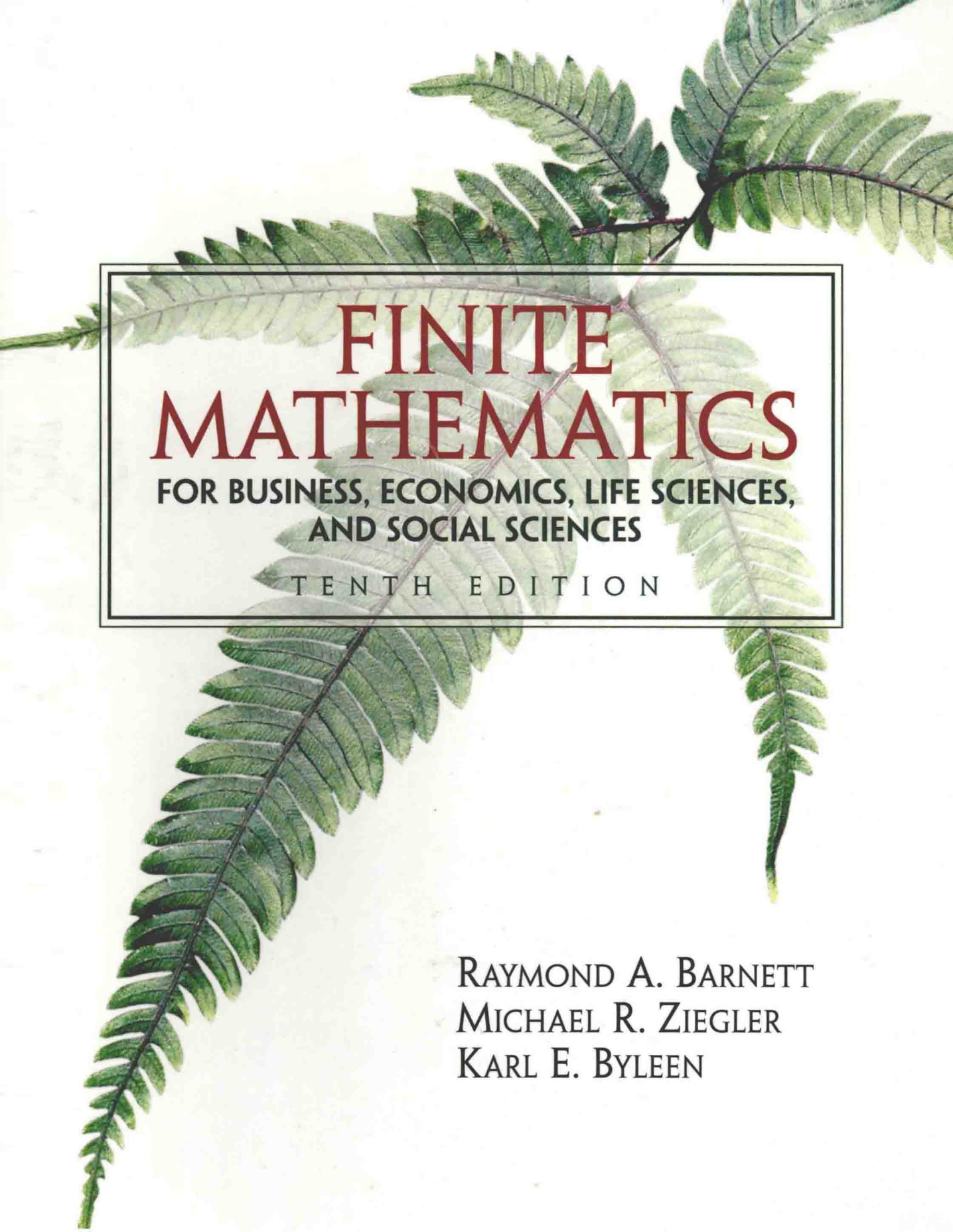

FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

TENTH EDITION

RAYMOND A. BARNETT
MICHAEL R. ZIEGLER
KARL E. BYLEEN

FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

Tenth Edition

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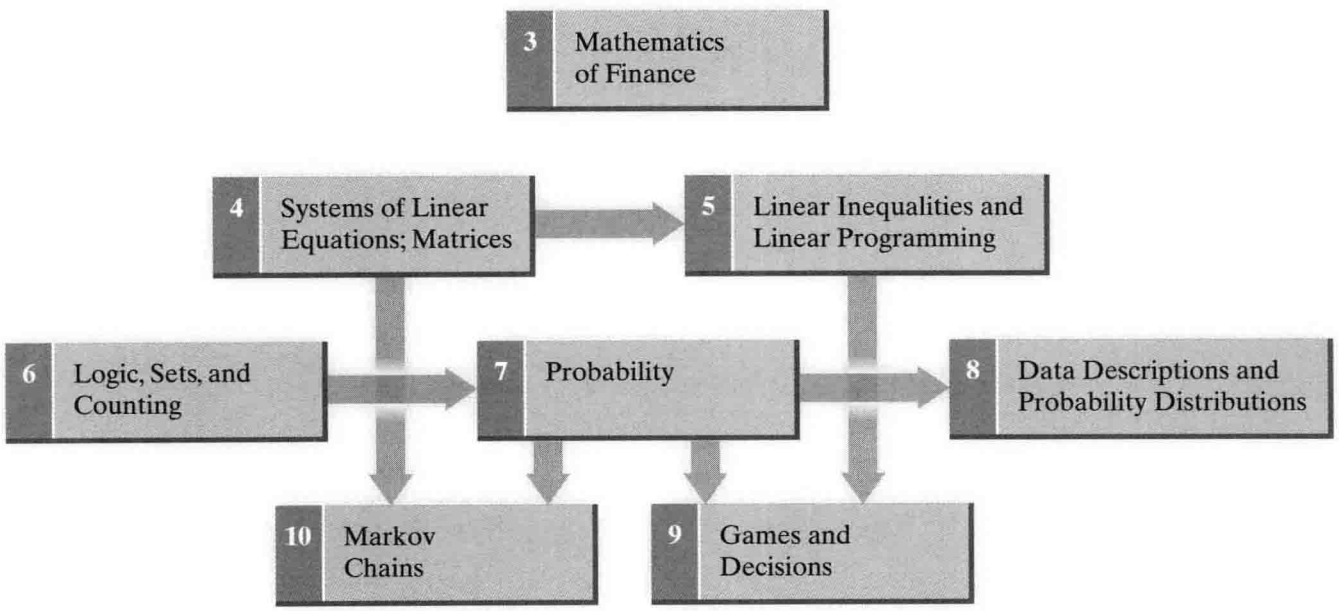
FINITE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

CHAPTER DEPENDENCIES

PART ONE A LIBRARY OF ELEMENTARY FUNCTIONS*

PART TWO FINITE MATHEMATICS



APPENDICES

* Selected topics from Part One may be referred to as needed in Part Two or reviewed systematically before starting Part Two.

PREFACE

The tenth edition of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in finite mathematics and for students who have had 1–2 years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the Chapter Dependency Chart on page ix). It is one of five books in the authors' college mathematics series.

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its tenth edition, *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of having a substantial amount of both.

Examples and Matched Problems

Over 280 completely worked examples, many of them updated for this edition, are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are **annotated** and the problem-solving steps are clearly identified. **Think Boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 1-4).

Each example is followed by a similar **matched problem** for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Exercise Sets

The book contains **over 3,700 carefully selected and graded exercises, and over 15% of these are new.** Many problems have multiple parts, significantly increasing the total number of exercises. Each exercise set is designed so that every student will experience success. Exercise sets are divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

Applications

A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included

to convince even the most skeptical student that mathematics is really useful (see the Applications Index inside the back cover). Worked examples involving applications are identified by  .

Almost **every exercise set contains application problems,** usually divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

Explore and Discuss

Every section contains **Explore–Discuss problems** interspersed at appropriate places to encourage the student to think about a relationship or process before a result is stated, or to investigate additional consequences of a development in the text. Verbalization of mathematical concepts, results, and processes is encouraged in these Explore–Discuss problems, as well as in some matched problems, and in some problems in almost every exercise set. This serves to foster critical thinking skills. The Explore–Discuss material also can be used as in-class or out-of-class group activities. In addition, at the end of every chapter, we have included two special chapter group activities that involve several of the concepts discussed in the chapter. Problems in the exercise sets that require verbalization are indicated by color problem numbers.

Technology

The generic term graphing utility is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. Although access to a graphing utility is not assumed, it is likely that many students will want to make use of one of these devices. To assist these students, optional graphing utility activities are included in appropriate places in the book. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, exercises for the student to solve, and a group activity that involves the use of technology at the end of each chapter. In the group activity at the end of Chapter 1, and continuing through Chapter 2, linear regression on a graphing utility is used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing utility material is clearly identified by either  or  and can be omitted without loss of continuity, if desired.

All **graphs** are computer-generated to ensure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.

Emphasis and Style

The text is written for student comprehension. Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

Additional Pedagogical Features

Boxes are used to highlight important **definitions, results,** and **step-by-step processes** (see Sections 1-1 and 1-4). **Caution statements** appear throughout the text where student errors often occur (see Sections 4-3 and 7-3). An **insight feature,** new to this tenth edition and appearing in nearly every section, makes explicit connections with previous knowledge. **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-1 and 4-2). **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review sections** include a comprehensive summary of important terms, symbols, and concepts, followed by a comprehensive set of review exercises. Answers to most review exercises, keyed to appropriate sections, are included in the back of the book. Answers to all other odd-numbered problems are also in the back of the book. Answers to application problems in linear programming include both the mathematical model and the numeric answer.

Content

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and uses. We encourage students to investigate mathematical ideas and processes graphically and numerically, as well as algebraically. This development lays a firm foundation for studying mathematics both in this book and in future endeavors. Depending on the syllabus for the course and the background of the students, some or all of this material can be covered at the beginning of a course, or selected portions can be referred to as needed later in the course.

The material in Part Two (Finite Mathematics) can be thought of as four units: mathematics of finance (Chapter 3); linear algebra, including matrices, linear systems, and linear programming (Chapters 4 and 5); probability and statistics (Chapters 6, 7, and 8); and applications of linear algebra and probability to game theory and Markov chains (Chapters 9 and 10). The first three units are independent of each other, while the last two chapters are dependent on some of the earlier chapters (see the Chapter Dependency Chart preceding this Preface).

Chapter 3 presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix B contains a section on arithmetic and geometric sequences that can be covered in conjunction with this chapter, if desired.

Chapter 4 covers linear systems and matrices with an emphasis on using row operations and Gauss–Jordan elimination to solve systems and to find matrix inverses. This chapter also contains numerous applications of mathematical modeling utilizing systems and matrices. To assist students in formulating solutions, all the answers in the back of the book to application problems in Exercises 4-3, 4-5, and the chapter Review Exercise contain both the mathematical model and its solution. The row operations discussed in Sections 4-2 and 4-3 are required for the simplex method in Chapter 5. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 10.

Chapter 5 provides broad and flexible coverage of linear programming. The first two sections cover two-variable graphing techniques. Instructors who wish to emphasize techniques can cover the basic simplex method in Sections 5-3

and 5-4 and then discuss any or all of the following: the dual method (Section 5-5), the big M method (Section 5-6), or the two-phase simplex method (Group Activity 1). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 5-4, 5-5, and 5-6, and either omit the solution or use software to find the solution (see the description of the software that accompanies this text in Student and Instructor Resources). To facilitate this approach, all the answers in the back of the book to application problems in Exercises 5-4, 5-5, 5-6, and the chapter Review Exercise contain both the mathematical model and its solution. Geometric, simplex, and dual solution methods are required for portions of Chapter 9.

Chapter 6 provides a foundation for probability with a treatment of logic, sets, and counting techniques. Chapter 7 covers basic probability, including Bayes' formula and random variables. Chapter 8 deals with basic descriptive statistics and more advanced probability distributions, including the important normal distribution. Appendix B contains a short discussion of the binomial theorem that can be used in conjunction with the development of the binomial distribution in Section 8-4.

Each of the last two chapters ties together concepts developed in earlier chapters and applies them to two interesting topics: game theory (Chapter 9) and Markov chains (Chapter 10). Either chapter provides an excellent unifying conclusion to a finite mathematics course.

Appendix A contains a self-test and a concise review of basic algebra that also may be covered as part of the course or referred to as needed. As mentioned above, Appendix B contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Error Check

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or by e-mail, to: byleen@execpc.com

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R. A. Barnett
M. R. Ziegler
K. E. Byleen

STUDENT AND INSTRUCTOR RESOURCES

Student Resources

Student Study-Pack

Everything a student needs to succeed in one place. Free packaged with the book, or available for purchase stand-alone. Study-Pack contains:

- *Student Solutions Manual*
  Fully worked solutions to odd-numbered exercises. This also contains a CD that contains:
  - *Explorations in Finite Mathematics*
    Software that contains over twenty routines that provide additional insight into the topics in the text.
  - *Decision Making for Finite Mathematics*
    Provides a tutorial on how to use the modern spreadsheet as a tool for solving problems and making decisions (this is contained within the *Student Solutions Manual*).
  - *Math Anxiety Videos*
- *Pearson Tutor Center*
  Tutors provide one-on-one tutoring for any problem with an answer at the back of the book. Students access the Tutor Center via toll-free phone, fax, or e-mail.

Instructor Resources

Content Distribution Center

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  Fully worked solutions to all textbook exercises.
- *TestGen*
  Test-generating software—create tests from textbook section objectives.
- *Test Item File*
  A printed test bank derived from TestGen.
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MathXL® is a powerful online homework, tutorial, and assessment system that accompanies your textbook. Instructors can create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook. Student work is tracked in an online gradebook. Students can take chapter tests and receive personalized study plans based on their results. The study plan diagnoses weaknesses and links students to tutorial exercises for objectives they need to study. Students can also access video clips from selected exercises. MathXL is available to qualified adopters. For more information, visit our Website at www.mathxl.com, or contact your Prentice Hall sales representative for a demonstration.

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- *Additional Resources for Instructors and Students* Access a multimedia textbook, online lecture videos, solutions manuals, online graphing calculator manuals, a video on overcoming math anxiety, PowerPoint lecture slides, and more.

CONTENTS

Preface xi

Student and Instructor Resources xvi

Part One **A Library of Elementary Functions** 1

1 *A Beginning Library of Elementary Functions* 2

1-1 *Functions* 3
1-2 *Elementary Functions: Graphs and Transformations* 21
1-3 *Linear Functions and Straight Lines* 35
1-4 *Quadratic Functions* 53
Important Terms, Symbols, and Concepts 68
Review Exercise 70
Group Activity 1: Introduction to Regression Analysis 75
Group Activity 2: Mathematical Modeling in Business 76

2 *Additional Elementary Functions* 78

2-1 *Polynomial and Rational Functions* 79
2-2 *Exponential Functions* 95
2-3 *Logarithmic Functions* 109
Important Terms, Symbols, and Concepts 122
Review Exercise 123
Group Activity 1: Comparing the Growth of Exponential and Polynomial Functions, and Logarithmic and Root Functions 126
Group Activity 2: Comparing Regression Models 126

Part Two **Finite Mathematics** 129

3 *Mathematics of Finance* 130

3-1 *Simple Interest* 131
3-2 *Compound Interest* 139
3-3 *Future Value of an Annuity; Sinking Funds* 152
3-4 *Present Value of an Annuity; Amortization* 161
Important Terms, Symbols, and Concepts 174
Review Exercise 175
Group Activity 1: Reducing Interest Payments on a Home Mortgage 178
Group Activity 2: Yield to Maturity and Internal Rate of Return 178

4 *Systems of Linear Equations; Matrices 180*
4-1 Review: Systems of Linear Equations in Two Variables 182
4-2 Systems of Linear Equations and Augmented Matrices 196
4-3 Gauss–Jordan Elimination 205
4-4 Matrices: Basic Operations 220
4-5 Inverse of a Square Matrix 234
4-6 Matrix Equations and Systems of Linear Equations 245
4-7 Leontief Input–Output Analysis 255
Important Terms, Symbols, and Concepts 264
Review Exercise 267
Group Activity 1: Using Matrices to Find Cost, Revenue, and Profit 271
Group Activity 2: Direct and Indirect Operating Costs 272

5 *Linear Inequalities and Linear Programming 274*
5-1 Systems of Linear Inequalities in Two Variables 275
5-2 Linear Programming in Two Dimensions: Geometric Approach 289
5-3 Geometric Introduction to the Simplex Method 303
5-4 Simplex Method: Maximization with Problem Constraints of the Form ≤ 311
5-5 Dual Problem: Minimization with Problem Constraints of the Form ≥ 330
5-6 Maximization and Minimization with Mixed Problem Constraints 346
Important Terms, Symbols, and Concepts 364
Review Exercise 368
Group Activity 1: Two-Phase Method: Alternative to the Big M Method 371
Group Activity 2: Production Scheduling 373

6 *Logic, Sets, and Counting 374*
6-1 Logic 375
6-2 Sets 385
6-3 Basic Counting Principles 391
6-4 Permutations and Combinations 401
Important Terms, Symbols, and Concepts 414
Review Exercise 415
Group Activity 1: Combinations with Repetition Allowed 417
Group Activity 2: Subset Lattices of Finite Sets 418

7 *Probability 420*
7-1 Sample Spaces, Events, and Probability 422
7-2 Union, Intersection, and Complement of Events; Odds 437
7-3 Conditional Probability, Intersection, and Independence 451
7-4 Bayes' Formula 467
7-5 Random Variable, Probability Distribution, and Expected Value 474
Important Terms, Symbols, and Concepts 484
Review Exercise 485
Group Activity 1: Car and Crying Towels 489
Group Activity 2: Simulation: Draft Lottery for Professional Sports 490

8 Data Description and Probability Distributions 492
8-1 Graphing Data 493
8-2 Measures of Central Tendency 507
8-3 Measures of Dispersion 517
8-4 Bernoulli Trials and Binomial Distributions 525
8-5 Normal Distributions 536
Important Terms, Symbols, and Concepts 548
Review Exercise 550
Group Activity 1: Analysis of Data on Student Lifestyle 552
Group Activity 2: Survival Rates for a Heart Transplant 552

9 Games and Decisions 554
9-1 Strictly Determined Games 556
9-2 Mixed Strategy Games 563
9-3 Linear Programming and 2×2 Games: Geometric Approach 574
9-4 Linear Programming and $m \times n$ Games: Simplex Method and the Dual Problem 580
Important Terms, Symbols, and Concepts 586
Review Exercise 587
Group Activity 1: Baseball Strategy 589
Group Activity 2: Simulation of Matrix Games 590

10 Markov Chains 592
10-1 Properties of Markov Chains 593
10-2 Regular Markov Chains 606
10-3 Absorbing Markov Chains 617
Important Terms, Symbols, and Concepts 633
Review Exercise 634
Group Activity 1: Social Mobility 637
Group Activity 2: Gambler's Ruin 638

A Basic Algebra Review 639
Self-Test on Basic Algebra 639
A-1 Algebra and Real Numbers 641
A-2 Operations on Polynomials 647
A-3 Factoring Polynomials 655
A-4 Operations on Rational Expressions 662
A-5 Integer Exponents and Scientific Notation 668
A-6 Rational Exponents and Radicals 674
A-7 Linear Equations and Inequalities in One Variable 681
A-8 Quadratic Equations 691

B *Special Topics* 703
B-1 Sequences, Series, and Summation Notation 703
B-2 Arithmetic and Geometric Sequences 710
B-3 Binomial Theorem 718

C *Tables* 723
Table I Area Under the Standard Normal Curve 723
Table II Basic Geometric Formulas 724

Answers A-1
Index I-1

Part One

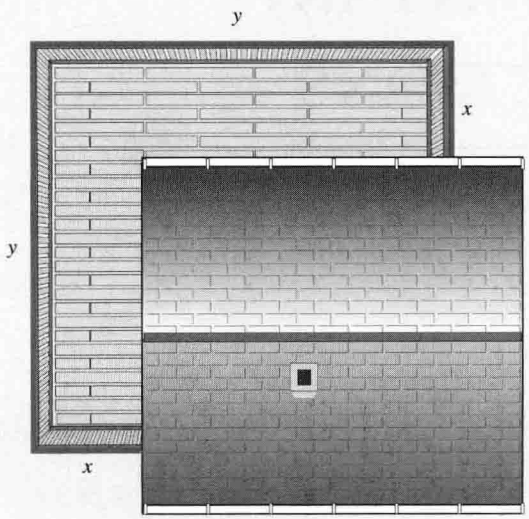
A Library of Elementary Functions

OBJECTIVES

1. Sketch graphs and give domains and ranges of the identity, absolute value, square, cube, square root, and cube root functions.
2. Find formulas for functions whose graphs are translations, expansions, contractions, or reflections of the graph of a given function.
3. Write the equation of the line through two given points in slope-intercept and point-slope form.
4. Find the vertex and axis of symmetry of the graph of a given quadratic function.
5. Calculate break-even points using linear and quadratic functions to model cost, revenue, and profit.

CHAPTER PROBLEM

The owner of a house perched on the top of an ocean side cliff wants to add a deck to two sides of the house, but only if the railing for the deck is made of a rare Australian hardwood. The contractor who is building the deck can only locate enough rare hardwood to provide 60 feet of railing for the deck. Let x and y represent the width and length of the deck (see the figure).



(A) Find the dimensions x and y if the deck has an area of 225 square feet.

(B) Find the dimensions x and y that will produce a deck with the maximum area.