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Edited by A. Dold and B. Eckmann

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F O R E W O R D

The Institute of Mathematics of Wrocław Technical University organized the Second International Conference on Probability Theory on Vector Spaces in Błażejewko from September 17 to September 23, 1979. The first Conference had been organized by the Institute in 1977. At the present Conference there were 74 registered participants from 10 countries, 44 among them from Poland. This Conference was sponsored by the Wrocław Technical University and was organized by the following committee: S.Gładysz, J.Górniak (Secretary), C.Ryll-Nardzewski and A.Weron (Chairman). Mrs. T.Cieślik and Mrs. O.Olak acted as Organizing Secretaries for the Conference.

It was the purpose of this meeting to bring together mathematicians working in Probability Theory on Vector Spaces to discuss the functional analysis aspects of this field. The following (non-disjoint) topics were covered:

Gaussian Processes and Stable Measures	–	Geometry of Banach Spaces, Special Class of Operators.
Limit Theorems (CLT, LIL, IP)	–	Topological Spaces, Geometry of Vector Spaces; C(S), D(0,1)
Random Fields, Stationary and Vector Valued Processes	–	Hilbert Space Methods, Dilation Theory and Reproducing Kernels.
Brownian Motion, Integrability of Random Vectors and Cylindrical Processes	–	Infinite-Dimensional Calculus and Differential Equations, Semigroups of Measures.

This volume containes 30 contributions - the written and often extended versions of most lectures gõven at the Conference. A great majority of papers present new results in the field and the rest are expository in nature. The material in this volume complements the material in the earlier volume Probability Theory on Vector Spaces, Proceedings Lecture Notes in Math. vol. 656, 1978, Springer Verlag.

While I take the responsibility for any mistakes in the organization of the Conference and the editing of the Proceedings, I wish to express my gratitude to several persons for their valuable help. On behalf of the Organizing Committee I wish to thank the authorities of the Wrocław Technical University for providing facilities which made it possible to hold the Conference in Błażejewko. I am indepted to Professors S.Gładysz and C.Ryll-Nardzewski for their help in the organization of the program. I wish to thank to Professors: A.Badrikian, S.D.Chatterji, Z.Ciesielski, X.Fernique, S.Gładysz, P.Masani, V.Mandrekar, V.Paulauskas, C.Ryll-Nardzewski and F.H.Szafraniec for presiding over the sessions. I am grateful to my colleagues from the Institute of Mathematics, in particular to Drs. J.Górniak and P.Kajetanowicz, for their help in various administrative matters. Special thanks are due to the contributors to this volume, to those who reviewed the papers and to Springer-Verlag for their excellent cooperation.

Aleksander Weron

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C O N T E N T S

HILBERT SPACES OF HILBERT SPACE VALUED FUNCTIONS

by

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1. Introduction

The genesis of this paper lies in two results of Rovnyak [9] asserting the positive definiteness of dually related operator-valued kernels on the Carterian product $D_+ \times D_+$ of the open unit disk D_+ in $\mathbb{C}$[1]. These kernels $L_o(\cdot,\cdot)$, $K_o(\cdot,\cdot)$ involve complex Hilbert spaces W_1, W_2 and a holomorphic function $Y(\cdot)$ on D_+, the values $Y(z)$ of which are linear contractions on W_1 to W_2; their definitions are

$$(1.1)\qquad \begin{aligned} L_o(z,\zeta) &\underset{d}{=} \frac{I_{W_2} - Y(\zeta)\cdot Y(z)^*}{1 - \zeta\bar{z}}, \quad z,\zeta \in D_+, \\ M_o(z,\zeta) &\underset{d}{=} \frac{I_{W_1} - Y(\zeta)^*\cdot Y(z)}{1 - \bar{\zeta}z}, \quad z,\zeta \in D_+. \end{aligned}$$

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*Alexander von Humboldt Senior Visiting Scientist at the University of Erlangen, Winter, 1979-1980.

1) In this paper $\mathbb{C}$ is the complex number field, $\mathbb{R}$ the real number field, and $\mathbb{F}$ refers to anyone of these. $\mathbb{N}$ is the set of integers.

The Szegö kernel

(1.2) $$k(z,\zeta) \underset{d}{=} 1/(1 - \zeta\bar{z}), \quad z,\zeta \in D_+,$$

is of course positive definite (PD). But it is easily seen that for non-constant $Y(\cdot)$, the kernel $D(\cdot,\cdot)$ such that $D(z,\zeta) = I_{W_2} - Y(\zeta)\cdot Y(z)^*$ is not PD. Hence the positive definiteness of $L_o(\cdot,\cdot)$ is not trivially deducible from the known result that the product of PD kernels is PD. The same remark applies also to $M_o(\cdot,\cdot)$.

The proofs that $L_o(\cdot,\cdot)$ and $M_o(\cdot,\cdot)$ are PD given by Rovnyak [9], and subsequently by Nagy [6] and Nagy & Foais [7, pp. 231-233] lean heavily on the analytic implications of the kernels, and exploit the theory of holomorphic functions of the Hardy class H_2. In this paper we shall show that analyticity plays only a marginal role: it entails the fulfillment of certain general premises. Formulable in terms of these premises are more abstract and far reaching results on positive-definiteness, which subsume the original.

A convenient way to see this is to ask a general question. Let Λ be any set, let $K_1(\cdot,\cdot)$, $K_2(\cdot,\cdot)$ be PD kernels on $\Lambda\times\Lambda$ to $CL(W_1, W_1)$, $CL(W_2, W_2)$,[2] respectively, and let $Y(\cdot)$ be a function on Λ to $CL(W_1, W_2)$. Under what conditions will the kernel $L(\cdot,\cdot)$ defined $\forall\, \lambda,\lambda' \in \Lambda$ by

(1.3) $$L(\lambda,\lambda') \underset{d}{=} K_2(\lambda,\lambda') - Y(\lambda')\cdot K_1(\lambda,\lambda')\cdot Y(\lambda)^*$$

be PD on $\Lambda\times\Lambda$ to $CL(W_2, W_2)$?

Our answer involves the <u>function Hilbert spaces</u> $\mathcal{F}_1, \mathcal{F}_2$ for which the <u>reproducing kernels</u> are $K_1(\cdot,\cdot)$, $K_2(\cdot,\cdot)$, and also the multiplication operator M_Y from $\mathcal{F}_1$ to $\mathcal{F}_2$ induced by the given function $Y(\cdot)$. Briefly, we find (Thm. 3.4) that

(1.4) $L(\cdot,\cdot)$ is PD iff M_Y is a contraction.

Rovnyak's assertion that $L_o(\cdot,\cdot)$ is PD follows at once from this general result on taking

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2) $CL(\mathcal{X}\ \mathcal{Y})$ stands for the space of continuous linear operators on the Banach space $\mathcal{X}$ to the Banach space $\mathcal{Y}$.

(1.5) $\Lambda = D_+$, $K_j(z,\zeta) = k(z,\zeta)\cdot I_{W_j}$, $j = 1,2,$

where $k(\cdot,\cdot)$ is the Szegö kernel of (1.2). We have only to check the validity of the premises of our general theorem (3.4) in this special case. This verification naturally calls for some lemmas which lean (infact heavily) on the analytic side of the problem (§7).

We should point out that the spaces F_1, F_2 consist of functions on Λ taking values in the Hilbert spaces W_1, W_2, respectively, and that a preliminary question we have to consider is the extension of the Aronszajn concepts of "function Hilbert space" and "reproducing kernel" to this vectorial setting. We must also demonstrate that every operator-valued PD kernel is the reproducing kernel of some Hilbert space valued function Hilbert space (Thm. 2.7). In essence we have to adapt the theory given in Pedrick's unpublished report [8].

The abstract treatment of the dual kernel $M_o(\cdot,\cdot)$ is slightly more complex. We have to start with a set Λ endowed with an involution $*$ and let $Y^*(\lambda) \underset{d}{=} \{Y(\lambda^*)\}^*$ and $K_j^*(\lambda,\lambda') \underset{d}{=} K_j(\lambda^*,\lambda'^*)$, and to consider the kernel $M(\cdot,\cdot)$ defined $\forall \lambda,\lambda' \in \Lambda$ by

(1.6) $M(\lambda,\lambda') = K_1^*(\lambda,\lambda') - Y(\lambda')^*\cdot K_2^*(\lambda,\lambda')\cdot Y(\lambda).$

We then have, cf. Thm. 4.4, the exact analogue of (1.4), viz.

(1.7) $M(\cdot,\cdot)$ is PD iff M_{Y^*} is a contraction.

The assertion that $M_o(\cdot,\cdot)$ is PD follows at once from (1.7) on taking Λ, K_1, K_2 as in (1.5). We do not as yet know whether our recourse to an involutory Λ reflects an intrinsic aspect of the problem or merely paucity of understanding.

The idea to consider the multiplication operators M_Y and M_{Y^*} was suggested by the proof of Nagy & Foias [7, pp. 231-233], and indeed as (1.4) and (1.7) show, their role is intrinsic. It is, however, a drawback of these results that they place restraints on M_Y and M_{Y^*} rather than on $Y(\cdot)$ and $Y^*(\cdot)$ themselves. To remove this blemish it is reasonable, following Rovnyak, to impose on $Y(\cdot)$ the condition that its values $Y(\lambda)$ be contractions on W_1 to W_2. But this is not enough, and we faced with the problem of filling the