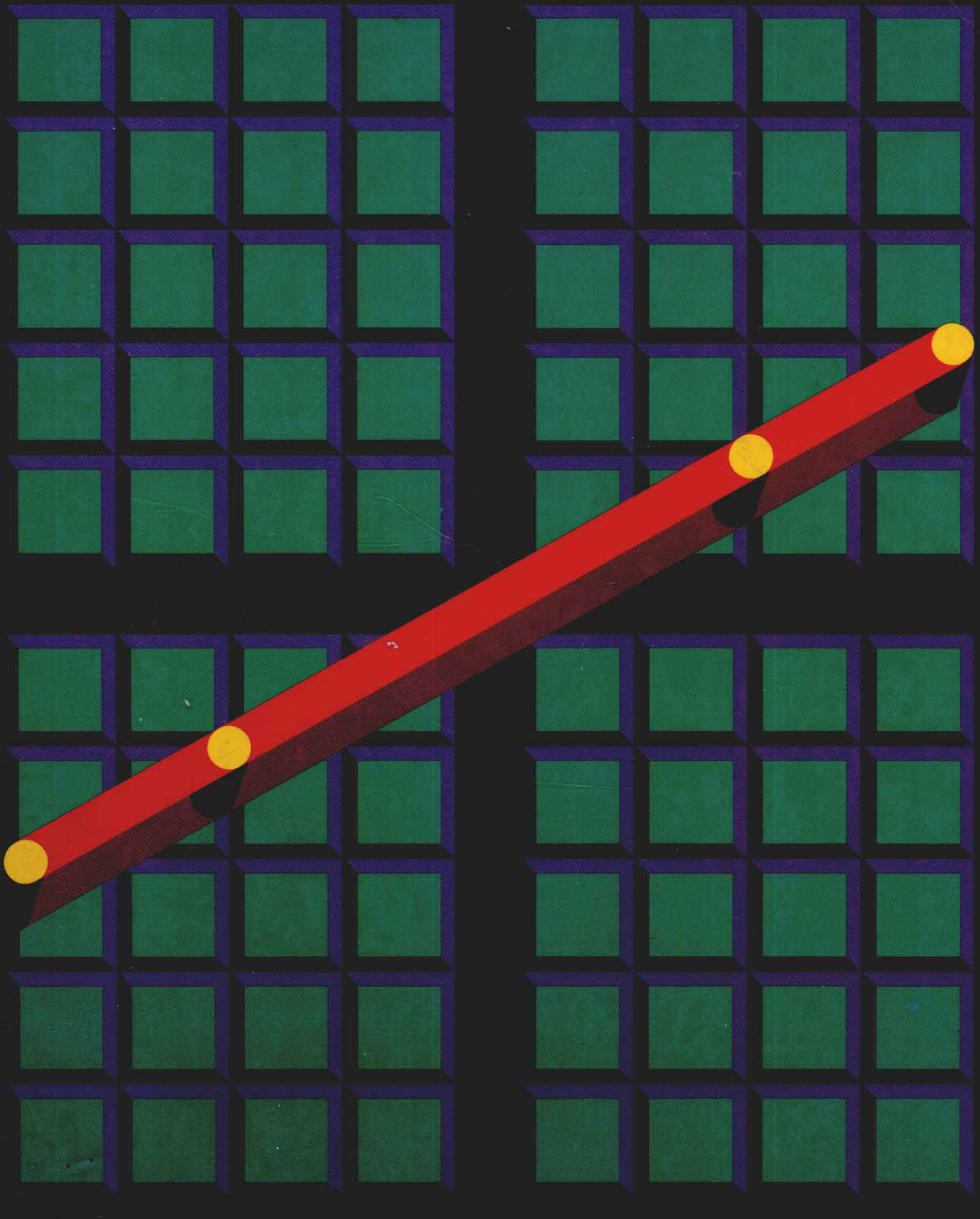
Raymond A. Barnett & Thomas J. Kearns

ELEMENTARY ALGEBRA

STRUCTURE AND USE

FIFTH EDITION



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ELEMENTARY ALGEBRA
Structure and Use

Raymond A. Barnett
Merritt College

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PREFACE

This is an introductory text in algebra written for students with no background in algebra and for students who need a review before proceeding further. The improvements in this fifth edition evolved out of generous responses from users of the fourth edition. Most of the changes in this edition have been made with an eye toward making the text even more accessible to students with minimal background and to provide a better transition to material covered in subsequent courses, intermediate algebra courses in particular.

PRINCIPAL CHANGES FROM THE FOURTH EDITION

1. **Sets** are treated informally in the text. A more detailed treatment is provided in Appendix A.
2. **Word problems** are more evenly dispersed throughout the text, rather than concentrated in one or two chapters. **Rate–time** and **mixture problems** are deferred until after systems of equations have been intro-

duced and both one- and two-variable methods are used to solve them (Sections 4-7, 4-8).

3. The review of **fractions**, **decimals**, and **percent** has been moved from the appendix to the text (Sections 3-1, 3-7), and **percent problems** have been added (Section 3-7).
4. The method of **solving equations by factoring** has been moved to an earlier section of the text (Section 6-7) and used thereafter when appropriate.
5. New sections on **solving radical equations** (Section 8-7) and **graphing quadratic equations** (Section 9-4) have been added.
6. **Order and inequality** are introduced briefly and informally early in the text (Sections 1-3, 2-4, 3-4). The more detailed treatment of **inequalities** is consolidated in a new chapter (Chapter 5), which can be covered anytime after Chapter 4. Material in subsequent chapters is not dependent on the material on inequalities.
7. **Calculators** are assumed to be available to students. Many of the problems in the text lend themselves naturally to calculator use and are made easier by such use. However, with the exception of very few exercises, the problems do not *require* use of a calculator and are not marked specifically as "calculator exercises."
8. A new appendix is included to provide an additional approach to **setting up word problems** (Appendix B).
9. The use of interval notation has been deferred to intermediate algebra.
10. Many **exercise sets** have been replaced and/or expanded. More problems involve **fractions** and **decimals**.
11. There are more **worked-out examples** with **matched problems**. There is more **boxed material** for emphasis, and **schematics** have been added for clarity. **Applications** have been kept current.

CAUTION

12. Areas that are a source of **common student errors** are highlighted with a special "caution" symbol.

IMPORTANT FEATURES RETAINED AND EXPANDED FROM THE FOURTH EDITION

1. The text is still **written for student comprehension**. Each concept is illustrated with an example, followed by a parallel problem with an answer (given at the end of the section) so that a student can immediately check his or her understanding of the concept. These follow-up problems also encourage active rather than passive reading of the text.
2. The order of topics has been chosen to provide a **smooth transition from arithmetic to algebra**. The beginning chapters gradually develop algebraic concepts and applications based upon known properties of number systems. The last half of the text extends this development.
3. An **informal style** is used for exposition. Definitions are illustrated with simple examples. There are no formal statements of theorems in this text.

4. The text includes **more than 3,200 carefully selected and graded problems**. The exercises are divided into A, B, and C groupings. The A problems are easy and routine, the B problems more challenging but still emphasizing mechanics, and the C problems a mixture of theoretical and difficult mechanics. In short, the text is designed so that an average or below-average student will be able to experience success and a very capable student will be challenged.
5. The subject matter is related to the real world through many carefully seletcted **realistic applications** from the physical sciences, business and economics, life sciences, and social sciences. Thus, the text is equally suited for students interested in any of these areas.
6. The text continues to use **spiraling techniques** for difficult topics; that is, a topic is introduced in a relatively simple framework and then is returned to one or more times in successively more complex forms. Consider the following:

   Factoring: Sections 1-5, 6-2 to 6-7, 7-1 to 7-3, and 9-1
   Word problems: Sections 1-2, 1-3, 2-7, 2-8, 3-6 to 3-9, 4-7 to 4-9, 5-2, 5-4, 7-4, 8-3, 8-7, and 9-6
   Fractional forms: Chapters 3 and 7
   Order and inequality: Sections 1-3, 2-4, 3-4, and Chapter 5

   The use of this spiraling technique continues into the companion text *Intermediate Algebra: Structure and Use (Fourth Edition).*
7. **Answers** to all chapter review exercises and to all odd-numbered problems from the other exercises are in the back of the book.
8. **Historical comments** are included for interest.
9. **Chapter review sections** include a summary of the chapter with all important terms and symbols, and a comprehensive review exercise. Answers to all review exercises are included in the back of the book and are keyed (with numbers in italics) to corresponding text sections.

ADDITIONAL STUDENT AIDS

1. **Common student errors** are clearly identified at places where they naturally occur (see Sections 2-7, 3-3, 3-4, 3-5).
2. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-4, 1-5, 2-6, 2-7).
3. **Annotation** of examples and developments is found throughout the text to help students through critical stages (see Sections 1-2, 2-4, 2-6, 2-7).
4. **Functional use of a second color** guides students through critical steps (see Sections 1-5, 2-6, 2-7).
5. **Summaries** of formulas and symbols (keyed to the sections in which they are introduced) and the metric system are inside the front and back covers of the text for convenient reference.

6. A **solutions manual** is available at a nominal cost through a bookstore. The manual includes detailed solutions to all odd-numbered problems and all chapter review exercises.

INSTRUCTOR'S AIDS

This supplements package contains a wide and varied assortment of useful instructor's aids. They include:

1. A **student's solutions manual** contains detailed solutions to every chapter review exercise as well as all other odd-numbered problems. This supplement is available to students at a nominal fee.
2. An **answer manual** (which slips inside the back of the text) contains answers to the even-numbered problems not answered in the text. This supplement is available to adopters without charge.
3. An **instructor's resource manual** provides sample tests (chapter, midterm, and final), transparency masters, and additional teaching suggestions and assistance.
4. A **computer testing system** is also available to adopters without cost. This system provides the instructor with numerous test questions from the text. Several test question types are available including multiple choice, open-ended, matching, true-false, and vocabulary. The testing system enables the instructor to find these questions by several different criteria. In addition, instructors may edit their own questions.
5. A printed and bound **test bank** is also available. This bank is a hard copy listing of the questions found in the computerized version.

ERROR CHECK

Because of the careful checking and proofing by a number of very competent people (acting independently), the authors and publisher believe this book to be substantially error-free. If any errors remain, the authors would be grateful if corrections were sent to: Mathematics Editor, College Division, 43rd floor, McGraw-Hill Book Company, 1221 Avenue of the Americas, New York, New York 10020.

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Thomas J. Kearns

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1

NATURAL NUMBERS

1-1 The Set of Natural Numbers
1-2 Algebraic Expressions—Their Formulation and Evaluation
1-3 Equality and Inequality
1-4 Properties of Addition and Multiplication; Exponents
1-5 Distributive Properties
1-6 Combining Like Terms
1-7 Chapter Review

Arithmetic, as you have studied it thus far in school, involves numbers, certain operations on numbers, and problems in which these operations are used. The numbers involved are whole numbers 0, 1, 2, . . . , fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, . . . , negatives of these, and perhaps other numbers such as $\sqrt{2}$ or π. The operations are the familiar operations of addition, subtraction, multiplication, division, and possibly the taking of square roots.

Algebra extends the concepts of arithmetic. In addition to specific numbers, algebra also involves symbols that represent unspecified or unknown numbers. These objects—numbers and symbols—are manipulated by the same basic operations used in arithmetic.

When arithmetic is extended to algebra, where the objects manipulated are not just specific numbers but unknown quantities as well, a wider range of problems can be attacked. For example, arithmetic alone can solve the following problem:

A rectangular field is 110 yards long and 65 yards wide. What is its area?

You can routinely calculate the area to be 7,150 square yards, by multiplying 110 by 65. A problem only slightly different, however, requires an algebraic approach:

A rectangular field is twice as long as it is wide and has area 6,962 square yards. What are its dimensions?

The answer, that the field is 59 yards wide by 118 yards long, is unlikely to be found quickly by guessing and arithmetic. It is, however, easily found by using simple algebra.

In this text you will encounter not only the basic objects and operations of algebra, but also many practical problems in which these objects and operations are used in their solutions. Since algebra uses many kinds of numbers, and symbols for these numbers, it is important that we go back and take a careful look at some of the properties of numbers that you may have previously taken for granted. We begin in this chapter using only the counting numbers 1, 2, 3, ... Algebraic ideas and methods will be extended to the integers . . . , −2, −1, 0, 1, 2, . . . in Chapter 2 and to fractions and the rational numbers in Chapter 3.

1-1
THE SET OF NATURAL NUMBERS

- The Set of Natural Numbers
- Important Subsets of the Set of Natural Numbers
- Least Common Multiple

We begin our development of algebra using only the simplest set of numbers, the **counting numbers** 1, 2, 3, ... These numbers are also referred to as the set of **natural numbers**, and the two names can be used interchangeably.

THE SET OF NATURAL NUMBERS

The word ''set'' here and throughout the text will be used as it is used in everyday language, meaning a collection. We want the collection to have the property that for any given object, it is either in the set or it is not. The word ''subset'' will also be used informally to mean part, or possibly all, of a set, much as a subcommittee is to a committee. We will often represent a set by listing its **elements** (objects in the collection) between braces { } or by giving it a capital letter name. Symbolically, the set of natural numbers will be represented by the letter N:

$N = \{1, 2, 3, \ldots\}$ Natural or counting numbers

The three dots tell us that the numbers go on without end, following the pattern indicated by the first three numbers. This is a useful way to represent certain infinite sets. (A set is called a **finite set** if it can be counted and has an end; otherwise it is an **infinite set**.)

Example 1 Select the natural numbers out of the following list:

$\frac{2}{3}, 1, \sqrt{2}, \pi, 5, 7.63, 17, 83\frac{7}{8}, 610$

Solution 1, 5, 17, and 610 are natural numbers.

Problem 1 Select the natural numbers out of the following list: $4, \frac{3}{4}, 19, 305, 4\frac{2}{3}, 7.32, \sqrt{3}$.†

Assumption

We assume that you know what natural numbers are, how to add and multiply them, and how to subtract and divide them when the result is a natural number.

---

† The answers to matched problems are found at the end of a section, just before the exercise set.

You may recall that the results of addition, multiplication, subtraction, and division of numbers are called the **sum**, **product**, **difference**, and **quotient**, respectively.

IMPORTANT SUBSETS OF THE SET OF NATURAL NUMBERS

The set of natural numbers can be separated into two subsets called even numbers and odd numbers. A natural number is an **even number** if it is exactly divisible by 2 (that is, divisible by 2 without a remainder). A natural number is an **odd number** if it is not exactly divisible by 2.

Example 2 Separate the set of natural numbers {1, 2, 3, . . .} into even and odd numbers.

Solution The set of even numbers: {2, 4, 6, . . .}
The set of odd numbers: {1, 3, 5, . . .}

Problem 2 Separate the following set into even and odd numbers: {8, 13, 7, 32, 57, 625, 532}.

When we add or subtract two or more numbers, the numbers are called **terms**; when we multiply two or more numbers, the numbers are called **factors**.

Terms Factors

$$\overset{\downarrow}{3} + \overset{\downarrow}{5} + \overset{\downarrow}{8} \qquad \overset{\downarrow}{3} \times \overset{\downarrow}{5} \times \overset{\downarrow}{8}$$

In mathematics, at the level of algebra and higher, parentheses ( ) or the dot "·" are usually used in place of the times sign ×, since the times sign is easily confused with the letter x, a letter that finds frequent use in algebra. Thus,

$$3 \times 5 \times 8$$

$$(3)(5)(8)$$

$$3 \cdot 5 \cdot 8$$

all represent the product of 3, 5, and 8.

The natural numbers, excluding 1, can also be separated into two other important subsets called composite numbers and prime numbers. A natural number is a **composite number** if it can be rewritten as a product of two or more natural numbers other than itself and 1 (8 is a composite number, since $8 = 2 \cdot 4$). Stated in a different but equivalent way, a natural number is a composite number if it can be divided exactly (no remainder) by a natural number other than itself and 1 (9 is a composite number, since it is exactly divisible by 3). A natural number, excluding 1, is a **prime number** if it is not a composite number (11 is a prime number, since it cannot be divided exactly by any natural number other than itself or 1). Equivalently, a number is prime if its only factors are 1 and itself. The number 1 is defined to be neither prime or composite. The natural number 2 is the only even prime number. It can be proved that there are infinitely many prime numbers.

Example 3 Separate the set {2, 3, 4, . . . , 18, 19} into prime and composite numbers.

Solution The numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are composite, since

$$\begin{array}{cccc} 4 = 2 \cdot 2 & 6 = 2 \cdot 3 & 8 = 2 \cdot 4 & 9 = 3 \cdot 3 \\ 10 = 2 \cdot 5 & 12 = 3 \cdot 4 & 14 = 2 \cdot 7 & 15 = 3 \cdot 5 \\ & \quad = 2 \cdot 6 & & \end{array}$$

The remaining numbers 2, 3, 5, 7, 11, 13, 17, and 19 are prime.

Problem 3 Separate the set {6, 9, 11, 21, 23, 25, 27, 29} into prime and composite numbers.

A fundamental theorem of arithmetic states that every composite number has, except for order, a unique (one and only one) set of prime factors. A natural number represented as a product of prime factors is said to be **completely factored**.

Example 4 Write each number in a completely factored form:

(A) 8 **(B)** 36 **(C)** 60

Solution **(A)** $8 = 2 \cdot 4 = 2 \cdot 2 \cdot 2$ We continue factoring using natural numbers until we can go no further.

(B) $36 = \underset{\substack{\nearrow \\ \text{36 factored}}}{6 \cdot 6} = \underset{\substack{\nwarrow \\ \text{36 completely factored}}}{2 \cdot 3 \cdot 2 \cdot 3}$

or

$$36 = 4 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3$$

or

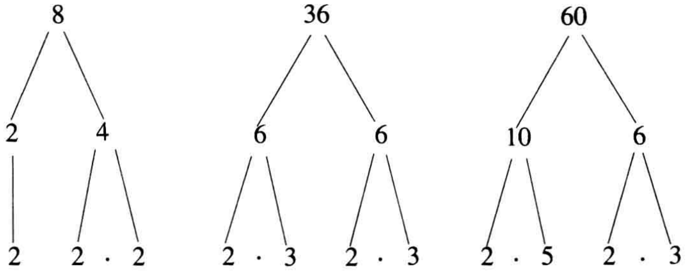
$$36 = 3 \cdot 12 = 3 \cdot 4 \cdot 3 = 3 \cdot 2 \cdot 2 \cdot 3$$

or

$$36 = 2 \cdot 18 = 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3$$

All four ways in which we factored 36 initially lead to the same set of prime factors: two 2s and two 3s. The order in which the factors are written makes no difference.

(C) $60 = 10 \cdot 6 = 2 \cdot 5 \cdot 2 \cdot 3 = 2 \cdot 2 \cdot 3 \cdot 5$
It may be easier to see the factorization schematically:



$8 = 2 \cdot 2 \cdot 2$ $\qquad 36 = 2 \cdot 2 \cdot 3 \cdot 3$ $\qquad 60 = 2 \cdot 2 \cdot 3 \cdot 5$

Problem 4 Write each number in completely factored form:

(A) 12 **(B)** 26 **(C)** 72

In factoring natural numbers it is easiest to look for small prime factors first. Numbers with 2 or 5 as factors are easily recognized, for example, 46 and 65. Also, it is useful to note that a number is exactly divisible by 3 when the sum of its digits is divisible by 3. For example, 51 and 177 are each divisible by 3, since $5 + 1 = 6$ and $1 + 7 + 7 = 15$ are each divisible by 3:

$$51 = 3 \cdot 17 \qquad \text{and} \qquad 177 = 3 \cdot 59$$

LEAST COMMON MULTIPLE

We can use the prime factors of numbers to aid us in finding the least common multiple (LCM) of two or more natural numbers, a process we will need to know later when dealing with fractions and certain types of equations.