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3000 SOLVED PROBLEMS IN

PHYSICS

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by

Alvin Halpern, Ph.D.
Brooklyn College

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Alvin Halpern, Ph.D., *Professor of Physics at Brooklyn College*
Dr. Halpern has extensive teaching experience in physics and is the chairman of the physics department at Brooklyn College. He is a member of the executive committee for the doctoral program in physics at CUNY and has written numerous research articles.

Other Contributors to This Volume

- Frederick Bueche, Ph.D., *University of Dayton*
- Don Chodrow, Ph.D., *James Madison University*
- John Cooper, Ph.D., *Naval Postgraduate School*
- Alan H. Cromer, Ph.D., *Northeastern University*
- Robert Eisberg, Ph.D., *University of California, Santa Barbara*
- Ronald Gautreau, Ph.D., *New Jersey Institute of Technology*
- Eugene Hecht, Ph.D., *Adelphi University*
- Roland H. Ingham, Jr., M.D., *Brigham Hospital, MA*
- Joseph J. Kepes, Ph.D., *University of Dayton*
- Lawrence S. Lerner, Ph.D., *University of California, Long Beach*
- Donald R. Pitts, Ph.D., *Tennessee Technological University*
- William Savin, Ph.D., *New Jersey Institute of Technology*
- Leighton E. Sissom, Ph.D., *Tennessee Technological University*
- Harold S. Slusher, Ph.D., *University of Dayton*
- Paul E. Tippens, Ph.D., *Southern Technical Institute*
- Dare A. Wells, Ph.D., *University of Texas, El Paso*

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To the Student

This book is intended for use by students of general physics, either in calculus- or noncalculus-based courses. Problems requiring real calculus (not merely calculus notation) are marked with a small superscript **c**.

The only way to master general physics is to gain ability and sophistication in problem-solving. This book is meant to make you a master of the art—and should do so if used properly. A problem is usually easiest solved if you have learned the ideas behind it; but, sometimes, these very ideas emerge in the course of a rash attack. If you are having difficulty with a topic, select a few solved problems at random in that area and examine them carefully. Then try to solve related problems without looking at the printed solutions. When you have done this, check your results.

There are numerous ways of posing a problem and, frequently, numerous ways of solving one. You should try to gain understanding of how to approach various classes of problems, rather than memorizing particular solutions. Understanding is better than memory for succeeding in physics.

The problems in this book cover every important topic, and they are drawn from a variety of expert sources. They range from the simple to the complex and offer the student in almost any college or university physics course plenty of practice and food for thought.

CONTENTS

CHAPTER 1

Mathematical Introduction

1.1 PLANAR VECTORS, SCIENTIFIC NOTATION, AND UNITS

1.1 What is a scalar quantity?

A scalar quantity has only magnitude; it is a pure number. Scalars, being simple numbers, are added, subtracted, etc., in the usual way.

1.2 What is a vector quantity?

A vector quantity has both magnitude and direction. For example, a car moving south at 40 km/h has a *vector velocity* of 40 km/h southward.

A vector quantity can be represented by an arrow drawn to scale. The length of the arrow is proportional to the magnitude of the vector quantity (40 km/h in the above example). The direction of the arrow represents the direction of the vector quantity.

1.3 What is the 'resultant' vector?

The resultant of a number of similar vectors, force vectors, for example, is that single vector which would have the same effect as all the original vectors taken together.

1.4 Describe the graphical addition of vectors.

The method for finding the resultant of several vectors consists in beginning at any convenient point and drawing (to scale) each vector arrow in turn. They may be taken in any order of succession. The tail end of each arrow is attached to the tip end of the preceding one.

The resultant is represented by an arrow with its tail end at the starting point and its tip end at the tip of the last vector added.

1.5 Describe the parallelogram method of addition of two vectors.

The resultant of two vectors acting at any angle may be represented by the diagonal of a parallelogram. The two vectors are drawn as the sides of the parallelogram and the resultant is its diagonal, as shown in Fig. 1-1. The direction of the resultant is away from the origin of the two vectors.

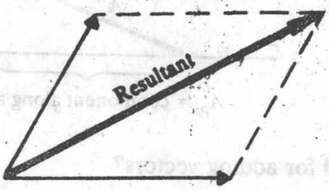

Fig. 1-1

1.6 How do you subtract vectors?

To subtract a vector B from a vector A, reverse the direction of B and add it vectorially to vector A, that is, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})$.

1.7 Describe the trigonometric functions.

For the right triangle shown in Fig. 1-2, by definition

$$\sin\theta=\frac{o}{h}\qquad \cos\theta=\frac{a}{h}\qquad \tan\theta=\frac{o}{a}$$

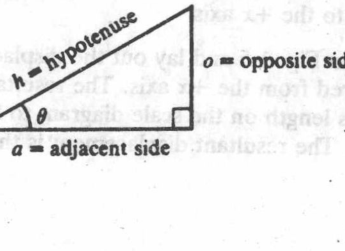

Fig. 1-2

1.8 Express each of the following in scientific notation: (*a*) 627.4, (*b*) 0.000365, (*c*) 20 001, (*d*) 1.0067, (*e*) 0.0067.

(*a*) 6.274×10^2. (*b*) 3.65×10^{-4}. (*c*) 2.001×10^4. (*d*) 1.0067×10^0. (*e*) 6.7×10^{-3}.

1.9 Express each of the following as simple numbers $\times 10^0$: (*a*) 31.65×10^{-3} (*b*) 0.415×10^6 (*c*) $1/(2.05 \times 10^{-3})$ (*d*) $1/(43 \times 10^3)$.

(*a*) 0.03165. (*b*) 415,000. (*c*) 488. (*d*) 0.0000233.

1.10 The diameter of the earth is about 1.27×10^7 m. Find its diameter in (*a*) millimeters, (*b*) megameters, (*c*) miles.

(*a*) $(1.27 \times 10^7 \text{ m})(1000 \text{ mm}/1 \text{ m}) = \underline{1.27 \times 10^{10} \text{ mm}}$. (*b*) Multiply meters by $1 \text{ Mm}/10^6$ m to obtain 12.7 Mm. (*c*) Then use (1 km/1000 m)(1 mi/1.61 km); the diameter is $\underline{7.89 \times 10^3 \text{ mi}}$.

1.11 A 100-m race is run on a 200-m-circumference circular track. The runners run eastward at the start and bend south. What is the displacement of the endpoint of the race from the starting point?

The runners move as shown in Fig. 1-3. The race is halfway around the track so the displacement is one diameter $= \underline{200/\pi = 63.7 \text{ m}}$ due south.

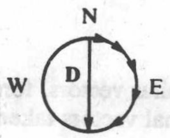

Fig. 1-3

1.12 What is a *component* of a vector?

A component of a vector is its "shadow" (perpendicular drop) on an axis in a given direction. For example, the *p*-component of a displacement is the distance along the *p* axis corresponding to the given displacement. It is a scalar quantity, being positive or negative as it is positively or negatively directed along the axis in question. In Fig. 1-4, A_p is positive. (One sometimes defines a vector component as a *vector* pointing along the axis and having the size of the scalar component. If the scalar component is negative the vector component points in the negative direction along the axis.) It is customary, and useful, to resolve a vector into components along *mutually perpendicular* directions (*rectangular components*).

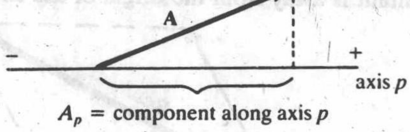

Fig. 1-4

1.13 What is the component method for adding vectors?

Each vector is resolved into its *x*, *y*, and *z* components, with negatively directed components taken as negative. The *x* component of the resultant R_x is the algebraic sum of all the *x* components. The *y* and *z* components of the resultant are found in a similar way.

1.14 Define the multiplication of a vector by a scalar.

The quantity $b\mathbf{F}$ is a vector having magnitude $|b|\,F$ (the absolute value of *b* times the magnitude of **F**); the direction of $b\mathbf{F}$ is that of **F** or $-\mathbf{F}$, depending on whether *b* is positive or negative.

1.15 Using the graphical method, find the resultant of the following two displacements: 2 m at 40° and 4 m at 127°, the angles being taken relative to the +*x* axis.

Choose *x*, *y* axes as shown in Fig. 1-5 and lay out the displacements to scale tip to tail from the origin. Note that all angles are measured from the +*x* axis. The resultant vector, **R**, points from starting point to endpoint as shown. Measure its length on the scale diagram to find its magnitude, 4.6 m. Using a protractor, measure its angle θ to be 101°. The resultant displacement is therefore 4.6 m at 101°.

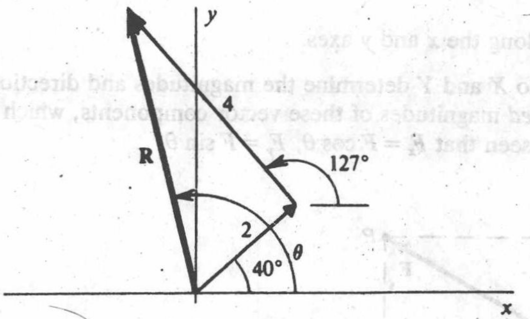

Fig. 1-5

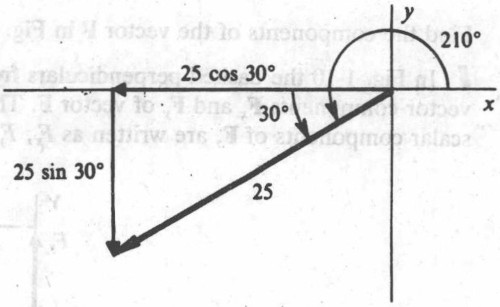

Fig. 1-6

1.16 Find the x and y components of a 25-m displacement at an angle of 210°.

▌ The vector displacement and its components are shown in Fig. 1-6. The components are

$$x \text{ component} = -25 \cos 30° = \underline{-21.7 \text{ m}} \qquad y \text{ component} = -25 \sin 30° = \underline{-12.5 \text{ m}}$$

Note in particular that each component points in the negative coordinate direction and must therefore be taken as negative.

1.17 Solve Prob. 1.15 by use of rectangular components.

▌ Resolve each vector into rectangular components as shown in Fig. 1-7(*a*) and (*b*). (Place a cross-hatch symbol on the original vector to show that it is replaced by its components.) The resultant has the components

$$R_x = 1.53 - 2.40 = -0.87 \text{ m} \qquad R_y = 1.29 + 3.20 = 4.49 \text{ m}$$

Note that components pointing in the negative direction must be assigned a negative value.

The resultant is shown in Fig. 1-7(*c*); we see that

$$R = \sqrt{(0.87)^2 + (4.49)^2} = \underline{4.57 \text{ m}} \qquad \tan \phi = \frac{4.49}{0.87}$$

Hence, $\phi = \underline{79°}$, from which $\theta = 180° - \phi = \underline{101°}$.

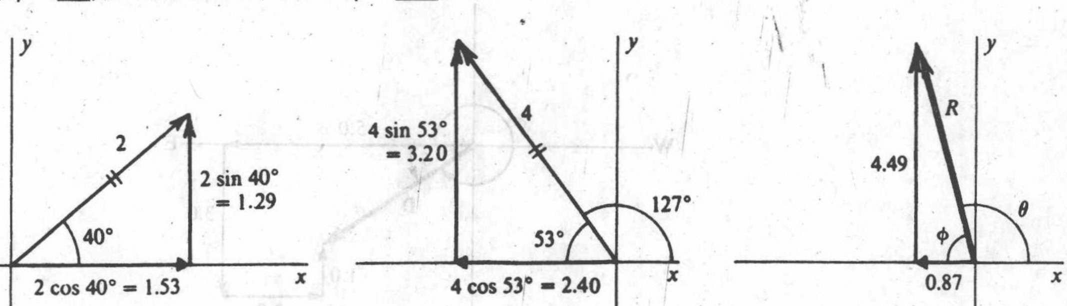

Fig. 1-7

1.18 Add the following two force vectors by use of the parallelogram method: 30 pounds at 30° and 20 pounds at 140°. (A *pound of force* is chosen such that a 1-kg object weighs 2.21 lb on earth. One pound is equivalent to a force of 4.45 N.)

▌ The force vectors are shown in Fig. 1-8. Construct a parallelogram using them as sides, as shown in Fig. 1-9. The resultant, **R**, is then shown as the diagonal. Measurement shows that **R** is $\underline{30 \text{ lb at } 72°}$.

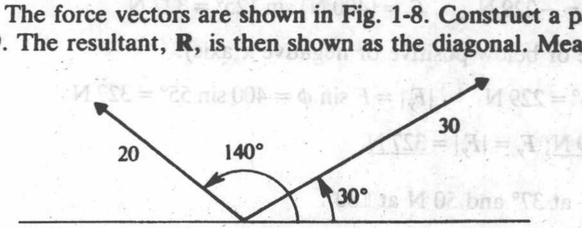

Fig. 1-8

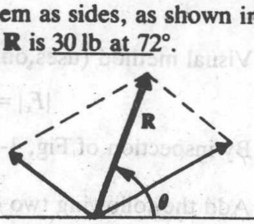

Fig. 1-9

1.19 Find the components of the vector **F** in Fig. 1-10 along the x and y axes.

In Fig. 1-10 the dashed perpendiculars from P to X and Y determine the magnitudes and directions of the vector components $\mathbf{F}_x$ and $\mathbf{F}_y$ of vector **F**. The *signed* magnitudes of these vector components, which are the scalar components of **F**, are written as F_x, F_y. It is seen that $F_x = F\cos\theta$, $F_y = F\sin\theta$.

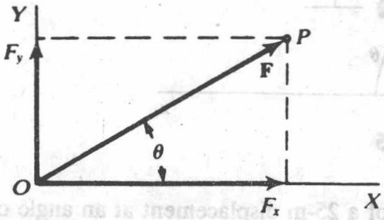

Fig. 1-10

1.20 **(a)** Let **F** have a magnitude of 300 N and make angle $\theta = 30°$ with the positive x direction. Find F_x and F_y. **(b)** Suppose that $F = 300$ N and $\theta = 145°$ (**F** is here in the second quadrant). Find F_x and F_y.

(a) $F_x = 300\cos 30° = \underline{259.8\text{ N}}$, $F_y = 300\sin 30° = \underline{150\text{ N}}$. **(b)** $F_x = 300\cos 145° = (300)(-0.8192) = \underline{-245.75\text{ N}}$ (in the negative direction of X), $F_y = 300\sin 145° = (300)(+0.5736) = \underline{172.07\text{ N}}$

1.21 A car goes 5.0 km east, 3.0 km south, 2.0 km west, and 1.0 km north. **(a)** Determine how far north and how far east it has been displaced. **(b)** Find the displacement vector both graphically and algebraically.

(a) Recalling that vectors can be added in any order we can immediately add the 3.0-km south and 1.0-km north displacement vectors to get a net 2.0-km south displacement vector. Similarly the 5.0-km east and 2.0-km west vectors add to a 3-km east displacement vector. Because the east displacement contributes no component along the north-south line and the south displacement has no component along the east-west line, the car is −2.0 km north and 3.0 km east of its starting point. **(b)** Using the head-to-tail method, we easily can construct the resultant displacement **D** as shown in Fig. 1-11. Algebraically we note that

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{2^2 + 3^2} = \underline{3.6\text{ km}} \qquad \tan\phi = -\tfrac{2}{3} \quad \text{or} \quad \tan\theta = \tfrac{2}{3} \qquad \theta = \underline{34°}\text{ south of east.}$$

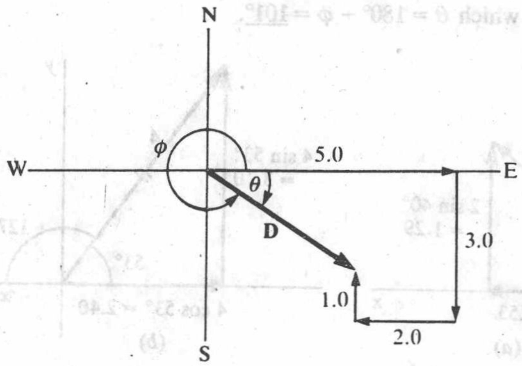

Fig. 1-11

1.22 Find the x and y components of a 400-N force at an angle of 125° to the x axis.

Formal method (uses angle above positive x axis):

$$F_x = (400\text{ N})\cos 125° = -229\text{ N} \qquad F_y = (400\text{ N})\sin 125° = 327\text{ N}$$

Visual method (uses only acute angles above or below positive or negative x axis):

$$|F_x| = F\cos\phi = 400\cos 55° = 229\text{ N} \qquad |F_y| = F\sin\phi = 400\sin 55° = 327\text{ N}$$

By inspection of Fig. 1-12, $F_x = -|F_x| = \underline{-229\text{ N}}$; $F_y = |F_y| = \underline{327\text{ N}}$.

1.23 Add the following two coplanar forces: 30 N at 37° and 50 N at 180°.

Split each into components and find the resultant: $R_x = 24 - 50 = -26$ N, $R_y = 18 + 0 = 18$ N. Then $R = \underline{31.6\text{ N}}$ and $\tan\theta = 18/-26$, so $\theta = \underline{145°}$.

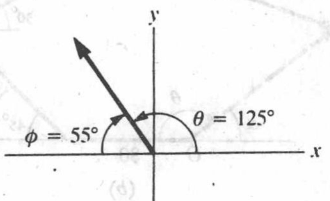

Fig. 1-12

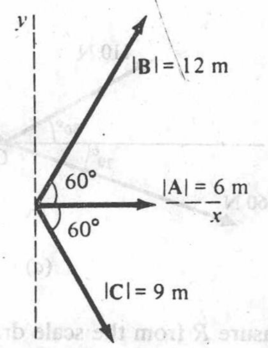

Fig. 1-13

1.24 For the vectors **A** and **B** shown in Fig. 1-13, find ***(a)*** **A + B**, ***(b)*** **A − B**, and ***(c)*** **B − A**.

▌ The components are $A_x = 6$ m, $A_y = 0$, $B_x = 12 \cos 60° = 6$ m, and $B_y = 12 \sin 60° = 10.4$ m. ***(a)*** $(\mathbf{A}+\mathbf{B})_x = 12$ m and $(\mathbf{A}+\mathbf{B})_y = 10.4$ m, so that $\mathbf{A}+\mathbf{B} = \underline{15.9 \text{ m}}$ at $\underline{40.9°}$, ***(b)*** $(\mathbf{A}-\mathbf{B})_x = 0$ and $(\mathbf{A}-\mathbf{B})_y = 0 - 10.4$ so $\mathbf{A}-\mathbf{B} = \underline{10.4 \text{ m}}$ at $\underline{-90°}$. ***(c)*** $(\mathbf{B}-\mathbf{A})_x = 0$ and $(\mathbf{B}-\mathbf{A})_y = 10.4 - 0$ so $\mathbf{B}-\mathbf{A} = \underline{10.4 \text{ m}}$ at $\underline{90°}$.

1.25 For the vectors shown in Fig. 1-13, find ***(a)*** **A + B + C** and ***(b)*** **A + B − C**.

▌ The x and y components of **C** are 4.5 m and −7.8 m. ***(a)*** The x component is $A_x + B_x + C_x = 16.5$ and for the y component we find 2.6, so the vector is $\underline{16.7 \text{ m}}$ at $\underline{9.0°}$. ***(b)*** $A_x + B_x - C_x = 7.5$ and the y component is $0 + 10.4 - (-7.8) = 18.2$; changing this to a magnitude and angle, we find $\underline{19.7 \text{ m}}$ at $\underline{68°}$.

1.26 For the vectors shown in Fig. 1-13, find ***(a)*** **A − 2C**, ***(b)*** **B − (A + C)**, and ***(c)*** **−A − B − C**.

▌ ***(a)*** The x component is $A_x - 2C_x = -3$ and the y component is $-2(-7.8) = 15.6$, giving $\underline{15.9 \text{ m}}$ at $\underline{101°}$. ***(b)*** The x component $= 6 - (6 + 4.5) = -4.5$; the y component $= 10.4 - [0 + (-7.8)] = 18.2$; therefore $(4.5^2 + 18.2^2)^{1/2} = \underline{18.7 \text{ m}}$ at $\underline{104°}$. ***(c)*** This is the negative of the vector of Prob. 1.25 ***(a)***, so that it $= \underline{16.7 \text{ m}}$ at $9.0° + 180° = \underline{189°} = \underline{-171°}$.

1.27 A displacement of 20 m is made in the xy plane at an angle of 70° (i.e., 70° counterclockwise from the $+x$ axis). Find its x and y components. Repeat if the angle is 120°; if the angle is 250°.

▌ In each case $s_x = s \cos\theta$ and $s_y = s \sin\theta$. The results are $\underline{6.8}$, $\underline{18.8 \text{ m}}$; $\underline{-10.0}$, $\underline{17.3 \text{ m}}$; $\underline{-6.8}$, $\underline{-18.8 \text{ m}}$.

1.28 It is found that an object will hang properly if an x force of 20 N and a y force of −30 N are applied to it. Find the single force (magnitude and direction) which would do the same job.

▌ Adding components of the forces yields $R_x = 20$ N and $R_y = -30$ N. $R = (400 + 900)^{1/2} = \underline{36 \text{ N}}$. Calling θ the counterclockwise angle from the $+x$ axis, $\tan\theta = -30/20$ and so $\theta = \underline{303.7°} = \underline{-56.3°}$.

1.29 Find the magnitude and direction of the force which has an x component of −40 N and a y component of −60 N.

▌ The resultant of these two forces is $R = (1600 + 3600)^{1/2} = \underline{72 \text{ N}}$. The angle θ is $180° + \tan^{-1}(6/4) = \underline{236.3°}$.

1.30 Find the magnitude and direction of the sum of the following two coplanar displacement vectors: 20 m at 0° and 10 m at 120°.

▌ Splitting each into components, $R_x = 20 - 5 = 15$ m and $R_y = 0 + 8.7 = 8.7$ m. Then $R = \underline{17.3 \text{ m}}$ with $\tan\theta = 8.7/15$ giving $\theta = \underline{30°}$.

1.31 Four coplanar forces act on a body at point O as shown in Fig. 1-14(*a*). Find their resultant graphically.

▌ Starting from O, the four vectors are plotted in turn as shown in Fig. 1.14(*b*). Place the tail end of one vector at the tip end of the preceding one. The arrow from O to the tip of the last vector represents the resultant of the vectors.

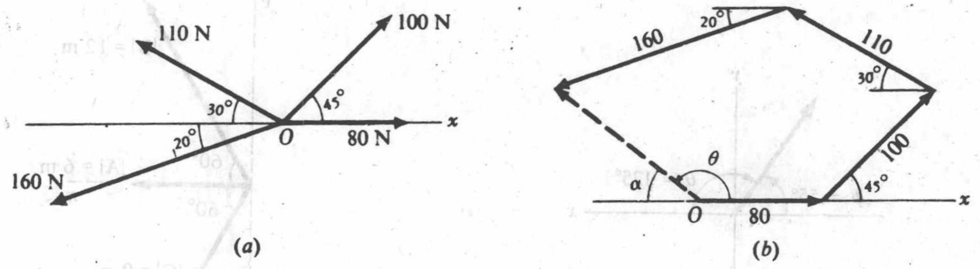

Fig. 1-14

Measure R from the scale drawing in Fig. 1.14(b) and find it to be 119 N. Angle α is measured by protractor and is found to be 37°. Hence the resultant makes an angle $\theta = 180° - 37° = 143°$ with the positive x axis. The resultant is 119 N at 143°.

1.32 Solve Prob. 1.31 by use of the rectangular component method.

▮ The vectors and their components are as follows.

magnitude, N	x component, N	y component, N
80	80	0
100	100 cos 45° = 71	100 sin 45° = 71
110	−110 cos 30° = −95	110 sin 30° = 55
160	−160 cos 20° = −150	−160 sin 20° = −55

Note the sign of each component. To find the resultant, we have

$$R_x = 80 + 71 - 95 - 150 = -94\ \text{N} \qquad R_y = 0 + 71 + 55 - 55 = 71\ \text{N}$$

The resultant is shown in Fig. 1-15; we see that $R = \sqrt{(94)^2 + (71)^2} = 118$ N. Further, $\tan\alpha = 71/94$, from which $\alpha = 37°$. Therefore the resultant is 118 N at $180 - 37 = 143°$.

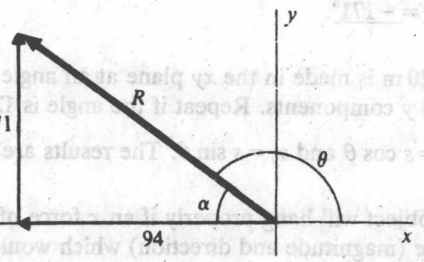

Fig. 1-15

1.33 Perform graphically the following vector additions and subtractions, where **A**, **B**, and **C** are the vectors shown in Fig. 1-16: (a) **A** + **B**. (b) **A** + **B** + **C**. (c) **A** − **B**. (d) **A** + **B** − **C**.

▮ See Fig. 1-16(a) through (d). In (c), **A** − **B** = **A** + (−**B**); that is, to subtract **B** from **A**, reverse the direction of **B** and add it vectorially to **A**. Similarly, in (d), **A** + **B** − **C** = **A** + **B** + (−**C**), where −**C** is equal in magnitude but opposite in direction to **C**.

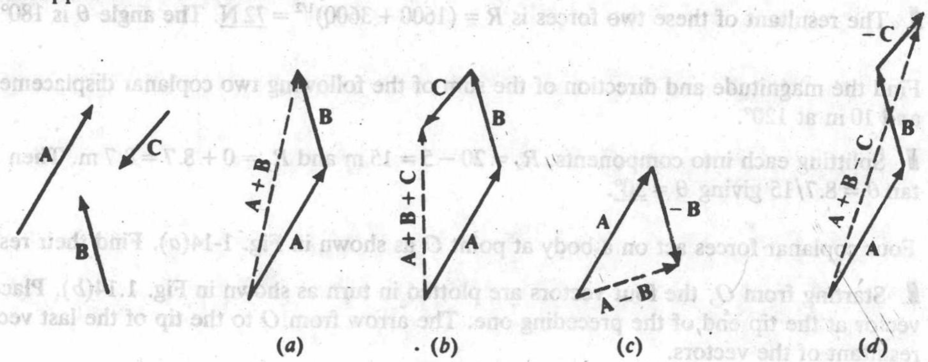

Fig. 1-16

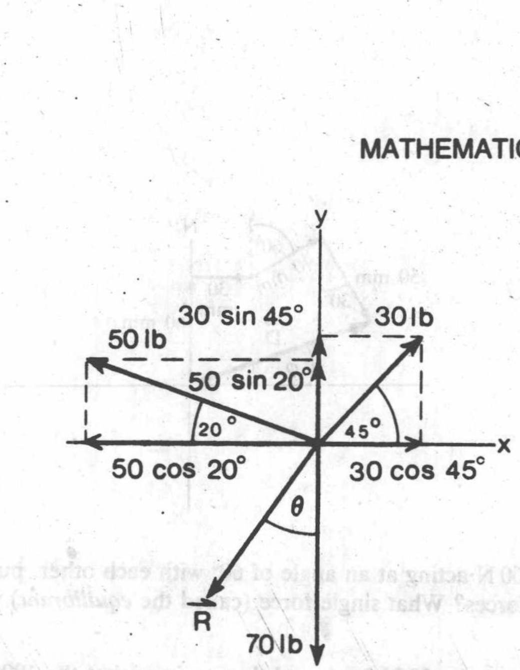

Fig. 1-17

1.34 Find the resultant **R** of the following forces all acting on the same point in the given directions: 30 lb to the northeast; 70 lb to the south; and 50 lb 20° north of west.

▮ Choose east as the positive x direction (Figure 1-17).

***x* components, lb**	***y* components, lb**
$30\cos 45° = 21.2$	$30\sin 45° = 21.2$
$-50\cos 20° = \underline{-47.0}$	$50\sin 20° = 17.1$
Total $= -25.8$ lb	$-70 = \underline{-70}$
	Total $= -31.7$ lb

$$R = \sqrt{(-25.8)^2 + (-31.7)^2} = \sqrt{665.8 + 1004.9} = \underline{40.9\text{ lb}} \qquad \tan\theta = \frac{-25.8}{-31.7} = 0.8139° \qquad \theta = \underline{39° \text{ west of south}}$$

1.35 Find the angle between two vector forces of equal magnitude, such that the resultant is one-third as much as either of the original forces.

▮ In the vector force diagram (Fig. 1-18), the diagonals of the rhombus bisect each other. Thus,

$$\cos\theta = \frac{F/6}{F} = \frac{1}{6} = 0.1667 \qquad \theta = 80.4° \qquad 2\theta = 160.8°$$

The angle between the two forces is $\underline{160.8°}$.

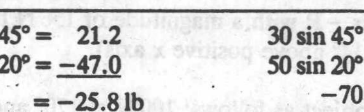

Fig. 1-18

1.36 Find the vector sum of the following four displacements on a map: 60 mm north; 30 mm west; 40 mm at 60° west of north; 50 mm at 30° west of south. Solve (*a*) graphically and (*b*) algebraically.

▮ (*a*) With ruler and protractor, construct the sum of vector displacements by the tail-to-head method as shown in Fig. 1-19. The resultant vector from tail of first to head of last is then also measured with ruler and protractor. *Ans.*: 97 mm at 67.7° W of N. (*b*) Let **D** = resultant displacement.

$$D_x = -30 - 40\sin 60° - 50\sin 30° = -89.6\text{ mm} \qquad D_y = 60 + 40\cos 60° - 50\cos 30° = +36.7\text{ mm}$$

$$D = \sqrt{D_x^2 + D_y^2} = \underline{96.8\text{ mm}} \qquad \tan\phi = \left|\frac{D_y}{D_x}\right| \Rightarrow \phi = \underline{22.3°} \quad \text{above negative } x \text{ axis}$$

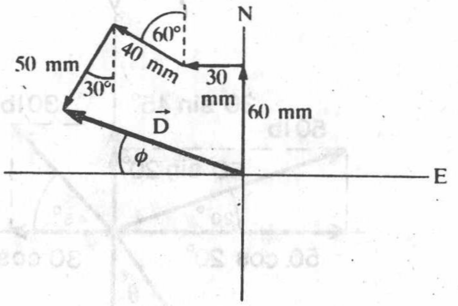

Fig. 1-19

1.37 Two forces, 80 N and 100 N acting at an angle of 60° with each other, pull on an object. What single force would replace the two forces? What single force (called the *equilibrant*) would balance the two forces? Solve algebraically.

▮ Choose the x axis along the 80-N force and the y axis so that the 100-N force 60° above the positive x axis has a positive y component. Then the single force **R** that replaces the two forces is the vector sum of these forces:

$$R_x = 80 + 100\cos 60° = 130\text{ N} \qquad R_y = 100\sin 60° = 87\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{156\text{ N}} \qquad \theta = \tan^{-1}\left|\frac{R_y}{R_x}\right| = \underline{34°}\text{ above positive } x \text{ axis.}$$

The force that balances **R** is $-\mathbf{R}$ with a magnitude of 156 N but pointing in the opposite direction to **R**: 34° below negative x axis (or 214° above positive x axis).

1.38 Two forces act on a point object as follows: 100 N at 170° and 100 N at 50°. Find their resultant.

▮ $\mathbf{F}_1 = 100$ N at 170° above x axis; $\mathbf{F}_2 = 100$ N at 50° above x axis.

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 \qquad R_x = 100\cos 170° + 100\cos 50° = -34.2\text{ N} \qquad R_y = 100\sin 170° + 100\sin 50° = 94.0\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{100\text{ N}} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

has two solutions: 290° and 110°. From a look at its components we see that **R** lies in the second quadrant, so the answer is 110° (or 70° above negative x axis). In a less formal approach we can find: $\phi = \tan^{-1}|R_y/R_x|$, which will give an acute angle solution, in this case of 70°, which always represents the angle with the positive or negative x axis, and either above or below that axis. Since we already know from the components which quadrant **R** lies in, we know the direction precisely. In our case the 70° is above the negative x axis.

1.39 A force of 100 N makes an angle of θ with the x axis and has a y component of 30 N. Find both the x component of the force and the angle θ.

▮ The data are sketched in Fig. 1-20. We wish to find F_x and θ. We know that

$$\sin\theta = \frac{o}{h} = \frac{30}{100} = 0.30$$

from which $\theta = \underline{17.5°}$. Then, since $a = h\cos\theta$, we have

$$F_x = 100\cos 17.5° = \underline{95.4\text{ N}}$$

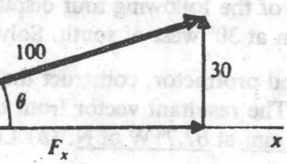

Fig. 1-20

1.40 A boat can travel at a speed of 8 km/h in still water on a lake. In the flowing water of a stream, it can move at 8 km/h relative to the water in the stream. If the stream speed is 3 km/h, how fast can the boat move past a tree on the shore in traveling (*a*) upstream? (*b*) downstream?

I **(a)** If the water were standing still, the boat's speed past the tree would be 8 km/h. But the stream is carrying it in the opposite direction at 3 km/h. Therefore the boat's speed relative to the tree is $8-3=\underline{5\text{ km/h}}$. **(b)** In this case, the stream is carrying the boat in the same direction the boat is trying to move. Hence its speed past the tree is $8+3=\underline{11\text{ km/h}}$.

1.41 A plane is traveling eastward at an airspeed of 500 km/h. But a 90 km/h wind is blowing southward. What are the direction and speed of the plane relative to the ground?

I The plane's resultant velocity is the sum of two velocities, 500 km/h eastward and 90 km/h southward. These component velocities are shown in Fig. 1-21. The plane's resultant velocity is found by use of

$$R=\sqrt{(500)^2+(90)^2}=\underline{508\text{ km/h}}$$

The angle α is given by

$$\tan\alpha=\frac{90}{500}=\underline{0.180}$$

from which $\alpha=\underline{10.2°}$. The plane's velocity relative to the ground is 508 km/h at 10.2° south of east.

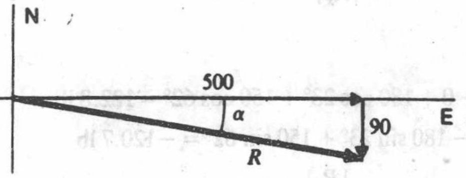

Fig. 1-21

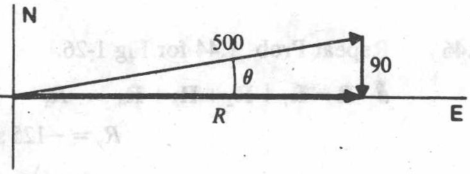

Fig. 1-22

1.42 With the same airspeed as in Prob. 1.41, in what direction must the plane head in order to move due east relative to the earth?

I The sum of the plane's velocity through the air and the velocity of the wind must be the resultant eastward velocity of the plane relative to the earth. This is shown in the vector diagram of Fig. 1-22. It is seen that $\sin\theta=90/500$, from which $\theta=10.4°$. The plane should head $\underline{10.4°}$ north of east if it is to move eastward on the earth.

If we wish to find the plane's eastward speed, Fig. 1-22 tells us that $R=500\cos\theta=\underline{492\text{ km/h}}$.

1.43 A child pulls a rope attached to a sled with a force of 60 N. The rope makes an angle of 40° to the ground. **(a)** Compute the effective value of the pull tending to move the sled along the ground. **(b)** Compute the force tending to lift the sled vertically.

I As shown in Fig. 1-23, the components of the 60 N force are 39 N and 46 N. **(a)** The pull along the ground is the horizontal component, $\underline{46\text{ N}}$. **(b)** The lifting force is the vertical component, $\underline{39\text{ N}}$.

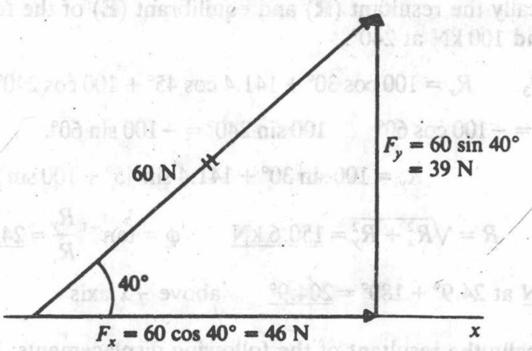

Fig. 1-23

1.44 Find the resultant of the coplanar force system shown in Fig. 1-24.

I $\mathbf{R}=\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3$ $\quad R_x=-40+80\cos 30°+0=29.3\text{ lb}$ $\quad R_y=0-80\sin 30°+60=20\text{ lb}$

$$R=\sqrt{R_x^2+R_y^2}=\underline{35.4\text{ lb}}\qquad \theta=\tan^{-1}\frac{R_y}{R_x}=\underline{34.3°}\quad\text{above }+x\text{ axis}$$

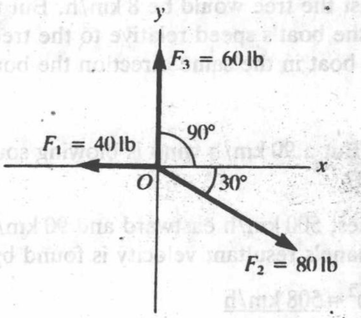

Fig. 1-24

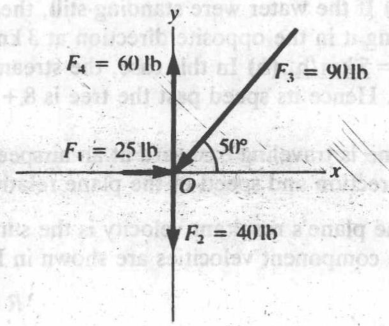

Fig. 1-25

1.45 Repeat Prob. 1.44 for Fig. 1-25.

▮ $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$ $\quad R_x = 25 + 0 - 90\cos 50° + 0 = -32.8$ lb $\quad R_y = 0 - 40 - 90\sin 50° + 60 = 19.2$ lb

$$R = \sqrt{R_x^2 + R_y^2} = \underline{38.0 \text{ lb}} \qquad \phi = \tan^{-1}\left|\frac{R_y}{R_x}\right| = \underline{30.3°} \quad \text{above } -x \text{ axis}$$

1.46 Repeat Prob. 1.44 for Fig 1-26.

▮ $\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3 + \mathbf{R}_4$ $\quad R_x = -125\cos 25° + 0 + 180\cos 23° + 150\cos 62° = 122.8$ lb

$$R_y = -125\sin 25° - 130 - 180\sin 23° + 150\sin 62° = -120.7 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{172.2 \text{ lb}}, \qquad \phi = \tan^{-1}\left|\frac{R_y}{R_x}\right| = \underline{44.5°} \quad \text{below } +x \text{ axis}$$

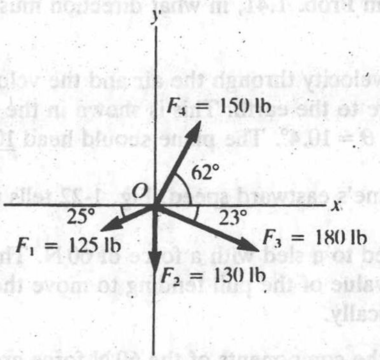

Fig. 1-26

1.47 Compute algebraically the resultant (**R**) and equilibrant (**E**) of the following coplanar forces: 100 kN at 30°, 141.4 kN at 45°, and 100 kN at 240°.

▮ $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ $\quad R_x = 100\cos 30° + 141.4\cos 45° + 100\cos 240° = 136.6$ kN.

Note: $100\cos 240° = -100\cos 60°$ $\quad 100\sin 240° = -100\sin 60°$.

$$R_y = 100\sin 30° + 141.4\sin 45° + 100\sin 240° = 63.4 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{150.6 \text{ kN}} \qquad \phi = \tan^{-1}\frac{R_y}{R_x} = \underline{24.9°} \quad \text{above } +x \text{ axis.}$$

$\mathbf{E} = -\mathbf{R} = \underline{150.6 \text{ kN}}$ at $24.9° + 180° = \underline{204.9°}$ above $+x$ axis

1.48 Compute algebraically the resultant of the following displacements: 20 m at 30°, 40 m at 120°, 25 m at 180°, 42 m at 270°, and 12 m at 315°.

▮ $\mathbf{D} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 =$ resultant displacement $\quad D_x = 20\cos 30° + 40\cos 120° - 25 + 0 + 12\cos 315° = -19.3$ m

Note: 180° is along $-x$ axis; 270° is along $-y$ axis; $\cos 120° = -\cos 60°$; $\sin 120° = \sin 60°$; $\cos 315° = \cos 45°$; $\sin 315° = -\sin 45°$.

$$D_y = 20\sin 30° + 40\sin 120° + 0 - 42 + 12\sin 315° = -5.8 \text{ m}$$