COLLEGE ALGEBRA

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TO MY CHILDREN
Anne Elizabeth Hungerford
Thomas Joseph Hungerford

TO MY SISTER
Susan Aileen Mercer

Preface

This book is designed to provide the essentials of algebra for students who have had two years of high school mathematics. It can be effectively used both by students who are preparing for calculus and by those for whom college algebra may be a terminal mathematics course. It follows the traditional college algebra order, beginning with a full chapter of review material and a chapter on solving equations.

Except for the order of topics and the absence of trigonometry, our general approach here is the same one used in our other books, *Precalculus Mathematics* and *Algebra & Trigonometry*. Our classroom experience has led to two inescapable conclusions:

(i) It is not sufficient merely to present the necessary technical tools, without explaining both how and why they work *in language that the student understands*.
(ii) A large number of students taking a course such as this at the college level simply do *not* understand the terse "definition-example-theorem-proof" style of many mathematics texts. For whatever reasons, such an approach does not appear to them as reasonable and logical, but rather as an arbitrary, unreasonable, and often incomprehensible foreign language.

We have attempted to write a book that deals effectively with these facts, one that helps students to discover that mathematics can be both comprehensible and reasonable. As a first step toward this goal, the text is designed to be readable and understandable by an average student, with a minimal amount of outside assistance. This has been accomplished without any sacrifice of rigor. But excessive and unnecessary formalism has been omitted in both format and content. We have done our best to present sound mathematics in an informal manner that stresses

detailed explanations of basic ideas, techniques and results;
extensive use of pictures and diagrams;
the fundamental reasons *why* a given technique works, as well as numerous examples showing *how* it is used;
the origins of many concepts (especially functions) in the real world and the ways that the common features of such real-life situations are abstracted to obtain a mathematical definition.

There is, alas, no way to do this in a short space. Consequently, a given topic may well occupy more space here than in some other texts. But this length is misleading. We have found that

because of the detailed explanations and numerous examples, a typical student can usually read the longer discussion here more easily and with greater understanding than a terse, compact presentation elsewhere.

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So in the long run the extra length should cause no serious scheduling problems. Most important, it provides the added benefit of greater understanding.

Another reason for the length of the book is that it has been designed for the instructor's convenience. It is extremely flexible and adaptable to a wide variety of courses. In many cases entire chapters or sections within chapters may be omitted, shortened, or covered in several different orders, *without* impairing the book's readability by students.

A complete discussion of the possible ways of using this text, including

> **the interdependence of the various chapters,**
> **the interdependence of the sections within each chapter,**
> **section-by-section pedagogical comments,**
> **numerous exam questions, and**
> **answers to exercises**

is given in the Instructor's Manual. It is available on request from the publisher.

Since calculators are now a fact of mathematical life, the book discusses both the uses and limitations of calculators in the study of functions. A student does not need to have a calculator to use the text, and exercises involving calculators are clearly labeled. But those students who do have one (the majority, in our experience) will benefit from seeing that calculators are not a substitute for learning the underlying theory, but can make that theory much easier to deal with in practical problems.

There are an unusually large number of exercises of widely varying types. The exercises labeled "A" are routine drill designed primarily to develop algebraic and manipulative skills. The "B" exercises are somewhat less mechanical and may occasionally require some careful thought. But any student who has read the text carefully should be able to do the majority of the "B" exercises. *The "C" label is used for exercises that are unusual* for one reason or another. Most of the time, "C" exercises are not difficult but are different from the sort of thing most students have seen before. A few of the "C" exercises are difficult mathematically. But most of them should be well within reach of most students.

Finally, there are scattered throughout the text sections labeled DO IT YOURSELF! Some of these include topics that are not absolutely essential, but which some instructors may want to include as a regular part of the course. Others provide interesting mathematical background or useful applications of the topics discussed in the text. Still others are needed by some students but not by others. Although they vary in level of difficulty, all of them can be read by students on their own.

Our earlier book, *Precalculus Mathematics*, contained almost half a page of acknowledgments. We are happy to thank all of those people once again. This book has benefited as much as the previous one from their contributions.

T. W. H.
R. E. M.

Cleveland, Ohio
Dayton, Ohio
August, 1981

To the Student

Read this—or you will turn into a frog!

If you want to succeed in this course, remember that *mathematics is not a spectator sport.* You can't learn math simply by listening to your instructor lecture or work problems, by looking at examples in the text, by reading the answers in the back of the book, or by borrowing your neighbor's homework. You have to take an active role, using pencil and paper and working out many problems yourself. And you *can* do this—even if you haven't taken math for a while or if you're a bit afraid of math—by making wise use of your chief resources: your instructor and this book.

When it comes to math textbooks, many students make a serious mistake, they use their books only for finding out what the homework problems are. If they get stuck on a problem, they page back through the text until they find a similar example. If the example doesn't clarify things, they may try reading part of the text (as little as possible). On a really bad day they may end up reading most of the section—piecemeal, from back to front. Rarely, if ever, do such students read through an entire section (or subsection) from beginning to end.

If this description fits you, don't feel guilty. Some mathematics texts *are* unreadable. But don't use your bad past experiences as an excuse for not reading this book. It has been classroom tested for years by students like yourself. Some parts were rewritten several times to improve their clarity. Consequently we can assure you that this book is readable and understandable by an average student, with a minimal amount of outside assistance. So if you want to get the most out of this course, we strongly suggest that you follow these guidelines:

1. Read the pages assigned by your instructor from beginning to end *before* starting the homework problems.
2. You can't read a math book as you would read a newspaper or a novel, so go slowly and carefully. If you find calculations you don't understand, take pencil and paper and try to work them out. If you don't understand a particular statement, reread the preceding material to see if you missed something.
3. Don't get bogged down on the first reading. If you have spent a reasonable amount of time trying to figure something out, mark the place with a question mark and continue reading.
4. When you've read through the assigned material once, go back and reread the parts you marked with question marks. You will often find that you can now understand many of them. Plan to ask your instructor about the rest.
5. *Now* do the homework problems. You should be able to do all, or almost all, of the assigned problems. After you've worked at the homework for a reasonable amount of time and answered as many problems as you can, mark the exercises that are still causing trouble. Plan to ask your instructor about them.

If you follow these five guidelines, you will get the most out of this book. But no book can answer all your questions. That's why your instructor is there. Unfortunately, many students are afraid to ask questions in class for fear that the questions will seem "dumb." Such students should remember this:

If you have honestly followed the five guidelines above and still have unanswered questions, then there are *at least* six other students in your class who have the same questions.

So it's not a dumb question. Furthermore, your instructor will welcome questions that arise from a serious effort on your part. In any case, your instructor is being paid (with your tuition money) to answer questions. So do yourself a favor and get your money's worth—*ask questions.*

Contents Overview

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Basic Algebra

Since it may have been some time since you had any algebra, you may be a bit rusty. This chapter is designed to help you remove the rust and to refresh your algebraic and manipulative skills. Although some students may remember enough to skip much of this chapter, others will need to spend considerably more time on it. Simply reading statements and examples is *not* enough. You must be able to handle these algebraic manipulations quickly and efficiently. The *only* way to achieve this level of skill is to do a large number of exercises.

1. THE REAL NUMBER SYSTEM

In grade school and high school mathematics you were introduced to the system of real numbers. These are the numbers used in everyday life. The most familiar ones are the **integers** (or whole numbers), that is, the numbers $0, 1, -1, 2, -2, 3, -3, \ldots$, and so on. The numbers $1, 2, 3, 4, \ldots$ are called the natural numbers or the positive integers.

The real number system also includes all **rational numbers,** that is, all fractions r/s, where r and s are integers and $s \neq 0$. Each of the following is a rational number:

$$\frac{1}{2} \qquad \frac{-4}{3} \qquad \frac{99{,}642}{716} \qquad \frac{-1400}{2} = \frac{-700}{1} = -700 \qquad 3\frac{5}{16} = \frac{53}{16} = 3.3125$$

The word "rational" in this context has no psychological implications. It refers to the ratio or quotient of two integers.

A crucial fact is that

some real numbers are *not* rational numbers

For example, consider a right triangle* with two equal sides of length 1, as shown in Figure 1-1.

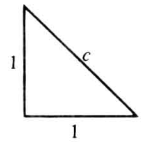

Figure 1-1

According to the Pythagorean Theorem,* the length of the hypotenuse* is a real number c that satisfies the equation $c^2 = 1^2 + 1^2 = 2$. (The number c is called the square root of 2 and is denoted $\sqrt{2}$.) We claim that

$$c = \sqrt{2} \text{ is } \textit{not} \text{ a rational number}$$

In other words, it is not possible to find integers a and b such that $(a/b)^2 = 2$.

A proof of this claim is given in Exercise C.1 on page 94. For now you can convince yourself of its *plausibility* by trying to find a rational number whose square is 2 (of course, you won't succeed). For example, if we square $\frac{1414}{1000}$ (= 1.414), we obtain 1.999396, which is *close* to 2, but is not exactly *equal* to 2. It doesn't take much of this to convince most people that no matter what rational number they square, the answer will never be *exactly* 2.† In other words, $\sqrt{2}$ cannot be a rational number.

A real number that is *not* a rational number (such as $\sqrt{2}$) is called an **irrational number** (ir-rational = not rational = not a ratio). Another well-known irrational number is the number π, which is used to calculate the area of a circle. In grade school you probably used $\frac{22}{7}$ or 3.1416 as the number π. But these rational numbers are just *approximations* of π (close to, but not quite *equal* to π).

Although only two irrational numbers have been mentioned, there are in fact infinitely many of them. Rationals and irrationals are further discussed in the DO IT YOURSELF! segment at the end of this section.

THE NUMBER LINE

The real number system can be pictured geometrically by using the real **number line** (or **coordinate line**). Take a straight line and choose a point on it; label this point 0 and call it the **origin.** Now choose some unit of measurement and label the point that is one unit to the *right* of 0 by the number 1. Using this unit length over and over, label the point one unit to the right of 1 as 2, the point one unit to the right of 2 as 3, and so on, as shown in Figure 1-2. Then do the same to the left of the origin. The point one unit to the left of 0 is labeled -1, the point one unit to the left of -1 is labeled -2, and so on. By now the scheme should be clear: The point $1\frac{1}{2}$ units to the *right* of 0 is labeled $1\frac{1}{2} = \frac{3}{2}$, the point 5.78 units to the *left* of 0 is labeled -5.78, and so on (see Figure 1-2).

*The terms "right triangle," "Pythagorean Theorem," and "hypotenuse" are explained in the Geometry Review at the end of this book.

† A calculator may display $\sqrt{2}$ as a decimal (rational number), such as 1.414213562. It may even display the square of this number as 2. But this happens because the calculator rounds off long decimals. If you perform the squaring by hand, you will find that the answer is *not* 2.

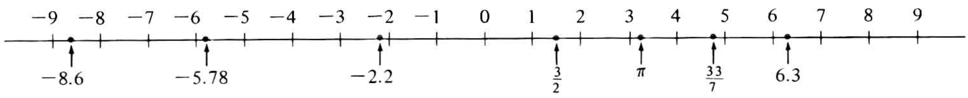

Figure 1-2

In the preceding construction of the number line, one fact is evident:

Every real number is the label of a unique point on the line.

We shall also assume the converse of this statement as a basic axiom:

Every point on the line has a unique real number label.

This correspondence between points on the line and real numbers will be used constantly. Strictly speaking, the number 3.6 and the point on the line labeled 3.6 are two different things. Nevertheless, we shall frequently use such phrases as "the point 3.6" or refer to a "number on the line." In context this usage will not cause any confusion. Indeed, the mental identification of real numbers with points on the line is often extremely helpful in understanding and solving various problems.

ORDER

The statement "a is less than b" (written $a < b$) and the statement "b is greater than a" (written $b > a$) mean exactly the same thing, namely, a lies to the *left* of b on the number line (or equivalently, b lies to the *right* of a on the number line).

EXAMPLE $-50 < 10$ and $10 < 30$ since on the number line -50 lies to the left of 10 and 10 lies to the left of 30 (see Figure 1-3).

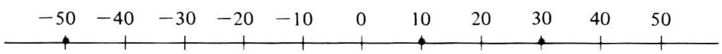

Figure 1-3

Often we shall write $a \leq b$ (or $b \geq a$), which means "a is less than or equal to b" (or "b is greater than or equal to a"). The statement $a \leq b \leq c$ means

$$a \leq b \quad \text{and} \quad b \leq c$$

Geometrically, the statement $a \leq b \leq c$ means that a lies to the left of c and b lies between a and c on the number line (and may possibly be equal to one or both of them). The preceding example shows that $-50 \leq 10 \leq 30$. Similarly, $a \leq b < c$ means $a \leq b$ *and* $b < c$ and so on.

The following facts are used frequently. For any real numbers a, b, and c

$a \leq a$;			
if $a \leq b$	*and*	$b \leq a$,	*then* $a = b$;
if $a \leq b$	*and*	$b \leq c$,	*then* $a \leq c$

These properties are easily verified by looking at the number line. The last property is also true if $\leq$ is replaced by $<$.

NEGATIVE NUMBERS AND NEGATIVES OF NUMBERS

The numbers to the right of 0 on the number line, that is,

$$\text{all numbers } a \text{ with } a > 0$$

are called **positive** numbers. The numbers to the left of 0, that is,

$$\text{all numbers } b \text{ with } b < 0$$

are called **negative** numbers. Finally, the **nonnegative** numbers are

$$\text{all numbers } a \text{ with } a \geq 0$$

Unfortunately for students, the word "negative" is used in two different ways in mathematics. As we just saw, *a negative* number is a number less than 0. The second usage of "negative" occurs in the phrase "*the negative of a* number":

The negative of a number *is the number obtained by changing the sign of the original number**

For example, the negative of 3 is the number -3 and the negative of -7 is the number 7. It is customary to consider 0 to be its own negative since $0 = -0$. Naturally, the use of the same word to describe two different situations can be confusing. Unfortunately, however, the usage is so widespread that we are stuck with it.

Observe that you can *always* change the sign of a given number by inserting a minus sign in front of it. For instance, to change the sign of 3 we insert a minus sign and get -3. To change the sign of the number -7 we put a minus sign in front and obtain $-(-7) = 7$. Consequently, the following statement is always true:

The negative of the number c *is the number* $-c$.

When you see a statement such as the one in the box above, it is important to remember that the number c may be either positive or negative or zero. As we have just seen, the negative of the positive number 3 is the negative number -3 and the negative of the negative number -7 is the positive number 7. Thus

If c *is a positive number, then* $-c$ *is a negative number.*
If c *is a negative number, then* $-c$ *is a positive number.*

*The negative of a number is sometimes called the additive inverse of the number.

ARITHMETIC

A summary of the important properties of addition, subtraction, multiplication, and division of real numbers is given below. You have been using these properties for years but may not have seen them stated in this manner. So if you don't understand a statement in one of the boxes, consult the numerical examples immediately after the box for clarification. Remember that the letters b, c, and d used in the boxes may represent positive or negative numbers or zero. Some of these properties (or "laws") have names. These names are convenient for reference purposes, but it is more important that you understand the meaning of each property and be able to use it.

NEGATIVES

(*i*) *The sum of a real number* c *and its negative is zero; that is,* $c + (-c) = 0$.

(*ii*) $-(-c) = c$;

(*iii*) $(-1) \cdot c = -c$

If b *and* c *are any real numbers, then*

(*iv*) $b + (-c) = b - c$;

(*v*) $-(b + c) = -b - c$;

(*vi*) $b - (-c) = b + c$

EXAMPLE (i) Let $c = 5$; then $5 + (-5) = 0$. Let $c = -\frac{3}{4}$; then $(-\frac{3}{4}) + [-(-\frac{3}{4})] = 0$.

EXAMPLE (ii) Let $c = 4$; then $-(-4) = 4$.

EXAMPLE (iii) Let $c = 5$; then $(-1) \cdot 5 = -5$. Let $c = -6$; then $(-1)(-6) = 6 = -(-6)$.

EXAMPLE (iv) Let $b = 5$ and $c = 7$; then $5 + (-7) = 5 - 7 = -2$. Let $b = -2$ and $c = 3$; then $-2 + (-3) = -2 - 3 = -5$.

EXAMPLE (v) Let $b = -7$ and $c = 3$; then $-(-7 + 3) = -(-7) - 3 = 7 - 3 = 4$.

EXAMPLE (vi) Let $b = \frac{3}{4}$ and $c = \frac{19}{4}$; then $\frac{3}{4} - (-\frac{19}{4}) = \frac{3}{4} + \frac{19}{4} = \frac{22}{4} = \frac{11}{2}$.

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