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Boundary Layer Theory

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Foreword

To the First English Edition

Boundary layer theory is the cornerstone of our knowledge of the flow of air and other fluids of small viscosity under circumstances of interest in many engineering applications. Thus many complex problems in aerodynamics have been clarified by a study of the flow within the boundary layer and its effect on the general flow around the body. Such problems include the variations of minimum drag and maximum lift of airplane wings with Reynolds number, wind tunnel turbulence, and other parameters. Even in those cases where complete mathematical analysis is at present impracticable, the boundary layer concept has been extraordinarily fruitful and useful.

The development of boundary layer theory during its first fifty years is a fascinating illustration of the birth of a new concept, its slow growth for many years in the hands of its creator and his associates, its belated acceptance by others, and the later almost exponential rise in the number of contributors to its further development.

The first decade following the classical paper of Prandtl in 1904 brought forth fewer than 10 papers by Prandtl and his students, a rate of about one paper per year. During the past year over 100 papers were published on various aspects of boundary layer theory and related experiments. The name of H. Schlichting first appears in 1930 with his doctoral thesis on the subject of wake flow. Shortly thereafter Schlichting devoted major effort to the problem of the stability of laminar boundary layer flow.

My own interest in the experimental aspects of boundary layer flow began in the late twenties. With the appearance of Schlichting's papers intensive attempts were made to find the amplified disturbances predicted by the theory. For 10 years the experimental results not only failed to confirm this theory but supported the idea that transition resulted from the presence of turbulence in the free air stream as described in a theory set forth by G. I. Taylor. Then on a well-remembered day in August, 1940, the predicted waves were seen in the flow near a flat plate in a wind tunnel of very low turbulence. The theory of stability described in the papers of Tollmien and Schlichting was soon confirmed quantitatively as well as qualitatively.

German periodicals available in the United States after the war referred to a series of lectures by Schlichting on boundary layer theory which had been published in 1942. This document of 279 pages with 116 figures was not available for some time. An English translation was given limited distribution as NACA Technical Memorandum No. 1217 in 1949. These lectures were completely rewritten to include material previously classified, confidential, or secret from Germany and other countries.

The result was the book of 483 pages and 295 figures published in 1951 in the German language. When this book became known to research workers and educators in the United States, there was an immediate request from several quarters for an English translation, since no comparable book was available in the English language.

The technical content of the present English edition is described in the author's preface. The emphasis is on the fundamental physical ideas rather than on mathematical refinement. Methods of theoretical analysis are set forth along with such experimental data as are pertinent to define the regions of applicability of the theoretical results or to give physical insight into the phenomena.

Aeronautical engineers and research scientists owe a debt of gratitude to Professor Schlichting for this timely review of the present state of boundary layer theory.

Washington D. C., December 13, 1954 — Hugh L. Dryden

Author's Preface to the Second English Edition

The first English edition of this book appeared in 1955. It was not a literal translation of the previous German edition. Taking into account the progress achieved in the meantime, I had completely rewritten two chapters and had made many smaller additions in various places throughout the book. The first English edition was very well received, particularly in the English-speaking countries, and there is little doubt that Dr. J. Kestin's success with the translation contributed to it in no small measure.

The first German edition of "Boundary Layer Theory" was reprinted without changes in 1954, but the new edition went out of print in 1957. Consequently, I have decided to publish a thoroughly revised new edition in the German language. This appeared in 1958 as the third edition of the book. Owing to the great volume of new research results obtained in the intervening period of time, the third edition had to be enlarged by some 120 pages and 80 figures. In particular, the chapters on compressible boundary layers (Chap. XV), on thermal boundary layers (Chap. XIV) and transition (Chaps. XVI and XVII) were considerably increased in size.

Since, in addition, the first English edition went out of print in 1958, I have gladly accepted the suggestion advanced by the two Publishers, Messrs. McGraw-Hill in New York and Messrs. G. Braun in Karlsruhe and prepared a new English edition which more nearly reflects the present status in the development of this branch of science.

I owe a great debt of gratitude to Professor J. Kestin who once more undertook the arduous task of preparing the translation. The present second English edition is based, substantially, on the contents of the third German edition of the book which appeared in 1958. While engaged on the work of translation, Dr. J. Kestin made many valuable suggestions for additions, particularly in Chapters XI and XIV, which I accepted gladly. One of my co-workers, Dr. K. Gersten, rewrote several sections on heat transfer in Chapters XIV and XIX, and I wish to record my thanks to him.

I hope that this new English edition will again be found useful by engineers working in industry and in research establishments and by students of mechanical engineering and machine design as well as by physicists and chemists who have to deal with problems of fluid flow.

H. Schlichting

Braunschweig, May 1960

Translator's Preface to the Second English Edition

It is once more my pleasure and privilege to record the immense benefits which I derived from my association with Professor H. Schlichting. A very close touch was maintained with the Author during the preparation of this new edition and, moreover, I was given the opportunity of spending two periods of time with Professor

H. Schlichting at his Institutes in Goettingen and Braunschweig during which I learned many things from him. I wish to thank Messrs. McGraw-Hill and the authorities of AGARD for making these memorable visits possible.

I owe a particular debt of gratitude to Dr. P. D. Richardson, Assistant Professor at Brown University and Mr. W. Pechau, Assistant at the Institute for Fluid Mechanics in Braunschweig who kindly consented to read the proofs and who made many valuable suggestions. Both publishers, Messrs. G. Braun of Karlsruhe and Messrs. McGraw-Hill of New York, spared no trouble in meeting our wishes regarding the production of the book.

Braunschweig, May 1960 J. Kestin

From Author's Preface to First German Edition

Since about the beginning of the current century modern research in the field of fluid dynamics has achieved great successes and has been able to provide a theoretical clarification of observed phenomena which the science of classical hydrodynamics of the preceding century failed to do. Essentially three branches of fluid dynamics have become particularly well developed during the last fifty years; they include boundary layer theory, gas dynamics, and aerofoil theory. The present book is concerned with the branch known as boundary layer theory. This is the oldest branch of modern fluid dynamics; it was founded by L. Prandtl in 1904 when he succeeded in showing how flows involving fluids of very small viscosity; in particular, water and air, the most important ones from the point of view of applications, can be made amenable to mathematical analysis. This was achieved by taking the effects of friction into account only in regions where they are essential, namely in the thin boundary layer which exists in the immediate neighbourhood of a solid body. This concept made it possible to clarify many phenomena which occur in flows and which had previously been incomprehensible. Most important of all, it has become possible to subject problems connected with the occurrence of drag to a theoretical analysis. The science of aeronautical engineering was making rapid progress and was soon able to utilize these theoretical results in practical applications. It did, on the other hand, pose many problems which could be solved with the aid of the new boundary layer theory. Aeronautical engineers have long since made the concept of a boundary layer one of everyday use and it is now unthinkable to do without it. In other fields of machine design in which problems of flow occur, in particular in the design of turbomachinery, the theory of boundary layers made much slower progress, but in modern times these new concepts have come to the fore in such applications as well.

The present book has been written principally for engineers. It is the outcome of a course of lectures which the Author delivered in the Winter Semester of 1941—42 for the scientific workers of the Aeronautical Research Institute in Brunswick. The subject matter has been utilized after the war in many special lectures held at the Engineering University in Brunswick for students of mechanical engineering and physics. Dr. H. Hahnemann prepared a set of lecture notes after the first series of lectures had been given. These were read and amplified by the Author. They were subsequently published in mimeographed form by the Office for Scientific Documentation (Zentrale für wissenschaftliches Berichtswesen) and distributed to a limited circle of interested scientific workers.

Several years after the war the Author decided completely to re-edit this older compilation and to publish it in the form of a book. The time seemed particularly propitious because it appeared ripe for the publication of a comprehensive book, and because the results of the research work carried out during the last ten to twenty years rounded off the whole field.

The book is divided into four main parts. The first part contains two introductory chapters in which the fundamentals of boundary layer theory are expounded without the use of mathematics and then proceeds to prepare the mathematical and physical justification for the theory of laminar boundary layers and includes the theory of thermal boundary layers. The third part is concerned with the phenomenon of transition from laminar to turbulent flow (origin of turbulence), and the fourth part is devoted to turbulent flows. It is now possible to take the view that the theory of laminar boundary layers is complete in its main outline. The physical relations have been completely clarified; the methods of calculation have been largely worked out and have, in many cases, been simplified to such an extent that they should present no difficulties to engineers. In discussing turbulent flows use has been made essentially only of the semi-empirical theories which derive from Prandtl's mixing length. It is true that according to present views these theories possess a number of shortcomings but nothing superior has so far been devised to take their place, nothing, which is useful to the engineer. No account of the statistical theories of turbulence has been included because they have not yet attained any practical significance for engineers.

As intimated in the title the emphasis has been laid on the theoretical treatment of problems. An attempt has been made to bring these considerations into a form which can be easily grasped by engineers. Only a small number of results has been quoted from among the very voluminous experimental material. They have been chosen for their suitability to give a clear, physical insight into the phenomena and to provide direct verification of the theory presented. Some examples have been chosen, namely those associated with turbulent flow, because they constitute the foundation of the semi-empirical theory. An attempt was made to demonstrate that essential progress is not made through an accumulation of extensive experimental results but rather through a small number of fundamental experiments backed by theoretical considerations.

Braunschweig, October 1950 H. Schlichting

From Author's Preface to Third (German) Edition

The first edition of this book which appeared in 1951 met with an unexpectedly good reception and the demand for it abroad, particularly in the U.S.A., was very great. Having received many requests from my professional colleagues, I have resolved to bring out an English translation of my book. This was prepared by Dr. J. Kestin of Brown University and appeared in 1955. The English translation contained numerous additions and modifications at that time, but I found it impossible to revise the whole manuscript thoroughly and in detail in order to bring it completely up to date.

In the present edition I have made an attempt to include all the significant papers which appeared in large numbers during the intervening years. I trust, however, that the revision left the principal aims of the book unaffected and that it still presents the theoretical considerations in a form which is accessible to engineers and that the voluminous experimental material is used principally as a substantiation of and illustration to the analysis. I have retained the original subdivision into Parts and Chapters and endeavoured to encompass all the revisions in the same framework, that is, without introducing any new chapter headings.

It might be useful to enumerate some of the additions: Chapters IX and XIII contain large new sections on exact solutions and the effect of suction; Chapter XV was considerably expanded and in the chapters on transition, particularly in Chapter XVII, I have included numerous examples of the application of the stability theory. The new results on the effect of heat transfer and compressibility on transition have been included. Chapter XXII on turbulent boundary layers has been completely rewritten and is now based on Truckenbrodt's method which leads to convenient quadratures and which has now become widely accepted. In discussing the effect of compressibility on turbulent boundary layers I have confined myself to a few short remarks about flat plates in Chapter XXI. It seems to me that these considerations are still in a fluid state and no definitive treatment can be given. I believe, just as in the previous edition, that I am still justified in not describing the statistical theory of turbulence. This, admittedly, has contributed materially to our better understanding of turbulent flows, but it has not yet acquired any importance for engineers.

The references to individual papers given at the end of each chapter as well as the bibliography at the end of the book have been expanded considerably. The book acquires about 120 new pages and 80 new figures. I trust that in spite of its increased volume, the book still provides a *comprehensive outline* of an important branch of fluid mechanics.

In preparing the manuscript I enjoyed the helpful assistance of several of my professional colleagues, and I wish to put on record my gratitude to them. In particular, I wish to thank Dr. E. Truckenbrodt who helped with the rewriting of Chapter XXII and to Dr. H. Görtler who advised me on the revision of Chapter IX. I also wish to thank my correspondents, too numerous to mention individually by name, who drew my attention to inexactitudes of expression and printing errors. I have made an effort to remove them all.

I would have found it impossible to digest the flood of recent papers without the able assistance which I received from my co-workers, Messrs. K. Gersten, E. Adams and E. G. Feindt. In this connection Mr. N. Scholtz's assistance with the original edition must not be forgotten.

Thanks are due to Messrs. Gersten, Pechau, Riegels and Rotta for their assistance with the reading of the proofs, to my secretary, Miss Behrens, and to my daughter Heike for the careful preparation of the final typescript. Messrs. Braun are to be thanked for their cooperation and for the excellent production of the book.

Braunschweig, March 1958 H. Schlichting

List of most commonly used symbols

In order not to depart too drastically from the conventions normally employed in papers on the subject it was found necessary to use the same symbol to denote several different quantities. Thus, for example, λ denotes the resistance coefficient of pipe flow, both laminar and turbulent, and in the theory of stability of laminar boundary layers it denotes the wavelength of a disturbance. Similarly, k denotes thermal conductivity in the theory of thermal boundary layers, and the height of a protuberance in the discussion of the influence of roughness o turbulent flow.

The following is a list of symbols most commonly used in the book.

I. General symbols

A = wetted area, or frontal area
d, D = diameter
g = acceleration due to gravity
h = channel width
l, L = length
p = pressure
$q = \frac{1}{2}\varrho V^2$ = dynamic head
r, ϕ, z = cylindrical co-ordinates
R = radius
u = mean velocity (in pipe)
$U(x)$ = velocity in potential flow
u, v, w = velocity components
V = free stream velocity
ϱ = density (mass per unit volume)
ω = angular velocity

II. Viscous flow, turbulence

A_τ = eddy viscosity
b = width of jet or wake
c_D = drag coefficient
c_f = skin friction coefficient
c_f' = local skin friction coefficient
L, H = shape factors of velocity profile
k = height of roughness element (protuberance)
k_S = height of grain for equivalent sand roughness
l = mixing length
R = Reynolds number
T = turbulence intensity (also degree or level of turbulence)
u', v', w' = components of turbulent velocity
$\overline{u'^2}, \overline{v'^2}, \overline{u'v'} \ldots$ = temporal means of turbulent velocities
U = maximum velocity at pipe centre

U_∞ = free stream velocity
$v_* = \sqrt{\tau_0/\varrho}$ = friction velocity
y = distance from wall
δ = boundary layer thickness
δ^* = displacement thickness
ε = apparent (virtual) kinematic viscosity in turbulent flow ("eddy viscosity")
$\eta = yv_*/\nu$ = dimensionless distance from wall
θ = momentum thickness
$\varkappa$ = empirical constant in turbulent flow; $l = \varkappa y$
λ = resistance coefficient of pipe flow
Λ = shape factor of laminar boundary layer velocity profile
μ = absolute viscosity
$\nu = \mu/\varrho$ = kinematic viscosity
τ = shearing stress (force per unit area)
τ_0 = shearing stress at a wall
$\phi = u/v_*$ = dimensionless velocity
ψ = stream function

III. Transition from laminar to turbulent flow

$c = \beta/\alpha = c_r + ic_i$
c_i = amplification (or damping) factor
c_r = wave propagation velocity of disturbance
$\mathsf{R}_{crit} = (U_m\delta^*/\nu)_{crit}$ = critical Reynolds number
u', v' = velocity components of disturbance
$U(y)$ = velocity profile in boundary layer
U_m = velocity in potential flow
$\beta = \beta_r + i\beta_i$
β_i = amplification (or damping) factor
β_r = circular frequency of disturbance
γ = intermittency factor
$\lambda = 2\pi/\alpha$ = wavelength of disturbance
$\phi(y)$ = amplitude of stream function of disturbance

IV. Thermal boundary layers and gasdynamics

$a = k/\varrho g c_p$ = thermal diffusivity
$c = \sqrt{\gamma p/\varrho}$ = velocity of sound
c_p, c_v = specific heats at constant pressure and volume, respectively
$\mathsf{E} = U_\infty^2/gc_p \Delta T$ = Eckert number (see p. 297)
$\mathsf{G} = g\beta l^3 \Delta T/\nu^2$ = Grashof number
k = thermal conductivity
$\mathsf{M} = V/c$ = Mach number
$\mathsf{N} = \alpha l/k$ = Nusselt number
$\mathsf{P} = \nu/a$ = Prandtl number
q = heat flux (quantity of heat per unit area and time)
$\mathsf{R}_i = -\{(g/\varrho)(d\varrho/dy)\}/(dU/dy)_W^2$ = Richardson number
$\mathsf{S} = \mathsf{N}/\mathsf{RP}$ = Stanton number

T = temperature
T_{ad} = adiabatic wall temperature
α = coefficient of heat transfer
β = coefficient of thermal expansion
$\gamma = c_p/c_v$ = isentropic exponent
δ_T = thickness of thermal boundary layer
ΔT = temperature difference
ω = exponent in viscosity-temperature relation

Abbreviations

The following abbreviations have been used throughout the book:

ARC	Aeronautical Research Council (Great Britain).
R & M	Reports and Memoranda
ASME	American Society of Mechanical Engineers (U.S.A.).
AVA	Aerodynamische Versuchsanstalt Göttingen (Germany).
ETH	Eidgenössische Technische Hochschule, Federal Institute of Technology, Zürich (Switzerland).
NACA	National Advisory Committee for Aeronautics (USA).
NASA	National Aeronautics and Space Administration (USA). Rep Report TM Technical Memorandum TN Technical Note
VDI	Verein Deutscher Ingenieure.
ZAMM	Zeitschrift für angewandte Mathematik und Mechanik.
ZAMP	Zeitschrift für angewandte Mathematik und Physik.
ZFM	Zeitschrift für Flugtechnik und Motorluftschiffahrt.
JAS	Journal of Aeronautical Sciences or Journal of Aero/Space Sciences.

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