# ASPECTS OF MATHEMATICS AND ITS APPLICATIONS

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Jorge Alberto BARROSO

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Leopoldo and Maria da Graça Nachbin married in Rio de Janeiro on July 28, 1956 and spent the two academic years 1956–1958 in the U.S.A. He was then a Rockefeller Foundation Fellow at the University of Chicago (1956–1957), and a Guggenheim Foundation Fellow at the Institute for Advanced Study in Princeton (1957–1958). This picture was taken in September 1956 at the University of Chicago campus.

The Organization of American States announced on July 28, 1982 in Washington, D.C., U.S.A. that its Houssay Prize in the Exact Sciences had been granted that day to Leopoldo Nachbin. This picture was taken on the same day in Rio de Janeiro by a newspaper photographer during an interview as a result of the press release announcing the prize.

J.A. BARROSO editor, Aspects of Mathematics and its Applications



# FOREWORD

This volume gathers together papers written by a number of mathematicians to celebrate the sixtieth birthday of Leopoldo Nachbin on January 7, 1982. The year 1982 proved to be exceptionally eventful in the life of this fine human being, this profound mathematician who is both a sensitive organizer and a devoted pioneer. On July 28, exactly on the day he celebrated his 26th wedding-anniversary with Maria da Graça Nachbin, the Organization of American States in Washington, D.C., U.S.A., honoured him with the Houssay Prize in the area of the Exact Sciences. A few days later, from August 2 to August 6, his sixtieth birthday and retirement from the Universidade Federal do Rio de Janeiro were commemorated by an International Mathematics Symposium at the University. On November 19 of the same year, however, his wife Maria died; she who had been an ideal companion and courageous friend.

Those familiar with the development of mathematics in Brazil during the last 40 years or so recognize the fundamental role that Leopoldo played therein. Besides making notable contributions to several branches of mathematics, Leopoldo had a basic role in creating a Brazilian school of mathematics, at first through serious and internationally known mathematical centres of excellence, such as the Centro Brasileiro de Pesquisas Físicas (CBPF) in Rio de Janeiro, the Instituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro, the Universidade de Brasília (UnB), and, later on, in the mathematical development of the Universidade Federal do Rio de Janeiro, as well as other centres in Brazil. His creative and missionary spirit is easy to prove when we look at the recent past and carefully observe the mathematical progress made by Brazil, and Leopoldo's influence in Latin-America, Portugal and Spain since around 1950.

Throughout a substantial part of his career Leopoldo lived, developed activities, and had influence in mathematics in the U.S.A. He also lived in France, and kept in close touch and collaboration with its mathematical school. Accordingly, Brazil, France, and the U.S.A. have been the most relevant countries in his life as a mathematician.

Born in Recife, Pernambucano, Brazil, on January 7, 1922, of a Polish Jewish father, Jacob Nachbin, and an Austrian Jewish mother, Léa Nachbin, from an early age Leopoldo revealed a strong taste for mathe-

matics, while at the same time his personality showed the powerful will to help and encourage others. He lost his father while he was still young, and found in his mother the necessary understanding and protection to dedicate himself to mathematics. At that time, Brazilian families only accorded value to three traditional careers: engineering, law and medicine.

Moving to Rio de Janeiro at the end of 1938, at the age of 17, he joined the Escola Nacional de Engenharia in Rio de Janeiro in 1939, and finished the engineering course in 1943, but his real interest was mathematics. Leopoldo Nachbin and the mathematician Professor Paulo Ribenboim are cousins, as their mothers were sisters. In 1984 Leopoldo's eldest son, André Nachbin, has entered the Courant Institute of Mathematical Sciences of New York University as a doctoral candidate.

Around the early 40s the Faculdade Nacional de Filosofia in Rio de Janeiro contracted Italian professors for long periods, among whom special mention should be made of mathematician Professor Gabrielle Mammana and the mathematical physicist Luigi Sobrero. At the same time, the Russian–Italian physicist Professor Gleb Wataghin was teaching at the Faculdade de Filosofia, Ciências e Letras in São Paulo. These Professors greatly stimulated and helped Leopoldo. They returned to Italy after Brazil joined the Second World War. Due to the influence of Luigi Sobrero and Gleb Wataghin, Leopoldo even vacillated between mathematics and physics, but he realized that his spirit was that of the mathematician. However, throughout his career Leopoldo kept in constant contact with physicists and had their sponsorship.

His first publication, 'Sobre a permutabilidade entre as operações de passagem ao limite e de integração de equações diferenciais', An. Acad. Brasil. Ciênc. 13 (1941) 327–335, was published when Leopoldo was 19 years old. By using ideas from functional analysis, it reveals his natural inclination for an area of mathematics to which he was to dedicate himself from that point on. Then begins a series of articles and books on harmonic analysis, Boolean algebras, topology, functional analysis, approximation theory and holomorphy, in which his spirit of innovation is shown in the creation and exploration of original ideas and new methods, in his constant interest in the basic aspects of mathematics rather than in extreme specialization.

During 1945–48 Leopoldo was under the direct influence of Professor Antonio Monteiro who, during 1945–49, visited the Faculdade Nacional de Filosofia in Rio de Janeiro, where Leopoldo passed his Livre Docência

dissertation, 'Combinação de topologias pseudo-metrisáveis e metrisáveis' in 1948, the results of which were announced in the note 'Sur la combinaison des topologies métrisables et pseudo-métrisables', C. R. Acad. Sci. Paris 223 (1946) 938–940. That was just before he left Brazil for the first time to visit the University of Chicago for two years.

The presence at the Faculdade de Filosofia, Ciências e Letras in São Paulo, of Professors André Weil (1945–47) and Jean Dieudonné (1946–47), as well as the visit to the Faculdade Nacional de Filosofia in Rio de Janeiro of Professors Adrian Albert (academic year 1947) and Marshall Stone (one trimester of 1947) proved to be decisive for the development of Leopoldo's career as a mathematician. When André Weil left for the University of Chicago in 1947, where Marshall Stone was Head of the Department of Mathematics and Adrian Albert a Professor, conditions became favourable for Leopoldo to spend a period of two years as a research associate at the University between 1948 and 1950. First with a U.S.A. State Department Fellowship and during the second year with a Guggenheim Foundation Fellowship. Thus began the international recognition of Leopoldo's worth as a mathematician and a long succession of fortunate events that marked his presence in the international scientific community. It should be mentioned that towards the end of his first visit to the U.S.A., Leopoldo met Professor Laurent Schwartz while taking part in the International Congress of Mathematicians held at Harvard University in 1950. Laurent Schwartz's influence on Leopoldo's career has been acknowledged as exceptional. It was Laurent Schwartz who promoted Leopoldo's two-year stay (1961–63) as a Visiting Professor at the Faculté des Sciences de Paris (Sorbonne), when Leopoldo was an invited speaker at the 1962 International Congress of Mathematicians held in Stockholm. In 1981, Leopoldo was the editor of the two-volume work 'Mathematical Analysis and Applications', published by Academic Press, to honour Laurent Schwartz for his 65th birthday in 1980 with a collection of essays dedicated to him. Leopoldo's early training underwent a significant Nicolas Bourbaki influence through André Weil, Jean Dieudonné and Laurent Schwartz.

Upon his return to Brazil in 1950, Leopoldo enrolled for a competition to occupy the Chair of Mathematical Analysis at the Faculdade Nacional de Filosofia in Rio de Janeiro. Oddly enough, this competition came to be held only in 1972 when Leopoldo became a Full Professor at the Universidade Federal do Rio de Janeiro, just after he resigned from the Instituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro, in

1971. The thesis he presented in 1950 for that competition was written in Portuguese. It was titled 'Topologia e Ordem' and the results were announced in three notes in French, 'Sur les espaces topologiques ordonnés', 'Sur les espaces uniformisables ordonnés', and 'Sur les espaces uniformes ordonnés', C. R. Acad. Sci. Paris 226 (1948) 381–382, 547, 774–775. This thesis would only gain a wider international audience after 1965, when Van Nostrand published it in an English translation with an added appendix, under the title 'Topology and Order'. Since then, the results contained therein have been applied by other researchers in differential equations, probability theory and mathematical economics.

Leopoldo married Maria da Graça Nachbin in Rio de Janeiro on July 28, 1956 and immediately left Brazil with her for a two-year visit to the U.S.A. He was a Rockefeller Foundation Fellow at the University of Chicago in 1956–57, and a Guggenheim Foundation Fellow for a second time at the Institute for Advanced Study in Princeton in 1957–58. Leopoldo and Graça had three children, André, Léa and Luís, born in Chicago, Rio de Janeiro and Rochester respectively; three cities that were very significant in his career.

One of the founders, in 1949, of the physics centre, Centro Brasileiro de Pesquisas Físicas (CBPF) in Rio de Janeiro (to which he is still associated as a Full Researcher), Leopoldo was the principal idealizer, in 1952, of the mathematics centre Instituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro. The importance that IMPA has achieved in national and international mathematics, and the extent to which Leopoldo is responsible for this prominence, are well known to the international scientific community.

Since the very beginning of his remarkable career as a researcher and a teacher, Leopoldo has received invitations to transfer permanently to universities outside Brazil, such as the University of Chicago and the University of Rochester, among others in Europe and the U.S.A. In 1966 Professor Adrian Albert, then Dean of the University of Chicago, offered him a permanent named endowed professorship, which Leopoldo very reluctantly declined for private reasons. After his stay as Visiting Professor at the Faculté des Sciences de Paris (Sorbonne) from 1961 to 1963, the University of Rochester, through Professor Leonard Gillman, offered him in 1963 a permanent Full Professorship that he accepted, and, in 1967, in order to match the offer of the University of Chicago, a promotion to a permanent George Eastman Professorship, which Leopoldo, for whom the Chair was specially created, holds since then.

Leopoldo's varied range of activities is also evident in the editorial area and in questions concerning mathematical education. In the latter area, to which he was attracted by Marshall Stone during several years, Leopoldo's presence has made itself felt through his active participation in several international meetings on the subject. In the editorial area among other commitments mention should be made of Leopoldo's role as editor of the collection of monographs 'Notas de Matemática', highly respected throughout the international mathematical community and published in Rio de Janeiro since 1948 until volume 47, whereafter North-Holland took over in 1973 to continue it under its editorship up to now.

Nor can we fail to mention Leopoldo's deep concern for the question of human rights in the scientific community, such a crucial problem in today's society. This concern of his reveals one of the many facets of the remarkable influence that Laurent Schwartz exerted on Leopoldo.

The International Mathematics Symposium held in honour of Leopoldo Nachbin in Rio de Janeiro in August (2 to 6, 1982), with the participation of mathematicians from Brazil, France, Ireland, the U.S.A., Venezuela and East Germany, was made possible by the sponsorship and financial support of the Universidade Federal do Rio de Janeiro and IBM-Brasil. We offer our heartfelt thanks to them. The Proceedings of that meeting were not published, as it was felt preferable to make available the present collection of writings in homage of him.

*Rio de Janeiro, July* 1984

*Jorge Alberto Barroso*

## CONTENTS

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# THE LIFE AND WORKS OF LEOPOLDO NACHBIN

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## 0. Introduction

The mathematician who is to be honoured by this volume was born on January 7 1922 in Recife, state of Pernambuco, in the north of Brazil. He started his secondary studies at the age of ten, after an examination, and with a special permission of the federal Ministry of Education (the minimum legal age to enter secondary school was eleven). It seems that during the first year in the new school he had difficulties with mathematics. Starting with the second year, however, he excelled in the subject, mainly under the influence of an extraordinary teacher: Luiz Ribeiro. He also took advantage of the very good library of the Ginásio Pernambucano, in which the works of many outstanding mathematicians could be found as a gift from the French government.

He moved to Rio de Janeiro, then the capital of Brazil, at the end of 1938 and in January 1939, at exactly the age of seventeen, entered the Engineering School of the University of Brazil (now the Federal University of Rio de Janeiro). From 1940 on, he also attended classes in mathematics at the Faculdade Nacional de Filosofia but it was not possible to study officially both engineering and mathematics at the same time. He was a teaching assistant for the calculus course in the School of Engineering starting in 1941. In 1943 he obtained a degree in civil engineering. His first mathematical publication [1][1] dates from 1941 and by the time he was twenty-four he had seven publications, one of them in the Transactions of the American Mathematical Society.

At his beginnings Leopoldo Nachbin was influenced by two Italian professors: the analyst Gabriele Mammana (a student of Mauro Picone) and the mathematical physicist Luigi Sobrero (a disciple of Tullio

[1] The numbers in square brackets refer to the list of publications of Leopoldo Nachbin, those in curly brackets to that of other authors at the end of this article.

Levi-Civitá, and one of the first mathematicians to apply hypercomplex numbers to partial differential equations {77}), who returned to Italy in 1942 after Brazil entered the second world war on the allied side. In 1945 the Portuguese analyst and logician Antonio A. Monteiro arrived in Rio de Janeiro. In the 1930's Monteiro studied in Paris, where he wrote a dissertation under the direction of Maurice Fréchet. Monteiro and Nachbin became friends, and Monteiro inspired Nachbin's works on ordered sets and Boolean algebras. Their friendship lasted even after Monteiro's departure for Argentina in 1949 until his death in 1980 (see [95c]). Between the spring of 1945 and September 1947 André Weil was in São Paulo and Jean Dieudonné spent the academic years 1946/47 and 1947/48 there. Nachbin had frequent contact with both of them in Rio de Janeiro, where the two went to lecture for longer periods, and in São Paulo, where Nachbin visited them. Weil was working in algebraic geometry but only a few years earlier introduced uniform spaces, which were to become the basis of some of Nachbin's work in topology. Dieudonné was interested at that time mainly in field theory and in topological vector spaces, and it is the latter which influenced Nachbin. He also obtained from them early drafts of the treatise of Bourbaki. In 1945 only four 'fascicules' had appeared (Théorie des ensembles: Fascicule des résultats, Topologie générale: Chap. I–IV, Algèbre: Chap. I), and Nachbin got acquainted with them through Monteiro who lent him his personal copies when he arrived in Rio. The fifth 'fascicule', Topologie générale: Chap. V–VIII, came out in 1947.

In the years after the war a number of distinguished mathematicians from the United States, many from the University of Chicago, visited Brazil for longer periods: A.A. Albert, W. Ambrose, M.H. Stone, O. Zariski, A. Zygmund. Nachbin got to know them, they played a role in his subsequent career, and Marshall Stone's three months long course in 1947 on rings of continuous functions had a profound influence on Nachbin's work in approximation theory.

## 1. Order

Before sketching Nachbin's results on Boolean algebras, let me recall some definitions concerning ordered sets. A *preorder* on a set $E$ is a relation $x \leqslant y$ which satisfies: (1) $x \leqslant x$ for all $x \in E$, (2) If $x \leqslant y$ and $y \leqslant z$, then $x \leqslant z$. If $x \leqslant y$ and $y \leqslant x$ imply $x = y$, then the preorder is an

*order.* Given $x, y \in E$, the *least upper bound* $z = \sup\{x, y\}$ of $x$ and $y$ is the element $z \in E$ such that $z \geq x$, $z \geq y$ and if $w \geq x$, $w \geq y$, then $w \geq z$; the *greatest lower bound* $t = \inf\{x, y\}$ of $x$ and $y$ is the element $t \in E$ such that $t \leq x$, $t \leq y$ and if $s \leq x$, $s \leq y$, then $s \leq t$. One defines similarly $\sup_{\iota \in I}\{x_\iota\}$ and $\inf_{\iota \in I}\{x_\iota\}$ of any family $(x_\iota)_{\iota \in I}$ of elements of $E$.

An ordered set $E$ in which $\sup\{x, y\}$ exists for any $x, y \in E$ is called a *sup-lattice*; the definition of an *inf-lattice* is analogous, and an ordered set which is at the same time a sup-lattice and an inf-lattice is said to be a *lattice.* A lattice $E$ is *complete* if for any family $(x_\iota)_{\iota \in I}$ of elements of $E$ both $\sup_{\iota \in I}\{x_\iota\}$ and $\inf_{\iota \in I}\{x_\iota\}$ exist. A lattice $E$ is *distributive* if for any $x, y, z \in E$ one has

$$\sup\{x, \inf\{y, z\}\} = \inf\{\sup\{x, y\}, \sup\{x, z\}\}$$

and

$$\inf\{x, \sup\{y, z\}\} = \sup\{\inf\{x, y\}, \inf\{x, z\}\}.$$

A distributive lattice $E$ having a smallest element 0 and a largest element $1 \neq 0$ is a *Boolean algebra* if for any $x \in E$ there exists a complement $x^c \in E$ such that $\sup\{x, x^c\} = 1$ and $\inf\{x, x^c\} = 0$.

A *filter* on a lattice $E$ having elements 0 and 1 is a subset $F$ of $E$ such that: (1) $0 \notin F$ and $1 \in F$, (2) If $x \in F$ and $y \in E$ is such that $y \geq x$, then $y \in F$, (3) If $x, y \in F$, then $\inf\{x, y\} \in F$. A filter is *maximal* if it is not contained properly in another filter. A filter $F$ on $E$ is said to be *prime* if $x, y \in E$, $x \notin F$, $y \notin F$ imply $\sup\{x, y\} \notin F$. On a distributive lattice every maximal filter is prime. Indeed, assume that $F$ is a maximal filter on $E$ and that there exist $x, y \in E$ such that $x \notin F$, $y \notin F$ but $\sup\{x, y\} \in F$. One easily checks that $G = \{z \in E | \sup\{x, z\} \in F\}$ is a filter containing both $F$ and $y$, which contradicts the maximality of $F$.

It was known that in a Boolean algebra every prime filter is maximal. Nachbin proves in [10] that this property characterized Boolean algebras: if $E$ is a distributive lattice having elements 0 and 1 in which every prime filter is maximal, then $E$ is a Boolean algebra.

An *ideal* on a sup-lattice $E$ is a subset $I$ of $E$ such that: (1) If $x \in I$, and $y \in E$ is such that $y \leq x$, then $y \in I$, (2) If $x, y \in I$, then $\sup\{x, y\} \in I$. The set $\mathcal{J}(E)$ of all ideals on $E$, ordered by inclusion, is a complete lattice having smallest element $\emptyset$ and largest element $E$.

Nachbin says that an element $x$ in a complete lattice $L$ is *compact* if