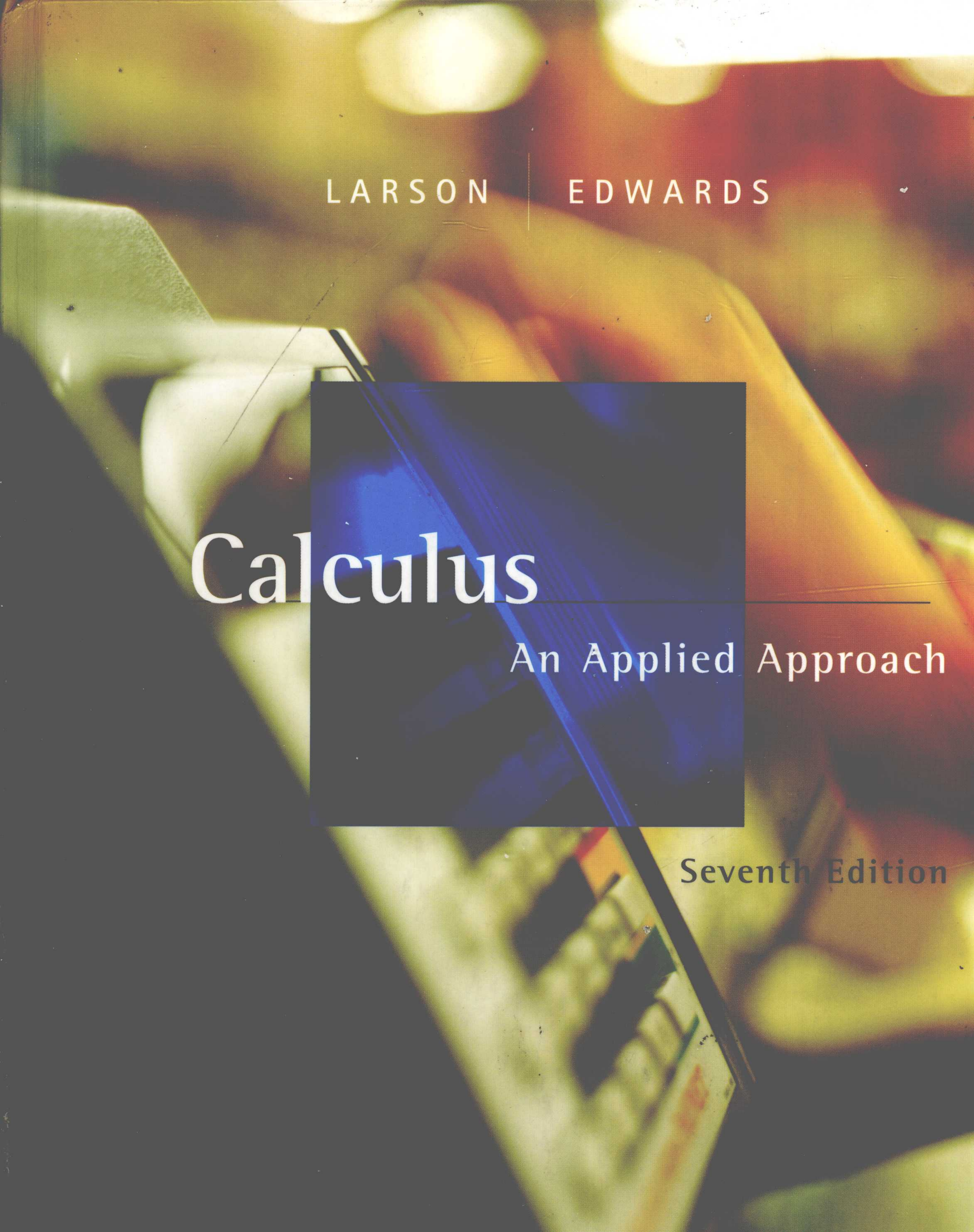

LARSON | EDWARDS

Calculus

An Applied Approach

Seventh Edition

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Calculus

An Applied Approach

RON LARSON
The Pennsylvania State University
The Behrend College

BRUCE H. EDWARDS
University of Florida

with the assistance of
DAVID C. FALVO
The Pennsylvania State University
The Behrend College

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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A Word from the Authors

Welcome to *Calculus: An Applied Approach*, Seventh Edition. In this revision, we have focused on making the text even more student-oriented. To encourage mastery and understanding, we have outlined a straightforward program of study with continual reinforcement and applicability to the real world.

Student-Oriented Approach

Each chapter begins with "What you should learn" and "Why you should learn it." The "What you should learn" is a list of *Objectives* that students will examine in the chapter. The "Why you should learn it" lists sample applications that appear throughout the chapter. Each section begins with a list of learning *Objectives*, enabling students to identify and focus on the key points of the section.

Following every example is a *Try It* exercise. The new problem allows for students to immediately practice the concept learned in the example.

It is crucial for a student to understand an algebraic concept before attempting to master a related calculus concept. To help students in this area, *Algebra Review* tips appear at point of use throughout the text. A two-page *Algebra Review* appears at the end of each chapter, which emphasizes key algebraic concepts discussed in the chapter.

Before students are exposed to selected topics, *Discovery* projects allow them to explore concepts on their own, making them more likely to remember the results. These optional boxed features can be omitted, if the instructor desires, with no loss of continuity in the coverage of the material.

Throughout the text, *Study Tips* address special cases, expand on concepts, and help students avoid common errors. *Side Comments* help explain the steps of a solution. State-of-the-art graphics help students with visualization, especially when working with functions of several variables.

Advances in *Technology* are helping to change the world around us. We have updated and increased technology coverage to be even more readily available at point of use. Students are encouraged to use a graphing utility, computer program, or spreadsheet software as a tool for exploration, discovery, and problem solving. Students are not required to have access to a graphing utility to use this text effectively. In addition to describing the benefits of using technology, the text also pays special attention to its possible misuse or misinterpretation.

Just before each section exercise set, the *Take Another Look* feature asks students to look back at one or more concepts presented in the section, using questions designed to enhance understanding of key ideas.

Each chapter presents many opportunities for students to assess their progress, both at the end of each section (*Prerequisite Review* and *Section Exercises*) and at the end of each chapter (*Chapter Summary*, *Study Strategies*, *Study Tools*, and *Review Exercises*). The test items in *Sample Post-Graduation Exam Questions* show the relevance of calculus. The test questions are representative of types of questions on several common post-graduation exams.

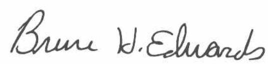
Business Capsules appear at the ends of numerous sections. These capsules and their accompanying exercises deal with business situations that are related to the mathematical concepts covered in the chapter.

Application to the Changing World Around Us

Students studying calculus need to understand how the subject matter relates to the real world. In this edition, we have focused on increasing the variety of applications, especially in the life sciences, economics, and finance. All real-data applications have been revised to use the most current information available. Exercises containing material from textbooks in other disciplines have been included to show the relevance of calculus in other areas. In addition, exercises involving the use of spreadsheets have been incorporated throughout.

We hope you enjoy the Seventh Edition. A readable text with a straightforward approach, it provides effective study tools and direct application to the lives and futures of calculus students.

Ron Larson



Bruce H. Edwards

Supplements

The integrated learning system for *Calculus: An Applied Approach*, Seventh Edition, addresses the changing needs of today's instructors and students, offering dynamic teaching tools for instructors and interactive learning resources for students in print, CD-ROM, and online formats.

Resources

Eduspace®, Houghton Mifflin's Online Learning Tool

Eduspace® is an online learning environment that combines algorithmic tutorials, homework capabilities, and testing. Text-specific content, organized by section, is available to help students understand the mathematics covered in this text.

For the Instructor

Instructor ClassPrep CD-ROM with HM Testing (Windows, Macintosh)

ClassPrep offers complete instructor solutions and other instructor resources. *HM Testing* is a computerized test generator with algorithmically generated test items.

Instructor Website (math.college.hmco.com/instructors)

This website contains pdfs of the *Complete Solutions Guide* and *Test Item File and Instructor's Resource Guide*. Digital Figures and Lessons are available (ppts) for use as handouts or slides.

For the Student

HM mathSpace® Student CD-ROM

HM mathSpace contains a prerequisite algebra review, a link to our online graphing calculator, and graphing calculator programs.

Excel Made Easy: Video Instruction with Activities CD-ROM

Excel Made Easy uses easy-to-follow videos to help students master mathematical concepts introduced in class. The CD-ROM includes electronic spreadsheets and detailed tutorials.

SMARTHINKING™ Online Tutoring

Instructional Video and DVD Series by Dana Mosely

The video and DVD series complement the textbook topic coverage should a student struggle with the calculus concepts or miss a class.

Student Solutions Guide

This printed manual features step-by-step solutions to the odd-numbered exercises. A practice test with full solutions is available for each chapter.

Excel Guide for Finite Math and Applied Calculus

The *Excel Guide* provides useful information, including step-by-step examples and sample exercises.

Student Website (math.college.hmco.com/students)

The website contains self-quizzing content to help students strengthen their calculus skills, a link to our online graphing calculator, graphing calculator programs, and printable formula cards.

Acknowledgments

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A special note of thanks goes to the instructors who responded to our survey and to all the students who have used the previous editions of the text.

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On a personal level, we are grateful to our spouses, Deanna Gilbert Larson and Consuelo Edwards, for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these comments very highly.

Ron Larson

Bruce H. Edwards

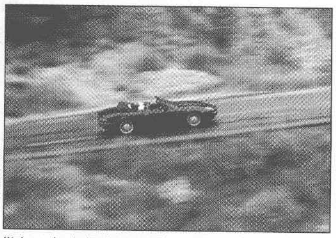
Features

CHAPTER OPENERS

Each chapter opens with *Strategies for Success*, a checklist that outlines what students should learn and lists several applications of those objectives. Each chapter opener also contains a list of the section topics and a photo referring students to an interesting application in the section exercises.

chapter

2 Differentiation



Higher-order derivatives are used to determine the acceleration function of a sports car. The acceleration function shows the changes in the car's velocity. As the car reaches its "cruising" speed, is the acceleration increasing or decreasing?

STRATEGIES FOR SUCCESS

WHAT YOU SHOULD LEARN:

- How to find the slope of a graph and calculate derivatives using the limit definition
- How to use the Constant Rule, Power Rule, Constant Multiple Rule, and Sum and Difference Rules
- How to find rates of change: velocity, marginal profit, marginal revenue, and marginal cost
- How to use the Product, Quotient, Chain, and General Power Rules
- How to calculate higher-order derivatives and derivatives using implicit differentiation
- How to solve related-rate problems and applications

WHY YOU SHOULD LEARN IT:

Derivatives have many applications in real life, as can be seen by the examples below, which represent a small sample of the applications in this chapter.

- Increasing Revenue, Example 10 on page 101
- Psychology: Migraine Prevalence, Exercise 62 on page 104
- Average Velocity, Exercises 15 and 16 on page 117
- Demand Function, Exercises 53 and 54 on page 129
- Quality Control, Exercise 58 on page 129
- Velocity and Acceleration, Exercises 41–44 and 50 on pages 145 and 146

81

SECTION 2.5 The Chain Rule 131

2.5 THE CHAIN RULE

- Find derivatives using the Chain Rule.
- Find derivatives using the General Power Rule.
- Write derivatives in simplified form.
- Use derivatives to answer questions about real-life situations.
- Use the differentiation rules to differentiate algebraic functions.

The Chain Rule

In this section, you will study one of the most powerful rules of differential calculus—the **Chain Rule.** This differentiation rule deals with composite functions and adds versatility to the rules presented in Sections 2.2 and 2.4. For example, compare the functions below. Those on the left can be differentiated without the Chain Rule, whereas those on the right are best done with the Chain Rule.

Without the Chain Rule	*With the Chain Rule*
$y = x^2 + 1$	$y = \sqrt{x^2 + 1}$
$y = x + 1$	$y = (x + 1)^{-1/2}$
$y = 3x + 2$	$y = (3x + 2)^5$
$y = \frac{x + 5}{x^2 + 2}$	$y = \left(\frac{x + 5}{x^2 + 2}\right)^2$

The Chain Rule

If $y = f(u)$ is a differentiable function of u, and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x, and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

Basically, the Chain Rule states that if y changes dy/du times as fast as u, and u changes du/dx times as fast as x, then y changes

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

times as fast as x, as illustrated in Figure 2.28. One advantage of the dy/dx notation for derivatives is that it helps you remember differentiation rules, such as the Chain Rule. For instance, in the formula

$$dy/dx = (dy/du)(du/dx)$$

you can imagine that the du's divide out.

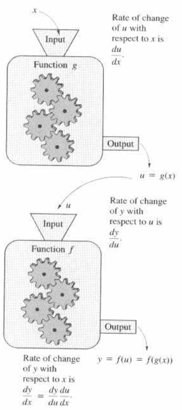


FIGURE 2.28

SECTION OBJECTIVES

Each section begins with a list of objectives covered in that section. This outline helps instructors with class planning and students in studying the material in the section.

DEFINITIONS AND THEOREMS

All definitions and theorems are highlighted for emphasis and easy reference.

EXAMPLES

To increase the usefulness of the text as a study tool, the Seventh Edition presents a wide variety of examples, each titled for easy reference. Many of these detailed examples display solutions that are presented graphically, analytically, and/or numerically to provide further insight into mathematical concepts. Side comments clarify the steps of the solution as necessary. Examples using real-life data are identified with a globe icon and are accompanied by the types of illustrations that students are used to seeing in newspapers and magazines.

TRY ITS

Appearing after every example, these new problems help students reinforce concepts right after they are presented.

112 **CHAPTER 2** Differentiation

The profit function in Example 5 is unusual in that the profit continues to increase as long as the number of units sold increases. In practice, it is more common to encounter situations in which sales can be increased only by lowering the price per item. Such reductions in price will ultimately cause the profit to decline.

The number of units x that consumers are willing to purchase at a given price per unit p is given by the **demand function**

$p = f(x)$. Demand function

The total revenue R is then related to the price per unit and the quantity demanded (or sold) by the equation

$R = xp$. Revenue function

EXAMPLE 6 Finding a Demand Function

A business sells 2000 items per month at a price of \$10 each. It is estimated that monthly sales will increase 250 units for each \$0.25 reduction in price. Use this information to find the demand function and total revenue function.

SOLUTION From the given estimate, x increases 250 units each time p drops \$0.25 from the original cost of \$10. This is described by the equation

$$x = 2000 + 250\left(\frac{10 - p}{0.25}\right)$$
$$= 2000 + 10{,}000 - 1000p$$
$$= 12{,}000 - 1000p.$$

Solving for p in terms of x produces

$p = 12 - \frac{x}{1000}$. Demand function

This, in turn, implies that the revenue function is

$R = xp$ Formula for revenue

$= x\left(12 - \frac{x}{1000}\right)$

$= 12x - \frac{x^2}{1000}$. Revenue function

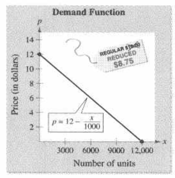


FIGURE 2.24

The graph of the demand function is shown in Figure 2.24. Notice that as the price decreases, the quantity demanded increases.

TRY IT 6

Find the demand function in Example 6 if monthly sales increase 200 units for each \$0.10 reduction in price.

290 **CHAPTER 4** Exponential and Logarithmic Functions

4.5 DERIVATIVES OF LOGARITHMIC FUNCTIONS

- Find derivatives of natural logarithmic functions.
- Use calculus to analyze the graphs of functions that involve the natural logarithmic function.
- Use the definition of logarithms and the change-of-base formula to evaluate logarithmic expressions involving other bases.
- Find derivatives of exponential and logarithmic functions involving other bases.

DISCOVERY

Sketch the graph of $y = \ln x$ on a piece of paper. Draw tangent lines to the graph at various points. How do the slopes of these tangent lines change as you move to the right? Is the slope ever equal to zero? Use the formula for the derivative of the logarithmic function to confirm your conclusions.

Derivatives of Logarithmic Functions

Implicit differentiation can be used to develop the derivative of the natural logarithmic function.

$y = \ln x$ Natural logarithmic function

$e^y = x$ Write in exponential form.

$\frac{d}{dx}[e^y] = \frac{d}{dx}[x]$ Differentiate with respect to x.

$e^y\frac{dy}{dx} = 1$ Chain Rule

$\frac{dy}{dx} = \frac{1}{e^y}$ Divide each side by e^y.

$\frac{dy}{dx} = \frac{1}{x}$ Substitute x for e^y.

This result and its Chain Rule version are summarized below.

Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$ 2. $\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$

EXAMPLE 1 Differentiating a Logarithmic Function

Find the derivative of

$f(x) = \ln 2x$.

SOLUTION Let $u = 2x$. Then $du/dx = 2$, and you can apply the Chain Rule as shown.

$$f'(x) = \frac{1}{u}\frac{du}{dx} = \frac{1}{2x}(2) = \frac{1}{x}$$

TRY IT 1

Find the derivative of $f(x) = \ln 5x$.

DISCOVERY

Before students are exposed to selected topics, *Discovery* projects allow them to explore concepts on their own, making them more likely to remember the results. These optional boxed features can be omitted, if the instructor desires, with no loss of continuity in the coverage of material.

ALGEBRA REVIEWS

Algebra Reviews appear throughout each chapter and offer students algebraic support at point of use. These smaller reviews are then revisited in the Algebra Review at the end of each chapter, where additional details of examples with solutions and explanations are provided.

176 CHAPTER 3 Applications of the Derivative

Not only is the function in Example 3 continuous on the entire real line, it is also differentiable there. For such functions, the only critical numbers are those for which $f'(x) = 0$. The next example considers a continuous function that has *both* types of critical numbers—those for which $f'(x) = 0$ and those for which f' is undefined.

ALGEBRA REVIEW

For help on the algebra in Example 4, see Example 2(d) in the *Chapter 3 Algebra Review*, on page 249.

EXAMPLE 4 Finding Increasing and Decreasing Intervals

Find the open intervals on which the function

$$f(x) = (x^2 - 4)^{2/3}$$

is increasing or decreasing.

SOLUTION Begin by finding the derivative of the function.

$$f'(x) = \frac{2}{3}(x^2 - 4)^{-1/3}(2x) \quad \text{Differentiate.}$$

$$= \frac{4x}{3(x^2 - 4)^{1/3}} \quad \text{Simplify.}$$

From this, you can see that the derivative is zero when $x = 0$ and the derivative is undefined when $x = \pm 2$. So, the critical numbers are

$$x = -2, \quad x = 0, \quad \text{and} \quad x = 2. \quad \text{Critical numbers}$$

This implies that the test intervals are

$$(-\infty, -2), \quad (-2, 0), \quad (0, 2), \quad \text{and} \quad (2, \infty). \quad \text{Test intervals}$$

The table summarizes the testing of these four intervals, and the graph of the function is shown in Figure 3.6.

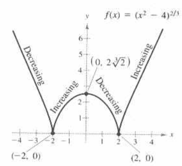


FIGURE 3.6

Interval	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < \infty$
Test value	$x = -3$	$x = -1$	$x = 1$	$x = 3$
Sign of $f'(x)$	$f'(-3) < 0$	$f'(-1) > 0$	$f'(1) < 0$	$f'(3) > 0$
Conclusion	Decreasing	Increasing	Decreasing	Increasing

TRY IT 4

Find the open intervals on which the function $f(x) = x^{2/3}$ is increasing or decreasing.

ALGEBRA REVIEW

To test the intervals in the table, it is not necessary to *evaluate* $f'(x)$ at each test value—you only need to determine its sign. For example, you can determine the sign of $f'(-3)$ as shown.

$$f'(-3) = \frac{4(-3)}{3(9-4)^{1/3}} = \frac{\text{negative}}{\text{positive}} = \text{negative}$$

320 CHAPTER 5 Integration and Its Applications

Finding Antiderivatives

The inverse relationship between the operations of integration and differentiation can be shown symbolically, as shown.

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x) \quad \text{Differentiation is the inverse of integration.}$$

$$\int f'(x)\,dx = f(x) + C \quad \text{Integration is the inverse of differentiation.}$$

This inverse relationship between integration and differentiation allows you to obtain integration formulas directly from differentiation formulas. The following summary lists the integration formulas that correspond to some of the differentiation formulas you have studied.

Basic Integration Rules

1. $\int k\,dx = kx + C$, k is a constant. — Constant Rule
2. $\int kf(x)\,dx = k\int f(x)\,dx$ — Constant Multiple Rule
3. $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$ — Sum Rule
4. $\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$ — Difference Rule
5. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ — Simple Power Rule

STUDY TIP

You will study the General Power Rule for integration in Section 5.2 and the Exponential and Log Rules in Section 5.3.

STUDY TIP

In Example 2(b), the integral $\int 1\,dx$ is usually shortened to the form $\int dx$.

Be sure you see that the Simple Power Rule has the restriction that n cannot be -1. So, you *cannot* use the Simple Power Rule to evaluate the integral

$$\int \frac{1}{x}\,dx.$$

To evaluate this integral, you need the Log Rule, which is described in Section 5.3.

TRY IT 2

Find each indefinite integral.

(a) $\int 5\,dx$

(b) $\int -1\,dr$

(c) $\int 2\,dt$

EXAMPLE 2 Finding Indefinite Integrals

Find each indefinite integral.

(a) $\int \frac{1}{2}\,dx$ (b) $\int 1\,dx$ (c) $\int -5\,dt$

SOLUTION

(a) $\int \frac{1}{2}\,dx = \frac{1}{2}x + C$ (b) $\int 1\,dx = x + C$ (c) $\int -5\,dt = -5t + C$

STUDY TIPS

Throughout the text, *Study Tips* help students avoid common errors, address special cases, and expand on theoretical concepts.

TAKE ANOTHER LOOK

Starting with Chapter 1, each section in the text closes with a *Take Another Look* problem asking students to look back at one or more concepts presented in the section, using questions designed to enhance understanding of key ideas. These problems can be completed as group projects in class or as homework assignments. Because these problems encourage students to think, reason, and write about calculus, they emphasize the synthesis or the further exploration of the concepts presented in the section.

352 **CHAPTER 5** Integration and Its Applications

Annuity

A sequence of equal payments made at regular time intervals over a period of time is called an **annuity.** Some examples of annuities are payroll savings plans, monthly home mortgage payments, and individual retirement accounts. The **amount of an annuity** is the sum of the payments plus the interest earned and can be found as shown below.

Amount of an Annuity

If c represents a continuous income function in dollars per year (where t is the time in years), r represents the interest rate compounded continuously, and T represents the term of the annuity in years, then the **amount of an annuity** is

$$\text{Amount of an annuity} = e^{rT}\int_0^T c(t)e^{-rt}\,dt.$$

EXAMPLE 9 **Finding the Amount of an Annuity**

You deposit \$2000 each year for 15 years in an individual retirement account (IRA) paying 10% interest. How much will you have in your IRA after 15 years?

SOLUTION The income function for your deposit is $c(t) = 2000$. So, the amount of the annuity after 15 years will be

$$\begin{aligned}\text{Amount of an annuity} &= e^{rT}\int_0^T c(t)e^{-rt}\,dt\\ &= e^{(0.10)(15)}\int_0^{15} 2000e^{-0.10t}\,dt\\ &= 2000e^{1.5}\left[-\frac{e^{-0.10t}}{0.10}\right]_0^{15}\\ &\approx \$69{,}633.78.\end{aligned}$$

TRY IT 9

If you deposit \$1000 in a savings account every year, paying 8% interest, how much will be in the account after 10 years?

TAKE ANOTHER LOOK

Using Geometry to Evaluate Definite Integrals

When using the Fundamental Theorem of Calculus to evaluate $\int_a^b f(x)\,dx$, remember that you must first be able to find an antiderivative of $f(x)$. If you are unable to find an antiderivative, you cannot use the Fundamental Theorem. In some cases, you can still evaluate the definite integral. For instance, explain how you can use geometry to evaluate

$$\int_{-2}^{2} \sqrt{4 - x^2}\,dx.$$

Use a symbolic integration utility to verify your answer.

SECTION 5.4 Area and the Fundamental Theorem of Calculus 353

PREREQUISITE REVIEW 5.4 The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the indefinite integral.

1. $\int (3x + 7)\,dx$
2. $\int \left(x^{3/2} + 2\sqrt{x}\right)dx$
3. $\int \frac{1}{5x}\,dx$
4. $\int e^{-6x}\,dx$

In Exercises 5 and 6, evaluate the expression when $a = 5$ and $b = 3$.

5. $\left(\frac{a}{5} - a\right) - \left(\frac{b}{5} - b\right)$
6. $\left(6a - \frac{a^3}{3}\right) - \left(6b - \frac{b^3}{3}\right)$

In Exercises 7–10, integrate the marginal function.

7. $\frac{dC}{dx} = 0.02x^{3/2} + 29{,}500$
8. $\frac{dR}{dx} = 9000 + 2x$
9. $\frac{dP}{dx} = 25{,}000 - 0.01x$
10. $\frac{dC}{dx} = 0.03x^2 + 4600$

EXERCISES 5.4

In Exercises 1–8, sketch the region whose area is represented by the definite integral. Then use a geometric formula to evaluate the integral.

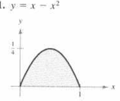
1. $\int_0^2 3\,dx$
2. $\int_0^4 2\,dx$
3. $\int_0^5 (x + 1)\,dx$
4. $\int_0^3 (2x + 1)\,dx$
5. $\int_{-2}^{3} |x - 1|\,dx$
6. $\int_{-1}^{4} |x - 2|\,dx$
7. $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$
8. $\int_0^2 \sqrt{4 - x^2}\,dx$

In Exercises 9 and 10, use the values $\int_0^5 f(x)\,dx = 8$ and $\int_0^5 g(x)\,dx = 3$ to evaluate the definite integral.

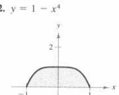
9. (a) $\int_0^5 [f(x) + g(x)]\,dx$ (b) $\int_0^5 [f(x) - g(x)]\,dx$
   (c) $\int_0^5 -4f(x)\,dx$ (d) $\int_0^5 [f(x) - 3g(x)]\,dx$
10. (a) $\int_0^5 2g(x)\,dx$ (b) $\int_5^0 f(x)\,dx$
   (c) $\int_5^5 f(x)\,dx$ (d) $\int_0^5 [f(x) - f(x)]\,dx$

In Exercises 11–18, find the area of the region.

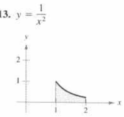
11. $y = x - x^2$



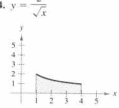
12. $y = 1 - x^4$



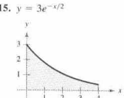
13. $y = \frac{1}{x^2}$



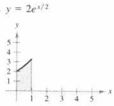
14. $y = \frac{2}{\sqrt{x}}$



15. $y = 3e^{-x/2}$



16. $y = 2e^{x/2}$



PREREQUISITE REVIEW

Starting with Chapter 1, each text section has a set of *Prerequisite Review* exercises. The exercises enable students to review and practice the previously learned skills necessary to master the new skills presented in the section. Answers to these sections appear in the back of the text.

EXERCISES

The text now contains almost 6000 exercises. Each exercise set is graded, progressing from skill-development problems to more challenging problems, to build confidence, skill, and understanding. The wide variety of types of exercises include many technology-oriented, real, and engaging problems. Answers to all odd-numbered exercises are included in the back of the text. To help instructors make homework assignments, many of the exercises in the text are labeled to indicate the area of application.

GRAPHING UTILITIES

Many exercises in the text can be solved using technology; however, the ⊕ symbol identifies all exercises for which students are specifically instructed to use a graphing utility, computer algebra system, or spreadsheet software.

TEXTBOOK EXERCISES

The Seventh Edition includes a number of exercises that contain material from textbooks in other disciplines, such as biology, chemistry, economics, finance, geology, physics, and psychology. These applications make the point to students that they will need to use calculus in future courses outside of the math curriculum. These exercises are identified by the icon and are labeled to indicate the subject area.

SECTION 6.2 Integration by Parts and Present Value 405

(a) Use a graphing utility to decide whether the board of trustees expects the gift income to increase or decrease over the five-year period.

(b) Find the expected total gift income over the five-year period.

(c) Determine the average annual gift income over the five-year period. Compare the result with the income given when $t = 3$.

61. ***Learning Theory*** A model for the ability M of a child to memorize, measured on a scale from 0 to 10, is

$$M = 1 + 1.6t \ln t, \quad 0 < t \le 4$$

where t is the child's age in years. Find the average value of this model between

(a) the child's first and second birthdays.

(b) the child's third and fourth birthdays.

62. ***Revenue*** A company sells a seasonal product. The revenue R (in dollars per year) generated by sales of the product can be modeled by

$$R = 410.5t^2e^{-t/30} + 25{,}000, \quad 0 \le t \le 365$$

where t is the time in days.

(a) Find the average daily receipts during the first quarter, which is given by $0 \le t \le 90$.

(b) Find the average daily receipts during the fourth quarter, which is given by $274 \le t \le 365$.

(c) Find the total daily receipts during the year.

Present Value In Exercises 63–68, find the present value of the income c (measured in dollars) over t_1 years at the given annual inflation rate r.

63. $c = 5000,\ r = 5\%,\ t_1 = 4$ years

64. $c = 450,\ r = 4\%,\ t_1 = 10$ years

65. $c = 150{,}000 + 2500t,\ r = 4\%,\ t_1 = 10$ years

66. $c = 30{,}000 + 500t,\ r = 7\%,\ t_1 = 6$ years

67. $c = 1000 + 50e^{t/2},\ r = 6\%,\ t_1 = 4$ years

68. $c = 5000 + 25te^{t/10},\ r = 6\%,\ t_1 = 10$ years

69. ***Present Value*** A company expects its income c during the next 4 years to be modeled by

$$c = 150{,}000 + 75{,}000t.$$

(a) Find the actual income for the business over the 4 years.

(b) Assuming an annual inflation rate of 4%, what is the present value of this income?

70. ***Present Value*** A professional athlete signs a three-year contract in which the earnings can be modeled by

$$c = 300{,}000 + 125{,}000t.$$

(a) Find the actual value of the athlete's contract.

(b) Assuming an annual inflation rate of 5%, what is the present value of the contract?

Future Value In Exercises 71 and 72, find the future value of the income (in dollars) given by $f(t)$ over t_1 years at the annual interest rate of r. If the function f represents a continuous investment over a period of t_1 years at an annual interest rate of r (compounded continuously), then the future value of the investment is given by

$$\text{Future value} = e^{rt_1}\int_0^{t_1} f(t)e^{-rt}\,dt.$$

71. $f(t) = 3000,\ r = 8\%,\ t_1 = 10$ years

72. $f(t) = 3000e^{0.05t},\ r = 10\%,\ t_1 = 5$ years

73. ***Finance: Future Value*** Use the equation from Exercises 71 and 72 to calculate the following. *(Source: Adapted from Garman/Forgue, Personal Finance, Fifth Edition)*

(a) The future value of $1200 saved each year for 10 years earning 7% interest.

(b) A person who wishes to invest $1200 each year finds one investment choice that is expected to pay 9% interest per year and another, riskier choice that may pay 10% interest per year. What is the difference in return (future value) if the investment is made for 15 years?

74. ***Consumer Awareness*** In 2004, the total cost to attend Pennsylvania State University for 1 year was estimated to be $19,843. If your grandparents had continuously invested in a college fund according to the model

$$f(t) = 400t$$

for 18 years, at an annual interest rate of 10%, would the fund have grown enough to allow you to cover 4 years of expenses at Pennsylvania State University? *(Source: Pennsylvania State University)*

75. Use a program similar to the Midpoint Rule program on page 366 with $n = 10$ to approximate

$$\int_1^4 \frac{4}{\sqrt{x} + \sqrt[3]{x}}\,dx.$$

76. Use a program similar to the Midpoint Rule program on page 366 with $n = 12$ to approximate the volume of the solid generated by revolving the region bounded by the graphs of

$$y = \frac{10}{\sqrt{xe^x}},\ y = 0,\ x = 1, \text{ and } x = 4$$

about the x-axis.

SECTION 3.5 Business and Economics Applications 219

36. ***Minimum Cost*** The ordering and transportation cost C of the components used in manufacturing a product is modeled by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \ge 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost. (*Hint:* Use the *root* feature of a graphing utility.)

37. ***Revenue*** The demand for a car wash is

$$x = 600 - 50p$$

where the current price is $5.00. Can revenue be increased by lowering the price and thus attracting more customers? Use price elasticity of demand to determine your answer.

38. ***Revenue*** Repeat Exercise 37 for a demand function of $x = 800 - 40p$.

39. ***Demand*** A demand function is modeled by $x = a/p^m$, where a is a constant and $m > 1$. Show that $\eta = -m$. In other words, show that a 1% increase in price results in an m% decrease in the quantity demanded.

40. ***Sales*** The sales S (in millions of dollars per year) for Lowe's for the years 1994 through 2003 can be modeled by

$$S = 201.556t^2 - 502.29t + 2622.8 + \frac{9286}{t},$$
$$4 \le t \le 13$$

where $t = 4$ corresponds to 1994. *(Source: Lowe's Companies)*

(a) During which year, from 1994 to 2003, were Lowe's sales increasing most rapidly?

(b) During which year were the sales increasing at the lowest rate?

(c) Find the rate of increase or decrease for each year in parts (a) and (b).

(d) Use a graphing utility to graph the sales function. Then use the *zoom* and *trace* features to confirm the results in parts (a), (b), and (c).

41. ***Revenue*** The revenue R (in millions of dollars per year) for Papa John's for the years 1994 through 2003 can be modeled by

$$R = \frac{-18.0 + 24.74t}{1 - 0.16t + 0.008t^2}, \quad 4 \le t \le 13$$

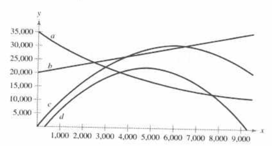
where $t = 4$ corresponds to 1994. *(Source: Papa John's Int'l.)*

(a) During which year, from 1994 to 2003, was Papa John's revenue the greatest? the least?

(b) During which year was the revenue increasing at the greatest rate? decreasing at the greatest rate?

(c) Use a graphing utility to graph the revenue function, and confirm your results in parts (a) and (b).

42. Match each graph with the function it best represents—a demand function, a revenue function, a cost function, or a profit function. Explain your reasoning. (The graphs are labeled a–d.)



BUSINESS CAPSULE

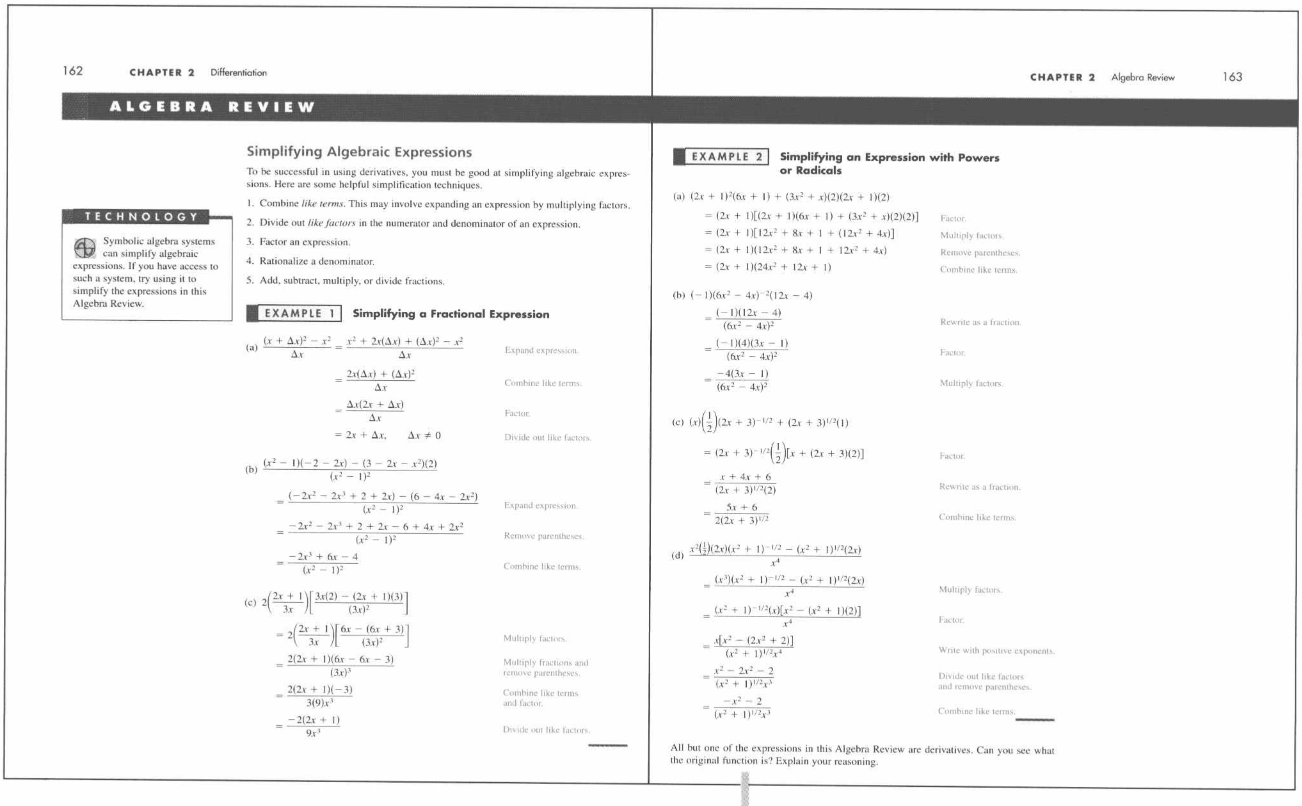
Courtesy of Transperfect Translations

While graduate students, Elizabeth Elting and Phil Shawe co-founded TransPerfect Translations in 1992. They used a rented computer and a $5000 credit card cash advance to market their service-oriented translation firm, now one of the largest in the country. Currently, they have a network of 4000 certified language specialists in North America, Europe, and Asia, which translates technical, legal, business, and marketing materials. In 2004, the company estimates its gross sales will be $35 million.

43. ***Research Project*** Choose an innovative product like the one described above. Use your school's library, the Internet, or some other reference source to research the history of the product or service. Collect data about the revenue that the product or service has generated, and find a mathematical model of the data. Summarize your findings.

BUSINESS CAPSULES

Business Capsules appear at the ends of numerous sections. These capsules and their accompanying exercises deal with business situations that are related to the mathematical concepts covered in the chapter.



162 CHAPTER 2 Differentiation

ALGEBRA REVIEW

Simplifying Algebraic Expressions

To be successful in using derivatives, you must be good at simplifying algebraic expressions. Here are some helpful simplification techniques.

1. Combine *like terms*. This may involve expanding an expression by multiplying factors.
2. Divide out *like factors* in the numerator and denominator of an expression.
3. Factor an expression.
4. Rationalize a denominator.
5. Add, subtract, multiply, or divide fractions.

TECHNOLOGY

Symbolic algebra systems can simplify algebraic expressions. If you have access to such a system, try using it to simplify the expressions in this Algebra Review.

EXAMPLE 1 Simplifying a Fractional Expression

(a) $\frac{(x+\Delta x)^2 - x^2}{\Delta x} = \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$ Expand expression.

$= \frac{2x(\Delta x) + (\Delta x)^2}{\Delta x}$ Combine like terms.

$= \frac{\Delta x(2x + \Delta x)}{\Delta x}$ Factor.

$= 2x + \Delta x, \quad \Delta x \neq 0$ Divide out like factors.

(b) $\frac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2)}{(x^2 - 1)^2}$

$= \frac{(-2x^2 - 2x^3 + 2 + 2x) - (6 - 4x - 2x^2)}{(x^2 - 1)^2}$ Expand expression.

$= \frac{-2x^2 - 2x^3 + 2 + 2x - 6 + 4x + 2x^2}{(x^2 - 1)^2}$ Remove parentheses.

$= \frac{-2x^3 + 6x - 4}{(x^2 - 1)^2}$ Combine like terms.

(c) $2\left(\frac{2x + 1}{3x}\right)\left[\frac{3x(2) - (2x + 1)(3)}{(3x)^2}\right]$

$= 2\left(\frac{2x + 1}{3x}\right)\left[\frac{6x - (6x + 3)}{(3x)^2}\right]$ Multiply factors.

$= \frac{2(2x + 1)(6x - 6x - 3)}{(3x)^3}$ Multiply fractions and remove parentheses.

$= \frac{2(2x + 1)(-3)}{3(9)x^3}$ Combine like terms and factor.

$= \frac{-2(2x + 1)}{9x^3}$ Divide out like factors.

CHAPTER 2 Algebra Review 163

EXAMPLE 2 Simplifying an Expression with Powers or Radicals

(a) $(2x + 1)^2(6x + 1) + (3x^2 + x)(2)(2x + 1)(2)$

$= (2x + 1)[(2x + 1)(6x + 1) + (3x^2 + x)(2)(2)]$ Factor.

$= (2x + 1)[12x^2 + 8x + 1 + (12x^2 + 4x)]$ Multiply factors.

$= (2x + 1)(12x^2 + 8x + 1 + 12x^2 + 4x)$ Remove parentheses.

$= (2x + 1)(24x^2 + 12x + 1)$ Combine like terms.

(b) $(-1)(6x^2 - 4x)^{-2}(12x - 4)$

$= \frac{(-1)(12x - 4)}{(6x^2 - 4x)^2}$ Rewrite as a fraction.

$= \frac{(-1)(4)(3x - 1)}{(6x^2 - 4x)^2}$ Factor.

$= \frac{-4(3x - 1)}{(6x^2 - 4x)^2}$ Multiply factors.

(c) $(x)\left(\frac{1}{2}\right)(2x + 3)^{-1/2} + (2x + 3)^{1/2}(1)$

$= (2x + 3)^{-1/2}\left(\frac{1}{2}\right)[x + (2x + 3)(2)]$ Factor.

$= \frac{x + 4x + 6}{(2x + 3)^{1/2}(2)}$ Rewrite as a fraction.

$= \frac{5x + 6}{2(2x + 3)^{1/2}}$ Combine like terms.

(d) $\frac{x^2(\frac{1}{2})(2x)(x^2 + 1)^{-1/2} - (x^2 + 1)^{1/2}(2x)}{x^4}$

$= \frac{(x^3)(x^2 + 1)^{-1/2} - (x^2 + 1)^{1/2}(2x)}{x^4}$ Multiply factors.

$= \frac{(x^2 + 1)^{-1/2}(x)[x^2 - (x^2 + 1)(2)]}{x^4}$ Factor.

$= \frac{x[x^2 - (2x^2 + 2)]}{(x^2 + 1)^{1/2}x^4}$ Write with positive exponents.

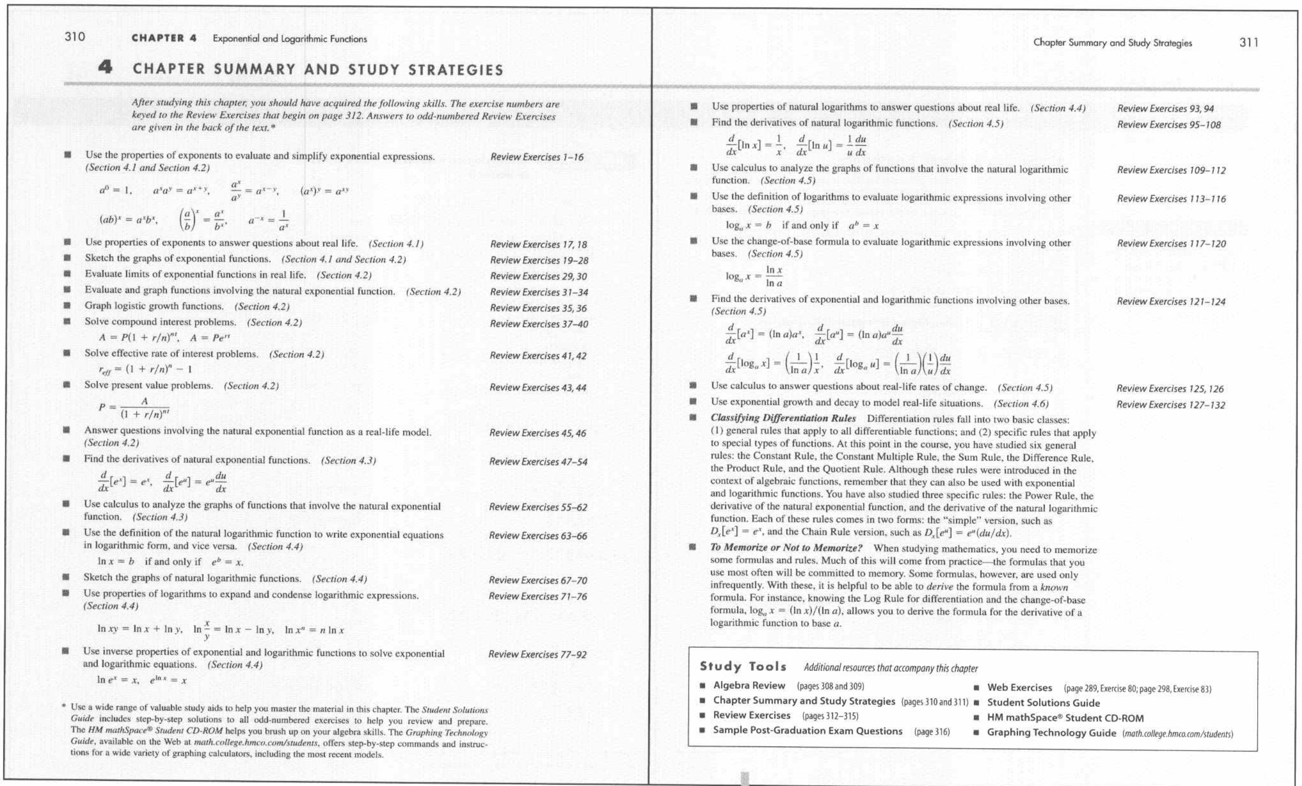
$= \frac{x^2 - 2x^2 - 2}{(x^2 + 1)^{1/2}x^3}$ Divide out like factors and remove parentheses.

$= \frac{-x^2 - 2}{(x^2 + 1)^{1/2}x^3}$ Combine like terms.

All but one of the expressions in this Algebra Review are derivatives. Can you see what the original function is? Explain your reasoning.

ALGEBRA REVIEW

At the end of each chapter, the *Algebra Review* illustrates the key algebraic concepts used in the chapter. Often, rudimentary steps are provided in detail for selected examples from the chapter. This review offers additional support to those students who have trouble following examples as a result of poor algebra skills.



4 CHAPTER SUMMARY AND STUDY STRATEGIES

*After studying this chapter, you should have acquired the following skills. The exercise numbers are keyed to the Review Exercises that begin on page 312. Answers to odd-numbered Review Exercises are given in the back of the text.**

- Use the properties of exponents to evaluate and simplify exponential expressions. *(Section 4.1 and Section 4.2)* — *Review Exercises 1–16*

  $a^0 = 1, \quad a^x a^y = a^{x+y}, \quad \frac{a^x}{a^y} = a^{x-y}, \quad (a^x)^y = a^{xy}$

  $(ab)^x = a^x b^x, \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}, \quad a^{-x} = \frac{1}{a^x}$
- Use properties of exponents to answer questions about real life. *(Section 4.1)* — *Review Exercises 17, 18*
- Sketch the graphs of exponential functions. *(Section 4.1 and Section 4.2)* — *Review Exercises 19–28*
- Evaluate limits of exponential functions in real life. *(Section 4.2)* — *Review Exercises 29, 30*
- Evaluate and graph functions involving the natural exponential function. *(Section 4.2)* — *Review Exercises 31–34*
- Graph logistic growth functions. *(Section 4.2)* — *Review Exercises 35, 36*
- Solve compound interest problems. *(Section 4.2)* — *Review Exercises 37–40*

  $A = P(1 + r/n)^{nt}, \quad A = Pe^{rt}$
- Solve effective rate of interest problems. *(Section 4.2)* — *Review Exercises 41, 42*

  $r_{eff} = (1 + r/n)^n - 1$
- Solve present value problems. *(Section 4.2)* — *Review Exercises 43, 44*

  $P = \frac{A}{(1 + r/n)^{nt}}$
- Answer questions involving the natural exponential function as a real-life model. *(Section 4.2)* — *Review Exercises 45, 46*
- Find the derivatives of natural exponential functions. *(Section 4.3)* — *Review Exercises 47–54*

  $\frac{d}{dx}[e^x] = e^x, \quad \frac{d}{dx}[e^u] = e^u \frac{du}{dx}$
- Use calculus to analyze the graphs of functions that involve the natural exponential function. *(Section 4.3)* — *Review Exercises 55–62*
- Use the definition of the natural logarithmic function to write exponential equations in logarithmic form, and vice versa. *(Section 4.4)* — *Review Exercises 63–66*

  $\ln x = b$ if and only if $e^b = x$.
- Sketch the graphs of natural logarithmic functions. *(Section 4.4)* — *Review Exercises 67–70*
- Use properties of logarithms to expand and condense logarithmic expressions. *(Section 4.4)* — *Review Exercises 71–76*

  $\ln xy = \ln x + \ln y, \quad \ln \frac{x}{y} = \ln x - \ln y, \quad \ln x^n = n \ln x$
- Use inverse properties of exponential and logarithmic functions to solve exponential and logarithmic equations. *(Section 4.4)* — *Review Exercises 77–92*

  $\ln e^x = x, \quad e^{\ln x} = x$

* Use a wide range of valuable study aids to help you master the material in this chapter. The *Student Solutions Guide* includes step-by-step solutions to all odd-numbered exercises to help you review and prepare. The *HM mathSpace® Student CD-ROM* helps you brush up on your algebra skills. The *Graphing Technology Guide*, available on the Web at *math.college.hmco.com/students*, offers step-by-step commands and instructions for a wide variety of graphing calculators, including the most recent models.

- Use properties of natural logarithms to answer questions about real life. *(Section 4.4)* — *Review Exercises 93, 94*
- Find the derivatives of natural logarithmic functions. *(Section 4.5)* — *Review Exercises 95–108*

  $\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad \frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$
- Use calculus to analyze the graphs of functions that involve the natural logarithmic function. *(Section 4.5)* — *Review Exercises 109–112*
- Use the definition of logarithms to evaluate logarithmic expressions involving other bases. *(Section 4.5)* — *Review Exercises 113–116*

  $\log_a x = b$ if and only if $a^b = x$
- Use the change-of-base formula to evaluate logarithmic expressions involving other bases. *(Section 4.5)* — *Review Exercises 117–120*

  $\log_a x = \frac{\ln x}{\ln a}$
- Find the derivatives of exponential and logarithmic functions involving other bases. *(Section 4.5)* — *Review Exercises 121–124*

  $\frac{d}{dx}[a^x] = (\ln a)a^x, \quad \frac{d}{dx}[a^u] = (\ln a)a^u \frac{du}{dx}$

  $\frac{d}{dx}[\log_a x] = \left(\frac{1}{\ln a}\right)\frac{1}{x}, \quad \frac{d}{dx}[\log_a u] = \left(\frac{1}{\ln a}\right)\left(\frac{1}{u}\right)\frac{du}{dx}$
- Use calculus to answer questions about real-life rates of change. *(Section 4.5)* — *Review Exercises 125, 126*
- Use exponential growth and decay to model real-life situations. *(Section 4.6)* — *Review Exercises 127–132*
- ***Classifying Differentiation Rules*** Differentiation rules fall into two basic classes: (1) general rules that apply to all differentiable functions; and (2) specific rules that apply to special types of functions. At this point in the course, you have studied six general rules: the Constant Rule, the Constant Multiple Rule, the Sum Rule, the Difference Rule, the Product Rule, and the Quotient Rule. Although these rules were introduced in the context of algebraic functions, remember that they can also be used with exponential and logarithmic functions. You have also studied three specific rules: the Power Rule, the derivative of the natural exponential function, and the derivative of the natural logarithmic function. Each of these rules comes in two forms: the "simple" version, such as $D_x[e^x] = e^x$, and the Chain Rule version, such as $D_x[e^u] = e^u(du/dx)$.
- ***To Memorize or Not to Memorize?*** When studying mathematics, you need to memorize some formulas and rules. Much of this will come from practice—the formulas that you use most often will be committed to memory. Some formulas, however, are used only infrequently. With these, it is helpful to be able to *derive* the formula from a *known* formula. For instance, knowing the Log Rule for differentiation and the change-of-base formula, $\log_a x = (\ln x)/(\ln a)$, allows you to derive the formula for the derivative of a logarithmic function to base a.

Study Tools *Additional resources that accompany this chapter*

- **Algebra Review** (pages 308 and 309)
- **Chapter Summary and Study Strategies** (pages 310 and 311)
- **Review Exercises** (pages 312–315)
- **Sample Post-Graduation Exam Questions** (page 316)
- **Web Exercises** (page 289, Exercise 80; page 298, Exercise 83)
- **Student Solutions Guide**
- **HM mathSpace® Student CD-ROM**
- **Graphing Technology Guide** (*math.college.hmco.com/students*)

CHAPTER SUMMARY AND STUDY STRATEGIES

The *Chapter Summary* reviews the skills covered in the chapter and correlates each skill to the *Review Exercises* that test those skills. Following each *Chapter Summary* is a short list of *Study Strategies* for addressing topics or situations specific to the chapter, and a list of *Study Tools* that accompany each chapter.

REVIEW EXERCISES

The *Review Exercises* offer students opportunities for additional practice as they complete each chapter. Answers to all odd-numbered *Review Exercises* appear at the end of the text.

544 **CHAPTER 7** Functions of Several Variables

7 CHAPTER REVIEW EXERCISES

In Exercises 1 and 2, plot the points.

1. $(2, -1, 4), (-1, 3, -3)$

2. $(1, -2, -3), (-4, -3, 5)$

In Exercises 3 and 4, find the distance between the two points.

3. $(0, 0, 0), (2, 5, 9)$

4. $(-4, 1, 5), (1, 3, 7)$

In Exercises 5 and 6, find the midpoint of the line segment joining the two points.

5. $(2, 6, 4), (-4, 2, 8)$

6. $(5, 0, 7), (-1, -2, 9)$

In Exercises 7–10, find the standard form of the equation of the sphere.

7. Center: $(0, 1, 0)$; radius: 5

8. Center: $(4, -5, 3)$; radius: 10

9. Diameter endpoints: $(3, 4, 0), (5, 8, 2)$

10. Diameter endpoints: $(-2, 5, 1), (4, -3, 3)$

In Exercises 11 and 12, find the center and radius of the sphere.

11. $x^2 + y^2 + z^2 + 4x - 2y - 8z + 5 = 0$

12. $x^2 + y^2 + z^2 + 4y - 10z - 7 = 0$

In Exercises 13 and 14, sketch the *xy*-trace of the sphere.

13. $(x + 2)^2 + (y - 1)^2 + (z - 3)^2 = 25$

14. $(x - 1)^2 + (y + 3)^2 + (z - 6)^2 = 72$

In Exercises 15–18, find the intercepts and sketch the graph of the plane.

15. $x + 2y + 3z = 6$

16. $2y + z = 4$

17. $6x + 3y - 6z = 12$

18. $4x - y + 2z = 8$

In Exercises 19–26, identify the surface.

19. $x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$

20. $16x^2 + 16y^2 - 9z^2 = 0$

21. $x^2 + \frac{y^2}{16} + \frac{z^2}{9} = 1$

22. $-x^2 + \frac{y^2}{16} + \frac{z^2}{9} = 1$

$\frac{x^2}{9} + y^2$

24. $-4x^2 + y^2 + z^2 = 4$

25. $z = \sqrt{x^2 + y^2}$

26. $z = 9x + 3y - 5$

In Exercises 27 and 28, find the function values.

27. $f(x, y) = xy^2$

(a) $f(2, 3)$ (b) $f(0, 1)$

(c) $f(-5, 7)$ (d) $f(-2, -4)$

28. $f(x, y) = \frac{x^2}{y}$

(a) $f(6, 9)$ (b) $f(8, 4)$

(c) $f(t, 2)$ (d) $f(r, r)$

In Exercises 29 and 30, describe the region *R* in the *xy*-plane that corresponds to the domain of the function. Then find the range of the function.

29. $f(x, y) = \sqrt{1 - x^2 - y^2}$

30. $f(x, y) = \frac{1}{x + y}$

In Exercises 31–34, describe the level curves of the function. Sketch the level curves for the given *c*-values.

31. $z = 10 - 2x - 5y, \; c = 0, 2, 4, 5, 10$

32. $z = \sqrt{9 - x^2 - y^2}, \; c = 0, 1, 2, 3$

33. $z = (xy)^2, \; c = 1, 4, 9, 12, 16$

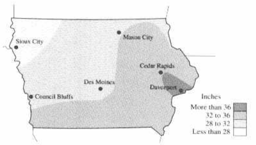
34. $z = 2e^{xy}, \; c = 1, 2, 3, 4, 5$

35. ***Meteorology*** The contour map shown below represents the average yearly precipitation for Iowa. *(Source: U.S. National Oceanic and Atmospheric Administration)*

(a) Discuss the use of color to represent the level curves.

(b) Which part of Iowa receives the most precipitation?

(c) Which part of Iowa receives the least precipitation?



316 **CHAPTER 4** Exponential and Logarithmic Functions

4 SAMPLE POST-GRADUATION EXAM QUESTIONS

CPA
GMAT
GRE
Actuarial
CLAST

The following questions represent the types of questions that appear on certified public accountant (CPA) exams, Graduate Management Admission Tests (GMAT), Graduate Records Exams (GRE), actuarial exams, and College-Level Academic Skills Tests (CLAST). The answers to the questions are given in the back of the book.

1. 10^x means that 10 is to be used as a factor x times, and 10^{-x} is equal to

$$\frac{1}{10^x}.$$

A very large or very small number, therefore, is frequently written as a decimal multiplied by 10^x, where x is an integer. Which, if any, are false?

(a) $470{,}000 = 4.7 \times 10^5$

(b) 450 billion $= 4.5 \times 10^{11}$

(c) $0.00000000075 = 7.5 \times 10^{-10}$

(d) 86 hundred-thousandths $= 8.6 \times 10^2$

2. The rate of decay of a radioactive substance is proportional to the amount of the substance present. Three years ago there was 6 grams of substance. Now there is 5 grams. How many grams will there be 3 years from now?

(a) 4 (b) $\frac{25}{6}$ (c) $\frac{125}{36}$ (d) $\frac{75}{36}$

3. In a certain town, 45% of the people have brown hair, 30% have brown eyes, and 15% have both brown hair and brown eyes. What percent of the people in the town have neither brown hair nor brown eyes?

(a) 25% (b) 35% (c) 40% (d) 50%

4. You deposit \$900 in a savings account that is compounded continuously at 4.76%. After 16 years, the amount in the account will be

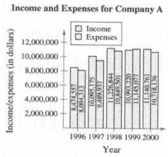
(a) \$1927.53 (b) \$1077.81 (c) \$943.88 (d) \$2827.53

5. A bookstore orders 75 books. Each book costs the bookstore \$29 and is sold for \$42. The bookstore must pay a \$4 service charge for each unsold book returned. If the bookstore returns seven books, how much profit will the bookstore make?

(a) \$975 (b) \$947 (c) \$856 (d) \$681

Figure for 6–9

Income and Expenses for Company A



For Questions 6–9, use the data given in the graph.

6. In how many of the years were expenses greater than in the preceding year?

(a) 2 (b) 4 (c) 1 (d) 3

7. In which year was the profit the greatest?

(a) 1997 (b) 2000 (c) 1996 (d) 1998

8. In 1999, profits decreased by x percent from 1998 with x equal to

(a) 60% (b) 140% (c) 340% (d) 40%

9. In 2000, profits increased by y percent from 1999 with y equal to

(a) 64% (b) 136% (c) 178% (d) 378%

POST-GRADUATION EXAM QUESTIONS

To emphasize the relevance of calculus, every chapter concludes with sample questions representative of the types of questions on certified public accountant (CPA) exams, Graduate Management Admission Tests® (GMAT®), Graduate Record Examinations® (GRE®), actuarial exams, and College-Level Academic Skills Tests (CLAST). The answers to all *Post-Graduation Exam Questions* are given in the back of the text.

A Plan for You as a Student

Study Strategies

Your success in mathematics depends on your active participation both in class and outside of class. Because the material you learn each day builds on the material you have learned previously, it is important that you keep up with your course work every day and develop a clear plan of study. This set of guidelines highlights key study strategies to help you learn how to study mathematics.

Preparing for Class The syllabus your instructor provides is an invaluable resource that outlines the major topics to be covered in the course. Use it to help you prepare. As a general rule, you should set aside two to four hours of study time for each hour spent in class. Being prepared is the first step toward success. Before class:

- Review your notes from the previous class.
- Read the portion of the text that will be covered in class.
- Use the objectives listed at the beginning of each section to keep you focused on the main ideas of the section.
- Pay special attention to the definitions, rules, and concepts highlighted in boxes. Also, be sure you understand the meanings of mathematical symbols and terms written in boldface type. Keep a vocabulary journal for easy reference.
- Read through the solved examples. Use the side comments given in the solution steps to help you in the solution process. Also, read the *Study Tips* given in the margins.
- Make notes of anything you do not understand as you read through the text. If you still do not understand after your instructor covers the topic in question, ask questions before your instructor moves on to a new topic.
- Try the *Discovery* and *Technology* exercises to get a better grasp of the material before the instructor presents it.

Keeping Up Another important step toward success in mathematics involves your ability to keep up with the work. It is very easy to fall behind, especially if you miss a class. To keep up with the course work, be sure to:

- Attend every class. Bring your text, a notebook, a pen or pencil, and a calculator (scientific or graphing). If you miss a class, get the notes from a classmate as soon as possible and review them carefully.
- Participate in class. As mentioned above, if there is a topic you do not understand, ask about it before the instructor moves on to a new topic.
- Take notes in class. After class, read through your notes and add explanations so that your notes make sense to *you*. Fill in any gaps and note any questions you might have.
- Reread the portion of the text that was covered in class. This time, work each example *before* reading through the solution.
- Do your homework as soon as possible, while concepts are still fresh in your mind. Allow at least two hours of homework time for each hour spent in class so you do not fall behind. Learning mathematics is a step-by-step process, and you must understand each topic in order to learn the next one.
- When you are working problems for homework assignments, show every step in your solution. Then, if you make an error, it will be easier to find where the error occurred.
- Use your notes from class, the text discussion, the examples, and the *Study Tips* as you do your homework. Many exercises are keyed to specific examples in the text for easy reference.