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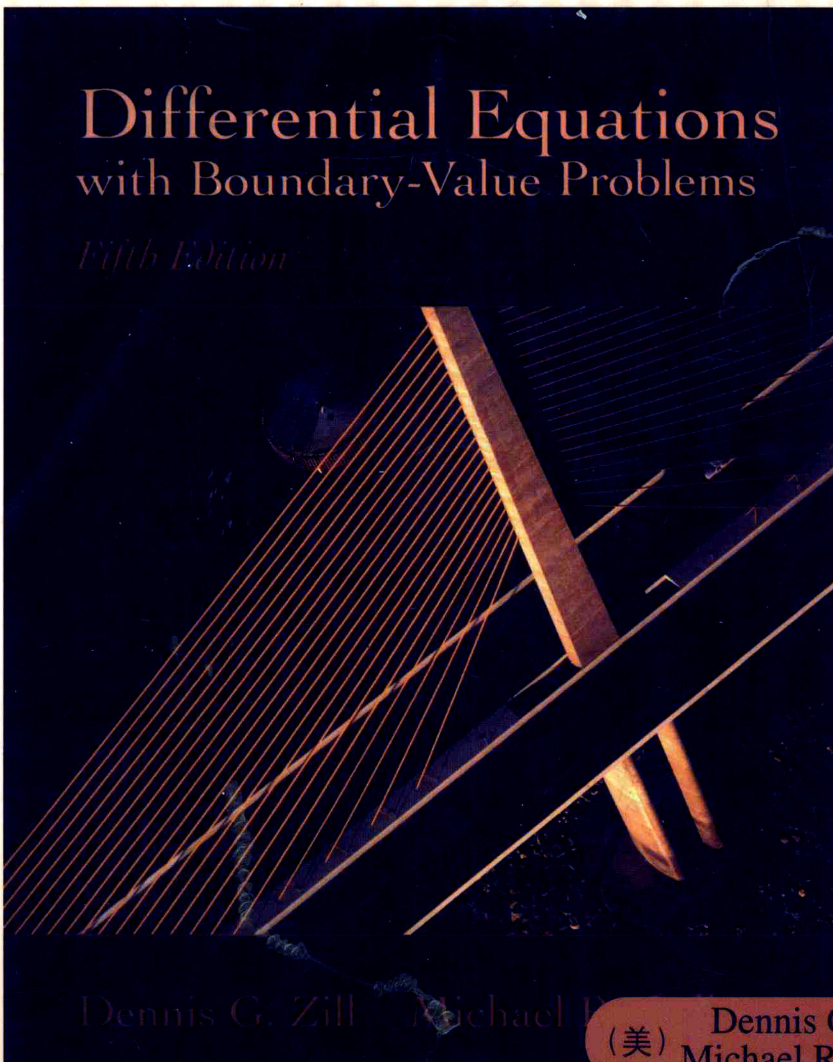
微分方程与边界值问题

(英文版·第5版)



(美) Dennis G. Zill
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（英文版·第5版）

Differential Equations with Boundary-Value Problems

(Fifth Edition)

（美） Dennis G. Zill
Michael R. Cullen 著

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PREFACE

In a revision of a textbook, the author is pulled by many influences in many directions. The author is expected to satisfy the requests from reviewers and editors to add things and to change things so that the text stays both current and competitive. At the same time the author is also expected not to increase the bulk of the book (a request to delete something is rare) and is certainly expected not to do anything that might upset the users of the previous edition. So yes, I have tried to meet, or at least strike a compromise with, all these demands. But no, I have not (in my opinion) deviated from the principle I have stated in prefaces over the many years that this text has been in print—that is, an undergraduate text should be written with the student's understanding kept firmly in mind, which means to me that the material should be presented in a straightforward, readable, and helpful manner, while the level of theory is kept consistent with the notion of a "first course."

In recent years, partially due to evolving technology and its expanding importance in pedagogy and to the reform movement in calculus, changes have occurred in the course called "Differential Equations." Instructors are questioning aspects of both the traditional teaching methods used and the traditional content of the course. This healthy introspection is important in making the subject matter not only more interesting for students but also more relevant to the world in which they live.

WHAT IS NEW TO THIS EDITION?

- This edition has a clearer delineation of the three major approaches to differential equations: analytical, qualitative, and numerical. Before launching into the search for analytic solutions of first-order differential equations, Chapter 2 starts with a section that is new to this book. In this section the qualitative behavior of solutions of first-order differential equations is examined using direction fields and phase-line analysis. There is no doubt in my mind that a reasonable amount of qualitative analysis should be, and will be, a permanent part of a typical first course.
- Since this is the 5th edition, an effort was made to liven up the exercise sets through the addition of new kinds of problems. Some of the problems that call for the use of a computer algebra system are new to this edition. With the realization that many colleges still lack the computational resources to incorporate problems of this sort into their curricula, I have placed the

majority of these problems at the end of the exercise sets under the heading "Computer Lab Assignments." This was done so that these kinds of problems would not "get in the way"; that is, they do not have to be "weeded out" of the more standard types of problems if an instructor wants to skip them entirely or postpone their coverage. Problems that require simpler technology such as a graphics calculator or graphing software have been marked by an icon in the margin. Finally, many conceptual and discussion problems have been added throughout most exercise sets. In some instances I have *slightly* reduced the number of routine "drill problems"—problems that usually require formal manipulation of some solution method—to accommodate these new problems. Like the Computer Lab Assignments, problems that emphasize understanding of concepts and problems that are suited to class or group discussion are, for convenience and recognition, placed near the end of the exercise sets.

- Three new *Project Modules,* all composed by Prof. Gilbert N. Lewis, appear after Chapters 3, 5, and 8. These modules explore mathematical models relating to the conservation of natural resources, the destruction of the Tacoma Narrows Bridge, and the modes of vibration of a multistory building during an earthquake. These modules are more than essays because each contains a set of questions that can be used as a computer lab assignment if desired.

WHAT HAS CHANGED IN THIS EDITION?

Although there are changes in almost all chapters, the greatest numbers of changes occur in Chapters 2, 6, and 7.

Chapter 2: *First-Order Differential Equations*

The topics in this chapter have been rearranged somewhat, and one new section has been added.

- Before the chase begins for solutions of first-order differential equations, some qualitative analysis is presented in Section 2.1, called "Solution Curves Without the Solution." How the behavior of solutions and the shape of solution curves can be discerned is discussed through the use of direction fields and phase-line analysis. Approximately half of the material in this section is new to this text; the direction field material appeared in Chapter 9 in the 4th edition. In testing the new material on autonomous first-order equations for several years in my own classes while using my alternative text *Differential Equations with Computer Lab Experiments,* I found that students actually enjoyed the brief excursion into the qualitative aspects of autonomous first-order differential equations because it did not involve complicated procedures and simply built upon the concept of interpreting a derivative—a concept already familiar to them from their study of differential calculus. This brief introduction to the notions of critical points, equilibrium solutions, and stability of classification of critical points as attractors, repellers, and semistable is carried out in a nontheoretical and nonthreatening manner.

- In this edition I have moved the discussion of linear equations (Section 2.3) before that on exact equations (Section 2.4). In Section 2.3 I have retained the "Property, Procedure, and Variation of Parameters" format, even though this approach to linear first-order differential equations did not meet with universal acclaim from the users of the 4th edition. Some texts present linear first-order operations (for example, obtaining the general form of the integrating factor from an appeal to exactness) as if linear first-order equations were somehow different from linear higher-order equations. Bear in mind that I am trying to get across early on the point that the same procedures presented in Chapter 4 for linear higher-order differential equations are applicable to linear first-order equations.
- In the discussion of exact equations in the 4th edition, the concept of an integrating factor was relegated to an exercise set. The procedure whereby an integrating factor can be determined for certain kinds of nonexact equations has now been added to Section 2.4.
- The intuitive Euler's method has been moved from Chapter 9 in the 4th edition to Section 2.6. This was done for two reasons: to strike a better balance among the analytical, qualitative, and numerical approaches to differential equations and to better illustrate the graphical aspects of computer software known generically as numerical solvers. Moreover, the presentation of Euler's method in Chapter 2 is consistent with the early consideration of this topic in most current calculus texts.

Chapter 6: *Series Solutions of Linear Equations*

- The lengthy discussion of the various cases of solving linear differential equations with variable coefficients using the method of Frobenius (Section 6.2) has been abbreviated to the essentials.

Chapter 7: *The Laplace Transform*

Chapter 7 has been completely reorganized in order to get to the point of the chapter sooner than in all previous editions—namely, that the Laplace transform is a useful tool for solving certain kinds of equations.

- The Laplace transform of derivatives, and how this result is used in solution of simple linear initial-value problems, is now introduced in Section 7.2.
- In the subsequent sections of this chapter initial-value problems of increasing difficulty, as well as the solution of other kinds of equations, are examined in conjunction with the unfolding of the various operational properties of the transform. In earlier editions all applications were considered in one, I admit, rather formidable section.

Chapter 8: ***Systems of Linear First-Order Differential Equations***

- More figures and graphs were added to Chapter 8. The graphs of solutions of plane systems are given in the *tx*-plane, *ty*-plane, and *xy*-plane. Although the words "phase plane" are introduced, no qualitative analysis of autonomous second-order equations or of autonomous plane systems is presented at this point in the text.
- A discussion of how the Laplace transform can be used to determine a matrix exponential has been added to Section 8.4.

WHAT REMAINS THE SAME?

The chapter lineup by topics and the basic underlying philosophy remain the same as in the previous edition. Like its predecessors, this edition contains a generous supply of examples, exercises, and applications. Nuances such as presenting applications of differential equations in separate chapters also continue in this revision. For every reviewer who states "incorporate applications with the discussion of solutions of differential equations" there is another reviewer who makes the plea to keep appliations separate from the methods of solution. I mostly agree with the latter viewpoint. I feel that presenting applications of first- and higher-order ordinary differential equations in separate chapters not only gives the greatest flexibility to the text but also allows the user to get his or her "feet on the ground" by focusing on fewer concepts at the beginning of the course. From the student's viewpoint the mixture of applications with methods of solution can be overwhelming, and from the instructor's viewpoint it makes the applied topics difficult to skip if they are not part of the course syllabus. But this philosophy is not carved in stone; I have now interwoven applications into the various sections of Chapter 7 on the Laplace transform. I was convinced to do this for two reasons: students have attained a certain comfort level with applied problems by the time Chapter 7 is usually covered, and, as noted above, the alternative (namely, lumping all applications into one section) is unpalatable. In conclusion, I have resisted the recommendations to present the solution of linear second-order differential equations and the solution of linear equations of order higher than two in separate sections.

SUPPLEMENTS AVAILABLE

For Students
Student Solutions Manual, by Warren S. Wright and Carol D. Wright (ISBN 0-534-38003-4), provides the solution to every third problem in each exercise set, with the exception of the Discussion Problems and the Computer Lab Assignments.

For Instructors
Complete Solutions Manual, by Warren S. Wright and Carol D. Wright (ISBN 0-534-38004-2), provides worked-out solutions to all problems in the text.

ACKNOWLEDGMENTS

A large measure of gratitude is owed to the following persons, who contributed to this revision through their help, suggestions, and criticisms:

Zaven Margosian, *Lawrence Technological University*
Brian M. O'Connor, *Tennessee Technological University*
Mohsen Razzaghi, *Mississippi State University*

A special acknowledgment is reserved for Gilbert Lewis, Michigan Technological University, who generously took time out of a busy schedule to craft the three new *Project Modules* in this edition. I also want to thank Barbara Lovenvirth for coordinating the work on these modules. Finally, I would like to express my sincere appreciation to my multitalented colleague Michael Berg for rendering the cartoon on page 31.

The task of compiling a text such as this one is time-consuming and difficult. Undoubtedly some errors have sifted through, as hundreds of manuscript pages have passed through many hands. I apologize for this in advance. To expedite the correction of an error please send it directly to my editor, Gary Ostedt, at Brooks/Cole.

Dennis G. Zill
Los Angeles

This edition is dedicated to my coauthor, colleague, and friend of 27 years, as well as an award-winning teacher, Michael R. Cullen, who passed away during its production.

Michael Cullen
1943–1999

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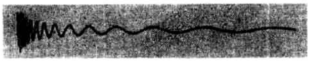

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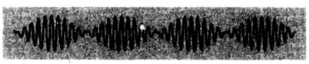

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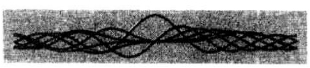

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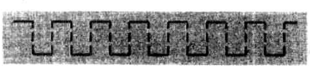

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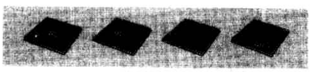

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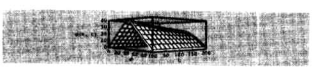

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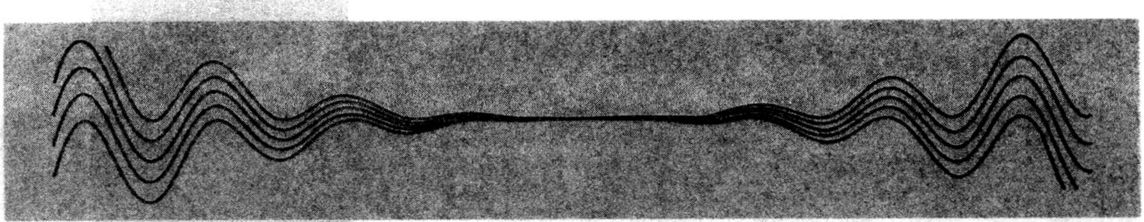

A family of solutions of a DE; see page 8.

1 INTRODUCTION TO DIFFERENTIAL EQUATIONS

1.1 Definitions and Terminology
1.2 Initial-Value Problems
1.3 Differential Equations as Mathematical Models

Chapter 1 in Review

INTRODUCTION

The words *differential* and *equations* certainly suggest solving some kind of equation that contains derivatives. Analogous to a course in algebra and trigonometry, where a good amount of time is spent solving equations such as $x^2 + 5x + 4 = 0$ for the unknown variable x, in this course one of our tasks will be to solve differential equations such as $y'' + 2y' + y = 0$ for the unknown function $y = \phi(x)$.

The first paragraph tells something, but not the complete story, about the course you are about to begin. As the course unfolds, you will see that there is more to the study of differential equations than just mastering methods someone has devised to solve them. But first, in order to read, study, and be conversant in a specialized subject, one first has to learn the terminology, the jargon, of that discipline.

1.1 DEFINITIONS AND TERMINOLOGY

• Ordinary and partial differential equations • Order of a DE • Linear and nonlinear DEs • Normal form • Solution of an ODE • Explicit and implicit solutions • Trivial solution • Family of solutions • Singular solution

In calculus you learned that the derivative dy/dx of a function $y = \phi(x)$ is itself another function $\phi'(x)$ found by an appropriate rule. The function $y = e^{0.1x^2}$ is differentiable on the interval $(-\infty, \infty)$, and its derivative is $dy/dx = 0.2xe^{0.1x^2}$. If we replace $e^{0.1x^2}$ on the right-hand side of the derivative by the symbol y, we obtain

$$\frac{dy}{dx} = 0.2xy. \tag{1}$$

Now imagine that a friend of yours simply hands you equation (1)—you have no idea how it was constructed—and asks: What is the function represented by the symbol y? You are now face to face with one of the basic problems in this course: How do you solve such an equation for the unknown function $y = \phi(x)$? The problem is loosely equivalent to the familiar reverse problem of differential calculus: Given a derivative, find an antiderivative.

Differential Equation The equation that we made up in (1) is called a **differential equation.** Before proceeding any further, let us consider a more precise definition of this concept.

DEFINITION 1.1 Differential Equation

An equation containing the derivatives of one or more dependent variables with respect to one or more independent variables is said to be a **differential equation (DE).**

In order to talk about them, we shall classify differential equations by **type, order,** and **linearity.**

Classification by Type If an equation contains only ordinary derivatives of one or more dependent variables with respect to a single independent variable, it is said to be an **ordinary differential equation (ODE).** For example,

$$\frac{dy}{dx} + 5y = e^x, \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} + 6y = 0, \quad \text{and} \quad \frac{dx}{dt} + \frac{dy}{dt} = 2x + y \tag{2}$$

are ordinary differential equations. An equation involving the partial derivatives of one or more dependent variables of two or more independent variables is called a **partial differential equation (PDE).** For example,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - 2\frac{\partial u}{\partial t}, \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \tag{3}$$

are partial differential equations.

Ordinary derivatives throughout this text will be written using either the **Leibniz notation** $dy/dx, d^2y/dx^2, d^3y/dx^3, \ldots$ or the **prime notation** $y', y'', y''', \ldots$. Using the latter notation, we can write the first two differential equations in (2) a little more compactly as $y' + 5y = e^x$ and $y'' - y' + 6y = 0$. Actually the prime notation is used to denote only the first three derivatives; the fourth derivative is written $y^{(4)}$ instead of y''''. In general, the nth derivative is written d^ny/dx^n or $y^{(n)}$. Although less convenient to write and to typeset, the Leibniz notation has an advantage over the prime notation in that it clearly displays both dependent and independent variables. For example, in the equation $d^2x/dt^2 + 16x = 0$ it is immediately seen that the symbol x now represents a dependent variable whereas the independent variable is t. You should also be aware that in physical sciences and engineering **Newton's dot notation** (derogatively referred to by some as the "flyspeck" notation) is sometimes used to denote derivatives with respect to time t. Thus the differential equation $d^2s/dt^2 = -32$ becomes $\ddot{s} = -32$. Partial derivatives are often denoted by a **subscript notation** indicating the independent variables. For example, with the subscript notation the second equation in (3) becomes $u_{xx} = u_{tt} - 2u_t$.

Classification by Order The **order of a differential equation** (either ODE or PDE) is the order of the highest derivative in the equation. For example,

$$\overset{\text{second-order}\ \downarrow}{\frac{d^2y}{dx^2}} + 5\left(\overset{\downarrow\ \text{first-order}}{\frac{dy}{dx}}\right)^3 - 4y = e^x$$

is a second-order ordinary differential equation. First-order ordinary differential equations are occasionally written in differential form $M(x, y)\,dx + N(x, y)\,dy = 0$. For example, if we assume that y denotes the dependent variable in $(y - x)\,dx + 4x\,dy = 0$, then $y' = dy/dx$, and so by dividing by the differential dx we get the alternative form $4xy' + y = x$. See the Remarks at the end of this section.

In symbols, we can express an nth-order ordinary differential equation in one dependent variable by the general form

$$F(x, y, y', \ldots, y^{(n)}) = 0, \tag{4}$$

where F is a real-valued function of $n + 2$ variables, $x, y, y', \ldots, y^{(n)}$, and where $y^{(n)} = d^ny/dx^n$. For both practical and theoretical reasons, we shall also make the assumption hereafter that it is possible to solve an ordinary differential equation in the form (4) uniquely for the highest derivative $y^{(n)}$ in terms of the remaining $n + 1$ variables. The differential equation

$$\frac{d^ny}{dx^n} = f(x, y, y', \ldots, y^{(n-1)}), \tag{5}$$