the multiplicand 4,363 and the multiplier 184 may be rounded to 4,000 and 200 respectively. The product of the two round numbers can thus be found mentally and used as a guide in checking the answer. Since the product of 4,000 and 200 is 800,000, the answer 802,792 ($4{,}363 \times 184 = 802{,}792$) is considered reasonable. If the answer had been calculated to be near 500,000, it would be considered unreasonable since it is apparently greatly different from the estimation. However, when the answer in multiplication must be checked precisely, one of the two above-mentioned methods should be used.

When there are decimals in multiplication, the number of decimal places in the product should equal the total number of decimal places in the factors.

EXAMPLE 6 Find the product of factors 4.23 and 1.1052.

Since 4.23 times 1.1052 equals 1.1052 times 4.23, the shorter factor is usually used as the multiplier. The multiplication for the example is given on page 5.

```
  1.1052
 ×4.23
 3 3156
 22 104
442 08
4.67 4996 (Product)
```

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there should be 6 decimal places in the product. The answer thus is 4.674996.

D. DIVISION. In Example 5, it can be seen that the process of division may be thought of as the process of finding an unknown factor in multiplication when one factor and the product are known.

In checking an answer in division, first find the product of the divisor and the quotient; then add the product to the remainder, if there is one. The result must be equal to the dividend. This method is based on the basic relationship in division:

$$\text{Divisor} \times \text{Quotient} + \text{Remainder of Division} = \text{Dividend}$$

EXAMPLE 7 Divide:

```
                  31 (Quotient)
(Divisor) 108)3,375 (Dividend)
          3 24
          ----
            135
            108
            ---
             27 (Remainder)
```

Check:

```
  108 (Divisor)
× 31 (Quotient)
  ---
  108
 324
 ---
3,348 (Product)
+  27 (Remainder)
 ----
3,375 (Dividend)
```

Complete quotient = $31\frac{27}{108}$

When the product of the divisor and the quotient is not the same amount as the dividend, there will be a remainder in division. The remainder may be expressed as a part of the quotient in two ways: (1) as a common fraction with the remainder as the numerator and the divisor as the denominator, such as the complete quotient $31\frac{27}{108}$ in the example above, or (2) as a decimal fraction. The division in Example 7 may be carried further by annexing zeros to the dividend until there is no remainder or until the number of decimal places in the quotient is as many as desired.

```
   31.25
108)3,375.00
  3 24
  ----
    135
    108
    ---
     27 0
     21 6
     ----
      5 40
      5 40
      ----
```

Notice that the decimal point in the quotient is always placed right above the decimal point in the dividend.

Complete quotient = 31.25

Check:

```
   108 (Divisor)
×31.25 (Quotient)
  ----
  5 40
  21 6
   108
 3 24
 ----
3,375.00 (Dividend)
```

When only the dividend contains decimal places, the decimal point in the quotient is placed directly above the decimal point in the dividend. Example 8 is used to illustrate this method.

When both the dividend and the divisor contain decimal places, the decimal point in the divisor is usually eliminated and the divisor becomes a