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LMI Approach to Analysis and Control of Takagi-Sugeno Fuzzy Systems with Time Delay

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Preface

Since initiated by Lotfi A. Zadeh in 1965, fuzzy set theory has triggered a considerably large body of areas to blossom. A fuzzy system is, in a very broad sense, any fuzzy logic-based system where fuzzy logic can be used either as the basis for the representation of different forms of system knowledge or the model for the interactions and relationships among the system variables. Fuzzy systems have proven to be an important tool for modeling complex systems for which, due to complexity or imprecision, classical tools are unsuccessful. There have been diverse fields of applications of fuzzy technology from medicine to management, from engineering to behavioral science, from vehicle control to computational linguistics, and so on. Fuzzy modeling is a conjunction to understand the system's behavior and build useful mathematical models. Different types of fuzzy models have been proposed in the literature, among which the Takagi-Sugeno (T-S) fuzzy model is a rule-based one suitable for the accurate approximation and identification of a wide class of nonlinear systems. There has been an increasing amount of work on analysis and synthesis of fuzzy systems based on T-S fuzzy models. Since 2000, T-S fuzzy model approach has been extended to tackle analysis and control problems of nonlinear systems with time delay. So far extensive results have been presented for investigating T-S fuzzy systems with time delay, many of which adopt an easy and popular scheme, say, linear matrix inequality (LMI) based method. However, there lacks of a monograph in this direction to provide the state-of-the-art of coverage of this new growing area.

This book serves as a comprehensive monograph on T-S fuzzy systems with time delay. It is not intended as a collection of existing results in the literature but to cover as many as possible interesting topics and establish systematic structures towards analysis and control methods. The book is mainly based on the recent research work carried on by the authors. It includes the latest developments and advances for analysis techniques and synthesis methods, brings out the characteristic systematism in them, and points out further insight to solve relevant problems. Topics on T-S fuzzy systems with time delay cover a wide range including stability analysis, stabilization, tracking control, variable structure control, observer design and filter design.

The book is a useful source of reference for all those, from graduate students to senior researchers, from mathematicians to human and social science scholars, interested in or working with fuzzy control methods. The prerequisites for the book are modest and they are fundamental knowledge of systems, control, matrix theory and basic fuzzy set theory.

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Symbols

$\mathbb{R}$	field of real numbers
$\mathbb{R}^n$	n-dimensional real Euclidean space
$\mathbb{R}^{n\times m}$	space of $n \times m$ real matrices
I	identity matrix
I_n	identity matrix of dimension $n \times n$
$0_{n\times m}$	zero matrix of dimension $n \times m$
A^T	transpose of matrix A
A^{-1}	inverse of matrix A
$\det(A)$	determinant of matrix A
$\deg(\cdot)$	degree of a polynomial
$\mathrm{rank}\,(A)$	rank of matrix A
$A > 0$	symmetric positive definite
$A \geq 0$	symmetric positive semi-definite
$A < 0$	symmetric negative definite
$A \leq 0$	symmetric negative semi-definite
$A^{1/2}$	symmetric square root of $A \geq 0$, i.e., $A^{1/2}A^{1/2} = A$
$\mathrm{diag}\,\{A_1, \cdots, A_n\}$	diagonal matrix with A_i as its ith diagonal element
$\lambda(A)$	eigenvalues of square matrix A
$\sigma_{max}(A)$	largest singular value of matrix A
$\sigma_{min}(A)$	smallest singular value of matrix A
$\rho(A)$	spectral radius of square matrix A

$\forall$	for all
$\in$	belong to
$\subseteq$	subset
$\sum$	sum
$\lvert \cdot \rvert$	absolute value (or modulus)
$\lVert \cdot \rVert$	spectral norm
$\lVert \cdot \rVert_\infty$	induced l_∞-norm
$:=$	defined as
$\rightarrow$	tend to, or mapping to (case sensitive)
lim	limit
$f(t_-)$	$= \lim_{\epsilon \to 0+} f(t - \epsilon)$
$f(t_+)$	$= \lim_{\epsilon \to 0+} f(t + \epsilon)$
sup	supremum
inf	infimum
LMI	linear matrix inequality
BMI	bilinear matrix inequality
PDC	parallel distributed compensation

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Contents

1. Introduction

1.1 History of Fuzzy Systems

Fuzzy theory is firmly grounded in the mathematics of fuzzy sets and fuzzy logic developed by Lofti A. Zadeh in 1965 [166]. Since its introduction, fuzzy set theory has grown to become a major scientific domain. A fuzzy system is referred to as any static or dynamic system which makes use of fuzzy logic and of the corresponding mathematical framework. Such a system allows a gradual and continuous transition, say, from 0 to 1, rather than a crisp and abrupt change between binary values of 0 and 1. It is known that in an ordinary set, an element of the universe either belongs to or does not belong to the set. This shows the membership of an element is crisp. A fuzzy set is a generalization of an ordinary set by allowing a degree of membership for each element, which is a real number on $[0, 1]$. The membership function of a set maps each element to its degree. Accordingly, fuzzy logic is an extension of ordinary logic, which can represent fuzzy implications. There are a lot of publications to address the details of fuzzy theory, see, e.g., [65][66][67] [71] [124][168][176].

Fuzzy theory also experienced a period of sparking controversy. In the 1960s, many scholars objected to the idea and viewed the new theory as against basic scientific principles. Some mathematicians claimed that probability theory could work equally well or better than fuzzy theory. Other criticisms claimed that fuzzy theory is something of an illusion because it merely hides the complexity of control system design in determining the rule set, or that fuzzy logic is little more than a restatement of traditional statistical methods in new and obscure nomenclature. In the late 1960s and early 1970s, fuzzy theory continued to growth as an independent field, and there appeared some other fundamental fuzzy concepts, like fuzzy algorithms[167], fuzzy decision making[2], etc. In 1973, Lofti A. Zadeh published a paper to establish the foundation for fuzzy control[168]. In this work, he introduced the concept of linguistic variables and proposed to use fuzzy IF-THEN rules to formulate human knowledge. Meanwhile, initial applications of fuzzy control for real systems, like the steam engine controller[102] and the cement kiln controller in Denmark, showed that the fuzzy field was promising. Although fuzzy theory was still ignored by some researchers because it was associated with artificial intelligence, a field resulting in a lack of credibility with industrial firms, the later big events largely encouraged people working at fuzzy control. In 1980s, Sugeno applied fuzzy control to a Fuji Electric water purification plant and pioneered the work on a fuzzy robot which is

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a self-parking car controlled by calling out commands [114][117]. Yasunobu and Miyamoto from Hitachi demonstrated the superiority of fuzzy control systems for the Sendai underground railway and used their idea to control accelerating, braking, and stopping when the line opened in 1987. During the Second Annual International Fuzzy Systems Association Conference held in Tokyo in 1987, Hirota displayed a fuzzy robot arm playing two-dimensional ping-pong game[60], and Yamakawa demonstrated the use of fuzzy control in balancing an inverted pendulum[155]. In 1990s, the fast development of fuzzy idea in Japan helped promote interest in fuzzy systems. Several scientific societies engaged in fuzzy field have been established. Practical applications were not only in industry but also in the manufacture of consumer products, like video camera, microwave cooker, and washing machine, etc [176].

Work on fuzzy systems was also proceeding in the United States and Europe. The US Environmental Protection Agency has investigated fuzzy control for energy-efficient motors, and NASA has studied fuzzy control for automated space docking. Large companies such as Boeing, General Motors, and Whirlpool have worked on fuzzy logic for use in low-power electrical appliances, energy-efficient electrical motors, and so on. In February 1992, the first IEEE International Conference on Fuzzy Systems was held in San Diego, and in 1993, the IEEE Transactions on Fuzzy Systems was inaugurated. These events symbolized the acceptance of fuzzy theory in a wide community of engineering society.

Up to now, fuzzy systems have been successfully used in a number of real-word applications, such as Penicillin-G conversion [1], prediction of river water flow [118], and many other examples in ecological systems and biomedical field [1][112]. Meanwhile, numerous publications have been reported in providing theoretical support. Various methodologies have been proposed for analysis, modeling, design, control and monitoring of fuzzy systems.

1.2 T-S Fuzzy Systems

Fuzzy ideas are useful for modeling complex nonlinear systems in which, due to complexity or imprecision, classical tools are unsuccessful. Like neural networks, fuzzy systems have been recognized as powerful universal approximators due to their capability of approximating a given system with arbitrary accuracy [68][69] [135][136][137]. Various techniques such as fuzzy clustering [1][161], neuro-fuzzy learning methods [92], and orthogonal least squares [118][136] have been proposed for data-driven fuzzy modeling. There are numerous other significant publications which discuss the fuzzy modeling [4][7][23] [25][91][111] [112][115][119][154].

Fuzzy models can be static or dynamic. Different types of fuzzy models have been reported in the literature. The widely used are rule-based fuzzy systems, in which the relationship between variables are represented by means of fuzzy IF-THEN rules of the form:

IF antecedent proposition THEN consequence proposition.

Rule-based fuzzy systems include Mamdani models (or linguistic fuzzy model) [168] [101], fuzzy relation models [109] [157], and Takagi-Sugeno (T-S) fuzzy models [119]. This book focuses on T-S fuzzy models, where the consequences are crisp functions of the antecedent variables rather than fuzzy propositions.

T-S fuzzy systems are popular and well used tools in recent years. A general T-S fuzzy model employs an affine fuzzy model with a constant term in the consequence[119]. It is known that smooth nonlinear dynamic systems can be approximated by affine T-S fuzzy models[5][159]. Most recent developments are based on T-S models with linear rule consequences (here and after, such models are generally called T-S fuzzy models). The main feature of T-S fuzzy models is to represent the nonlinear dynamics by simple (usually linear) models according to the so-called fuzzy rules and then to blend all the simple models into an overall single model through nonlinear fuzzy membership functions. Each simple model is called a local model or a sub-model. The output of the overall fuzzy model is calculated as a gradual activation of the local models by using proper defuzzification schemes [111][119][154]. It has been proved that T-S fuzzy models can approximate any smooth nonlinear dynamic systems[29][122].

A continuous-time T-S fuzzy model is of the following form:

Plant Rule i: IF θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + B_{wi} w(t) + B_{ui} u(t), \\
z(t) &= C_i x(t) + D_{wi} w(t) + D_{ui} u(t), \\
y(t) &= E_i x(t) + D_{ywi} w(t) + D_{yui} u(t), \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{m_1}$, $w \in \mathbb{R}^{m_2}$, $z \in \mathbb{R}^{n_1}$ and $y \in \mathbb{R}^{n_2}$ are the state, the control input, the disturbance, the controlled output and the measured output, respectively; r is the number of IF-THEN rules; $A_i, B_{wi}, B_{ui}, C_i, D_{wi}, D_{ui}, D_{ywi}, D_{yui}$ and E_i are real constant matrices with appropriate dimensions; $\theta_j(x)$ and μ_{ij} ($i = 1, \ldots, r$, $j = 1, \ldots, p$) are respectively the premise variables and the fuzzy sets.

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} h_i(\theta)[A_i x(t) + B_{wi} w(t) + B_{ui} u(t)], \\
z(t) &= \sum_{i=1}^{r} h_i(\theta)[C_i x(t) + D_{wi} w(t) + D_{ui} u(t)], \\
y(t) &= \sum_{i=1}^{r} h_i(\theta)[E_i x(t) + D_{ywi} w(t) + D_{yui} u(t)],
\end{aligned} \tag{1.2}$$

where

$$\theta = [\theta_1, \ldots, \theta_p],$$
$$h_i(\theta) = \nu_i(\theta) / \sum_{i=1}^{r} \nu_i(\theta),$$
$$\nu_i(\theta) = \prod_{j=1}^{p} \mu_{ij}(\theta_j), \tag{1.3}$$

and where $\mu_{ij}(\theta_j)$ is the grade of membership of θ_j in μ_{ij}. It is seen that ν_i maps $\mathbb{R}^p$ onto $[0,1]$, which is called the membership function corresponding to plant rule i, $i = 1, \ldots, r$. Moreover, the fuzzy weighting functions $h_i(\theta)$ satisfy

$$\sum_{i=1}^{r} h_i(\theta) = 1, \quad h_i(\theta) \geq 0. \tag{1.4}$$

The design of controllers to achieve the synthesis purpose is performed through the parallel distributed compensation (PDC) [134]. Details will be provided in related chapters.

Recent advances witness the extension of T-S fuzzy model approach to nonlinear time-delay systems. The analysis and synthesis method is originated in [9]. So far, Various methodologies have been proposed for investigation of T-S fuzzy systems with time-delay in a wide range of research topics, including stability and state-feedback stabilization [10][142][158], static output-feedback control [12], dynamic output-feedback control [73][150], linear quadratic state-feedback control [139], variable structure control [81], H_∞ control [73][150][160], and H_∞ filter design [149]. In case of unavailability of the states, fuzzy observers play an important role in analysis and synthesis of fuzzy systems. Accordingly, observer-based fuzzy control have also attracted much attention for T-S fuzzy systems with time-delay, see [9][10][138] for observer-based stabilization, [13] for observer-based reliable control and [14] for observer-based guaranteed cost control.

A continuous-time T-S fuzzy model with state delay is of the following form:

Plant Rule i: IF θ_1 is μ_{i1} and $\cdots$ and θ_p is μ_{ip} THEN

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_{wi} w(t) + B_{ui} u(t), \qquad (1.5)\\
z(t) &= C_i x(t) + C_{\tau i} x(t - \tau(t)) + D_{wi} w(t) + D_{ui} u(t),\\
y(t) &= E_i x(t) + E_{\tau i} x(t - \tau(t)) + D_{ywi} w(t) + D_{yui} u(t),\\
x(t) &= \phi(t), \quad t \in [-\tau_0, \ 0], \quad i = 1, 2, \ldots, r,
\end{aligned}$$

where notations are as in (1.1) and $A_{\tau i}, C_{\tau i}$ and $E_{\tau i}$ are real constant matrices corresponding to delay terms; ϕ is the initial condition; The time-delay $\tau(t)$ may be unknown but is assumed to be continuous function of time with an upper bound τ_0:

$$\tau(t) \leq \tau_0. \tag{1.6}$$