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Computer Algebra in Scientific Computing

10th International Workshop, CASC 2007
Bonn, Germany, September 2007
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Dedicated to

Prof. Vladimir P. Gerdt

on the occasion of his 60th birthday

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Preface

The outstanding feature of this CASC Workshop is that this is the tenth workshop in the series started in 1998. The general idea of this workshop was to bring together people working in the areas of computer algebra systems(CASs), computer algebra methods and algorithms, and various CA applications in natural sciences and engineering.

The nine earlier CASC conferences, CASC 1998, CASC 1999, CASC 2000, CASC 2001, CASC 2002, CASC 2003, CASC 2004, CASC 2005, and CASC 2006, were held, respectively, in St. Petersburg, Russia, in Munich, Germany, in Samarkand, Uzbekistan, in Konstanz, Germany, in Crimea, Ukraine, in Passau, Germany, in St. Petersburg, Russia, in Kalamata, Greece, and in Chişinău, Moldova, and they proved to be successful.

Since 1998, the topics of papers published in the CASC proceedings accounted both for the development of new excellent computer algebra systems and for expanding the scopes of application of CA methods and techniques. The present volume of the proceedings of CASC 2007 continues this tradition. Among the traditional topics, there are studies in polynomial and matrix algebra, quantifier elimination, and Gröbner bases.

One of the fruitful areas of the application of CA methods and systems is the derivation of new analytic solutions to differential equations, and several papers deal with this topic.

The application of CASs to stability investigation of both differential equations and difference methods for them is also the subject of a number of papers.

Several papers are devoted to the application of computer algebra methods and algorithms to the derivation of new mathematical models in biology and in mathematical physics.

In addition to the accepted submissions, this volume also includes two invited papers. The paper by F. Winkler and E. Shemyakova (RISC, Linz) addresses the theme of extending the range of analytically solvable PDEs with the aid of symbolic and algebraic methods. The key technique used here is the factorization of a differential operator. The authors have introduced the notion of *obstacle* for the factorization of a differential operator, i.e., conditions preventing a given operator from being factorizable.

The other invited lecture, by S. Fritzsche (Max-Planck Institute for Nuclear Physics, Heidelberg), is devoted to the problem of exploring decoherence and entanglement phenomena in quantum information theory. The author presents his Maple-based FEYNMAN program, which was developed recently to support the investigation of the above phenomena. One of the applications presented is the atomic photoionization, where the author shows how the polarization can be transferred from the incoming photons to the emitted photoelectrons, giving rise to a (spin-spin) entanglement between the photoelectron and the remaining (photo-)ion.

All the papers contained in this volume were accepted by the Program Committee after a thorough reviewing process.

The CASC 2007 workshop was supported financially by a generous grant from the Deutsche Forschungsgemeinschaft (DFG). Our particular thanks are due to the members of the CASC 2007 Local Organizing Committee at the University of Bonn: Andreas Weber (Computer Science Department) and Joachim von zur Gathen (B-IT), who ably handled local arrangements in Bonn. We are grateful to W. Meixner for his technical help in the preparation of the camera-ready manuscript for this volume.

July 2007

V.G. Ganzha
E.W. Mayr
E.V. Vorozhtsov

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Analytic Solutions of Linear Difference Equations, Formal Series, and Bottom Summation

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Abstract. We consider summation of consecutive values $\varphi(v), \varphi(v+1), \ldots, \varphi(w)$ of a meromorphic function $\varphi(z)$ where $v, w \in \mathbb{Z}$. We assume that $\varphi(z)$ satisfies a linear difference equation $L(y) = 0$ with polynomial coefficients, and that a summing operator for L exists (such an operator can be found – if it exists – by the Accurate Summation algorithm, or alternatively, by Gosper's algorithm when $\operatorname{ord} L = 1$).

The notion of *bottom summation* which covers the case where $\varphi(z)$ has poles in $\mathbb{Z}$ is introduced.

1 Introduction

Similarly to [8,3,5,1], this paper is concerned with the problem of summing the elements of a P-recursive sequence $f(k)$, $k \in \mathbb{Z}$, i.e., a sequence which satisfies a linear difference equation with polynomial coefficients.

Let E_k be the shift operator such that $E_k(f(k)) = f(k+1)$ for sequences $f(k)$ where $k \in \mathbb{Z}$. Let

$$L = a_d(k)E_k^d + \cdots + a_1(k)E_k + a_0(k) \in \mathbb{C}(k)[E_k]. \tag{1}$$

We say that an operator $R \in \mathbb{C}(n)[E_k]$ is a *summing* operator for L if

$$(E_k - 1) \circ R = 1 + M \circ L \tag{2}$$

for some $M \in \mathbb{C}(k)[E_k]$. It is easy to see that if there exists a summing operator for L, then there also exists one of order $< d$ (simply replace R by its remainder when divided by L from the right). Hence we can assume w.l.g. that $\operatorname{ord} R = \operatorname{ord} L - 1 = d - 1$:

$$R = r_{d-1}(k)E_k^{d-1} + \cdots + r_1(k)E_k + r_0(k) \in \mathbb{C}(k)[E_k]. \tag{3}$$

⋆ Partially supported by RFBR under grant 07-01-00482-a.
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If a summing operator exists, then it can be constructed by the Accurate Summation algorithm [3] or, when $d = 1$, by Gosper's algorithm [8]. In those cases where $R \in \mathbb{C}[k, E_k]$ exists, equality (2) gives an opportunity to use the discrete Newton-Leibniz formula

$$\sum_{k=v}^{w-1} f(k) = g(w) - g(v) \tag{4}$$

for all integers $v < w$, and for any sequence f such that $L(f) = 0$, taking $g = R(f)$.

However, it was shown in [5] that if R has rational-function coefficients which have poles in $\mathbb{Z}$, then this formula may give an incorrect result (see Example 5 of the present paper). This gives rise to defects in many implementations of summation algorithms. In [5,1] a way was proposed to construct a basis for the space $W_{L,R}$ of all solutions of $L(y) = 0$ for which (4) is valid for all integers $v < w$. It was also proved that $\dim W_{L,R} > 0$ in the case $d = 1$.

In the present paper we give a new sufficient condition for the correctness of definite summation by Gosper's algorithm and by the Accurate Summation algorithm.

In Section 3 below we prove that if a summing operator exists for L with $\operatorname{ord} L = d$, then $\dim W_{L,R} > 0$ regardless of the value of d.

In Section 4 we suppose that L acts on analytic functions:

$$L = a_d(z)E_z^d + \cdots + a_1(z)E_z + a_0(z) \in \mathbb{C}(z)[E_z], \tag{5}$$

where $E_z(\varphi(z)) = \varphi(z+1)$ for analytic functions $\varphi(z)$ where $z \in \mathbb{C}$. We consider the summing operator (if it exists) in the form

$$R = r_{d-1}(z)E_z^{d-1} + \cdots + r_1(z)E_z + r_0(z) \in \mathbb{C}(z)[E_z].$$

Let $\varphi(z)$ be a meromorphic solution of $L(y) = 0$. It turns out that if $\varphi(z)$ has no pole in $\mathbb{Z}$, then $R(\varphi)(z)$ has no pole in $\mathbb{Z}$ as well, and we can use (4) to sum values $\varphi(k)$ for $k = v, v+1, \ldots, w$. This follows from a stronger statement also proved in Section 4. The fact is that even if $\varphi(z)$ has some poles in $\mathbb{Z}$, the summation task can nevertheless be performed correctly. For any $k \in \mathbb{Z}$ the function $\varphi(z)$ can be represented as

$$\varphi(z) = c_{k,\rho_k}(z-k)^{\rho_k} + c_{k,\rho_k+1}(z-k)^{\rho_k+1} + \ldots$$

with $\rho_k \in \mathbb{Z}$ and $c_{k,\rho_k} \neq 0$. If $L(\varphi) = 0$, then there exists the minimal element ρ in the set of all ρ_k, $k \in \mathbb{Z}$. We associate with $\varphi(z)$ the sequence $f(k)$ such that $f(k) = c_{k,\rho_k}$ if $\rho_k = \rho$, and $f(k) = 0$ otherwise. Then the sequence $f(k)$ satisfies the equation $L(y) = 0$, if we use E_k instead of E_z in L. We associate a sequence $g(k)$ with $R(\varphi)$ in a similar way, and the value of ρ for $R(\varphi)$ will be the same as for φ. Now formula (4) is correct. We call this type of summation *bottom summation*.

Some important auxiliary statements (Section 2) on sequences of power series are based on the idea of the ε-deformation of a difference operator which was first used by M. van Hoeij in [7]; later this idea was used in [4] and in [2] as well.

2 Series-Valued Sequences

We start with some notations and definitions. Let ε be a variable (rather than a "small number"). As usual, $\mathbb{C}[[\varepsilon]]$ is the ring of formal power series in ε and $\mathbb{C}((\varepsilon)) = \mathbb{C}[[\varepsilon]][\varepsilon^{-1}]$ is its quotient field (the field of formal Laurent series in ε).

If $s \in \mathbb{C}((\varepsilon)) \setminus \{0\}$ then we define the *valuation* of s in the following way:

$$\nu(s) = -\min\{m \mid m \in \mathbb{Z},\ \varepsilon^m s \in \mathbb{C}[[\varepsilon]]\},$$

in addition we set $\nu(0) = \infty$. If $s \in \mathbb{C}((\varepsilon))$, $m \in \mathbb{Z}$ then $[\varepsilon^m]s$ is the coefficient of ε^m in the series s, and $[\varepsilon^\infty]0 = 0$. It follows from the definition of the valuation that if $s, t \in \mathbb{C}((\varepsilon))$ then

$$\nu(st) = \nu(s) + \nu(t), \qquad [\varepsilon^{\nu(st)}](st) = ([(\varepsilon^{\nu(s)}]s)([(\varepsilon^{\nu(t)}]t), \tag{6}$$

and

$$\nu(s+t) \geq \min\{\nu(s), \nu(t)\}. \tag{7}$$

If K is a ring, then $K^{\mathbb{Z}}$ denotes the ring of all maps $\mathbb{Z} \to K$, i.e., the ring of all two-sided K-valued sequences. Note that the operator E_k is a ring automorphism of $K^{\mathbb{Z}}$.

If $S \in \mathbb{C}((\varepsilon))^{\mathbb{Z}}$, then $\nu(S)$ denotes the sequence in $\mathbb{Z}^{\mathbb{Z}}$ whose kth element is $\nu(S(k))$. If $m \in \mathbb{Z}$, then $[\varepsilon^m]S$ denotes the sequence in $\mathbb{C}^{\mathbb{Z}}$ whose kth element is $[\varepsilon^m](S(k))$. We say that S is of *bounded depth* if the sequence $\nu(S)$ is bounded from below, i.e., there exists

$$m = \min_k \nu(S(k)). \tag{8}$$

If S is of bounded depth, then m in (8) is the *depth* of S. In this case the *bottom of* S, which is a sequence in $\mathbb{C}^{\mathbb{Z}}$, is defined by

$$\operatorname{bott}(S) = [\varepsilon^m]S.$$

An operator $\Lambda \in \mathbb{C}((\varepsilon))^{\mathbb{Z}}[E_k]$ of the form

$$\Lambda = S_d E_k^d + \cdots + S_1 E_k + S_0, \quad S_0, S_1, \ldots, S_d \in \mathbb{C}((\varepsilon))^{\mathbb{Z}}, \tag{9}$$

defines a map $\mathbb{C}((\varepsilon))^{\mathbb{Z}} \to \mathbb{C}((\varepsilon))^{\mathbb{Z}}$ where $(\Lambda S)(k) = \sum_{j=0}^d S_j(k) S(k+j)$. If each sequence S_j has bounded depth m_j for $j = 0, 1, \ldots, d$, then we say that Λ is of bounded depth $m = \min_{0 \leq j \leq d} m_j$. In this case the bottom of Λ is

$$\operatorname{bott}(\Lambda) = \sum_{j=0}^{d} ([\varepsilon^m]S_j) E_k^j \quad \in C^{\mathbb{Z}}[E_k].$$

Proposition 1. *Let Λ be an operator of the form (9), of bounded depth. Let $S \in \mathbb{C}((\varepsilon))$ satisfy $\Lambda(S) = 0$. If for all but finitely many $k \in \mathbb{Z}$ we have*

$$\nu(S_0(k)) = \nu(S_d(k)) = \min_{0 \leq j \leq d} \nu(S_j(k)), \tag{10}$$

then S is of bounded depth and $\tilde{\Lambda}(\operatorname{bott}(S)) = 0$, where $\tilde{\Lambda} = \operatorname{bott}(\Lambda)$.

Proof. Fix $k \in \mathbb{Z}$ and $i \in \{0, 1, \ldots, d\}$. From $\Lambda(S) = 0$ it follows that

$$\nu(S_i(k)S(k+i)) = \nu\left(-\sum_{0\le j\le d,\, j\ne i} S_j(k)S(k+j)\right),$$

so by (6) and (7) we have

$$\nu(S_i(k)) + \nu(S(k+i)) \ge \min_{\substack{0\le j\le d\\ j\ne i}} \nu(S_j(k)) + \min_{\substack{0\le j\le d\\ j\ne i}} \nu(S(k+j)). \tag{11}$$

Assume that $\nu(S_i(k)) = \min_{0\le j\le d} \nu(S_j(k))$. Then it follows from (11) that $\nu(S(k+i)) \ge \min_{\substack{0\le j\le d\\ j\ne i}} \nu(S(k+j))$. Specializing this to $i = 0$ and $i = d$ and using (10) we obtain that

$$\nu(S(k)) \ge \min_{1\le j\le d} \nu(S(k+j))$$

and

$$\nu(S(k+d)) \ge \min_{0\le j\le d-1} \nu(S(k+j))$$

for all but finitely many $k \in \mathbb{Z}$. Therefore, S is of bounded depth. The equality $\tilde{\Lambda}(\mathrm{bott}(S)) = 0$ now follows from (6). □

Example 1. Let

$$\Lambda = S_1E_k + S_0,\ S_1(k) = k + 1 + \varepsilon,\ S_0(k) = -k - \varepsilon$$

and

$$S(k) = \begin{cases} -\frac{1}{\varepsilon}, & \text{if } k = 0,\\ \sum_{i=0}^{\infty} \left(-\frac{1}{k}\right)^{i+1} \varepsilon^i, & \text{otherwise.} \end{cases}$$

Then $S_1(k)S(k+1) = -S_0(k)S(k) = -1$ for all k, and $\Lambda(S) = 0$ as a consequence. The depth of S is -1.

We see that

$$\mathrm{bott}(S)(k) = \begin{cases} -1, & \text{if } k = 0,\\ 0, & \text{otherwise,} \end{cases}$$

and $\mathrm{bott}(\Lambda) = (k+1)E_k - k$. It is easy to see that $(k+1)f(k+1) - kf(k) = 0$, where $f(k) = \mathrm{bott}(S)(k)$; so $\tilde{\Lambda}(\mathrm{bott}(S)) = 0$, where $\tilde{\Lambda} = \mathrm{bott}(\Lambda)$.

3 When a Summing Operator Exists

If $\varphi(z) \in \mathbb{C}(z)$, then we write $\hat{\varphi}(k)$ for the sequence $\varphi(k+\varepsilon)$, $k \in \mathbb{Z}$, of rational functions expanded into Laurent series about $\varepsilon = 0$. We associate with every operator

$$N = b_l(z)E_z^l + \cdots + b_1E_z + b_0(z) \quad \in \mathbb{C}(z)[E_z]$$

the operator

$$\hat{N} = \hat{b}_l(k)E_k^l + \cdots + \hat{b}_1(k)E_k + \hat{b}_0(k) \quad \in \mathbb{C}((\varepsilon))^{\mathbb{Z}}[E_k]$$

which acts on sequences from $\mathbb{C}((\varepsilon))^{\mathbb{Z}}$.

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