FOR ENGINEERING ECONOMIC ANALYSIS

MAX KURTZ

CULATIONS FOR ENGINEERING ECONOMIC ANALYSIS

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CALCULATIONS FOR ENGINEERING ECONOMIC ANALYSIS

ABOUT THE AUTHOR

Max Kurtz, P.E., is a consulting engineer, educator, and writer. He has taught courses for engineers preparing for the Professional Engineers' (P.E.) examination since 1961, and courses in CPM and operations research. He has received extremely high commendation for his teaching methods. He is also the author of McGraw-Hill's Handbook of Applied Mathematics for Engineers and Scientists, and is a major contributing author to the Second Edition of McGraw-Hill's Standard Handbook of Engineering Calculations.

PREFACE

To function successfully in today's complex and dynamic world, each organization engaged in economic or financial activity must plan its operations and formulate decisions in a completely methodical and expert manner. Consequently, it is mandatory that the organization be guided by individuals who possess a deep and thorough grasp of financial principles and of the techniques of economic analysis and decision making. In modern times, economic analysis has been placed on a rigorous basis by the development of new concepts and sophisticated investigative methods.

The objective of this book is to give the readers mastery over financial principles and the tools of economic analysis, thereby enabling them to examine each economic activity systematically and to formulate the best possible decisions. It is designed for use by all individuals who make decisions pertaining to economics: corporate executives, business managers, design engineers and architects, project managers, financial analysts, government planners, and a host of others.

This is an intensely practical book, and its main emphasis is on problem solving. This book presents the detailed numerical solution to 273 problems, with each step in the solution explained in a manner that makes it readily comprehensible. Thus, the book offers its readers intensive drill in problem solving and sharpens their analytic ability, thereby assisting them immeasurably in solving the problems in economic analysis with which they are likely to be confronted.

This book can also be extremely helpful to its readers in handling their personal finances. Example 2.28, where we calculate the interest rate paid where a commodity is purchased on the installment plan, provides an illustration.

This book is divided into two broad categories. Part 1 covers situations that revolve about the time value of money and therefore require decisions based on compound-interest calculations. Part 2 covers situations where the time value of money is not relevant or where it is merely incidental. Throughout the book, the modern methods of economic analysis, such as linear programming, dynamic programming, and critical path method, are developed and applied. Moreover, since the future is shrouded in uncertainty and decisions must often be based on probability, this book presents and illustrates thoroughly the techniques of probabilistic decision making.

I am grateful to my wife, Ruth Ingraham Kurtz, B.E. in E.E., formerly project manager in computer systems development, for her constant assistance in the preparation of the manuscript. I am also grateful to Margaret Lamb of the McGraw-Hill staff for her invaluable services on this project.

Max Kurtz

INTRODUCTION

This book has the following specific features:

1. It emphasizes the use of *simple logic* rather than abstruse mathematics (but without sacrificing mathematical rigor). The solution to Example 8.6 demonstrates how effective the use of simple logic can be. In this example, it is necessary to allocate production of a commodity among several alternative machines. This text demonstrates on a commonsense basis that the total cost of production is minimum when all incremental costs are equal.

Similarly, this book views probability as the relative frequency of a given event when the number of trials is vast. This simple device enables us to solve problems pertaining to probability in a simple and straightforward manner, thus obviating the need for invoking abstract laws of probability. The solution to Example 10.12 is illustrative.

- 2. In a vast number of cases, an example in financial mathematics is solved by multiple methods, and the multiplicity of solutions is a salient feature of this book. The solution to Example 6.21 is typical. Our motive in solving an example in multiple ways is not merely to demonstrate that the numerical result obtained by the first method is correct; on the contrary, our motive is far more profound. Each alternative method of solution stems from viewing the example from a different perspective. Thus, the reader learns that there are many ways of viewing a situation and thereby acquires a richer and deeper understanding of each subject. The reader also acquires the vision and flexibility of thought that are needed in coping with the diverse and original problems that arise in the real world.
- 3. Similarly, in many examples in financial mathematics, the numerical result obtained is verified in some simple but highly effective manner. Our motive in subjecting numerical solutions to verification is twofold. First, the verification imparts to the readers a very deep understanding of the definition, concept, or principle that lies at the core of the example, and it makes the example very vivid and highly interesting. Second, by demonstrating to the readers that it is possible to verify numerical results, this book encourages them to do so in their own work, and it gives them confidence that the numerical results they obtain are correct.

We shall present several illustrations of the use of verification as a device. In Example 6.7, we calculate the internal rate of return of an investment, and in Table 6.4 we verify the result. In Example 2.54, we construct the set of withdrawals that can be made from a fund in accordance with certain specifications, and in Table 2.10 we demonstrate that these specifications are satisfied. In Art. 5.2.2, we offer a simple technique for verifying the capitalized cost of an asset, and in the examples that follow we apply this technique.

- **4.** Each example in this book is given a descriptive *title*, and this title serves an important purpose. It calls the reader's attention to the specific character of the example and thereby emphasizes how this example is distinguished from all other examples. Thus, it brings out the uniqueness of the example very sharply. As an illustration, Example 2.34 has the title "Uniform Series with Periodic Omissions." This title alerts the reader to the fact that there are gaps in the payments, and consequently the reader will examine the given information with a very watchful eye to see what those gaps are. In summary, the title of an example enables the reader to absorb the statement of the example more effectively.
- 5. Whereas many other books in this field confine their study to standard situations of a textbook type, the present book analyzes in depth all types of situations, both standard and nonstandard. For example, in performing a cost comparison of alternative methods of accomplishing a task, many books consider only cases where the annual operating cost is uniform and consequently the cost comparison can be performed by applying a set equation. By contrast, this book analyzes situations where the annual operating cost varies or where some other unusual feature is present. In these nonstandard situations, no set equation is available to serve as a crutch; the engineering economist must draw upon his or her own understanding of financial mathematics and problem-solving ability.

Examples 5.3, 5.5, and 5.15 illustrate cost comparisons in nonstandard situations. Similarly, Examples 1.19, 1.20, 2.10, and 2.11 deal with situations where there is a change in the interest or investment rate. Thus, this book provides its readers with the skills that are needed for coping with real-life as distinguished from mere textbook problems.

- 6. This book employs numerical examples extensively to illustrate definitions and economic principles. It thereby reinforces these definitions and principles, making them far more meaningful and vivid than would otherwise be the case. Thus, the expression *equivalent sets of payments* is defined in Art. 1.4, and the definition is immediately illustrated by Example 1.9. Similarly, by calculating the mean rate of return of a set of related investments, Example 6.9 forcefully demonstrates that the internal rate of return is of limited utility in appraising an investment. By applying specific numerical data, Example 6.38 shows how the investment rate earned by stockholders of a corporation is strongly influenced by the extent to which the firm is trading on the equity.
- 7. Diagrams are used copiously throughout this book because they yield remarkable benefits. A diagram enables us to visualize a problem instantly by appealing to the eye as well as the intellect, and it illuminates relationships that would otherwise remain indistinct or completely invisible.

The use of diagrams in this book is illustrated by Figure 8.3, which pertains to Example 8.4. In this example, a machine must be serviced periodically to restore it to maximum efficiency, and it is necessary to determine how frequently the machine should be serviced. Figure 8.3 brings the problem into sharp focus by representing the average hourly profit that accrues from use of the machine by the slope of a straight line. Thus, the diagram trenchantly shows the relationship between the average hourly profit and the period between successive services. Moreover, it reveals pictorially how the average hourly profit responds to changes in the cost or duration of a service, which we investigate in Example 8.5. Thus, Figure 8.3 adds a vitality to Examples 8.4 and 8.5 and thereby makes these problems far more interesting than would otherwise be the case.

8. Markov processes play an important role in economic planning because future events are generally shaped by present conditions. For this reason, this book presents

a detailed exposition and application of Markov probability. Here too a very practical approach is used.

For example, a central problem in analyzing Markov processes is to establish the steady-state probabilities when the transition probabilities are known. This problem is solved in Example 10.28 by three methods. The third method is based on a simple practical device: constructing a Markov chain in which the relative frequency of a given transition coincides with the probability of that transition. The relative frequency of an individual outcome in this chain then equals the steady-state probability of that outcome. This device instantly verifies the results obtained mathematically and it stimulates the reader's interest by stripping the problem of intellectual abstraction.

9. One of the major subjects encompassed by linear programming is the famous product-mix problem. In this problem, a firm is capable of manufacturing several products, and it is necessary to determine how many units of each product should be manufactured in a given period. The conventional treatment of this problem is based on the narrowest possible criterion: maximizing the profit that accrues from this specific industrial operation.

In this book, by contrast, we apply a different and broader criterion by viewing the totality of the firm's investments. Every dollar that is invested in any venture must earn a certain minimum acceptable rate of return, and our objective is to maximize the *total income* that the firm derives from its available capital. Although the firm has a specific amount of capital allocated to this industrial operation, it is not obligated to expend it exclusively in this operation; part of the capital can be invested elsewhere.

The criterion applied in this book is formulated in Art. 7.1.2, and Example 7.7 demonstrates that the criterion of this book yields a more rational result than the conventional criterion. This example also shows that the simplex-tableau method requires merely a slight adaptation when our criterion is applied. Example 7.10 demonstrates that the criterion recognized in this book is the only logical one to apply in the situation where all points along a specific boundary of the feasible region yield the maximum profit.

Our solution to the product-mix problem illustrates the compelling need to view the activities of a firm in the broadest possible manner. Even when we are analyzing a single operation, it is imperative that we also examine the related and alternative operations.

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ECONOMIC ANALYSIS WITH COMPOUND INTEREST

NOMENCLATURE

The following list contains the symbols that appear frequently in the text. Where duplication occurs, the intended meaning of the symbol is readily apparent from the context.

a = annual rate of increase of cash-flow rate

A = periodic payment in a uniform series

 A_e = periodic payment in the uniform series that is equivalent to a conventional uniform-gradient series

 B_0 = first cost of asset

 B_r = book value of asset at end of rth year

C =annual operating (or maintenance) cost of asset

 $C_r = \cos t$ of commodity at end of rth year

 D_r = depreciation charge for rth year

F =future worth of a given sum of money

 F_c = future worth of a continuous cash flow

 F_{μ} = future worth of a uniform series

 F_{ug} = future worth of a uniform-gradient series

 F_{ur} = future worth of a uniform-rate series

 g = appreciation rate in a continuous cash flow (i.e., rate of increase of cashflow rate)

 $g_{\rm eff}$ = effective annual appreciation rate

G =gradient (constant difference) in a uniform-gradient series

 $H_r = r$ th payment in a series

i = interest (or investment) rate

 i_a = after-tax investment rate

 i_b = before-tax investment rate

 $i_{\rm eff} = {\rm effective \ interest \ rate}$

 $i_{\text{equiv}} = \text{equivalent interest rate}$

L =salvage value of asset

m = number of interest periods in one payment period, with discrete compounding

m = number of years in one payment period, with continuous compounding

n = number of interest periods

n =duration of continuous cash flow, years