

Quantum Theory of Finite Systems

Jean-Paul Blaizot and Georges Ripka

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Preface

This book introduces the quantum theory of finite many-body systems. Finite systems are met with in a variety of fields. Examples are nuclei, atoms, molecules and also solitons in particle physics, vortex lines in quantum liquids, etc. Despite the specialized terminologies many of the methods and approximation schemes used in different fields are strikingly similar. We give a unified presentation of the various methods, hoping that this will contribute to bridging the communication gaps between disciplines.

In contrast to most texts on the many-body problem, this book stresses the finite-system aspects of the theory. Thus special attention is given to the mean field approximations, to the ensuing broken symmetries, and to the associated collective motions such as rotations. The formalism is, of course, quite general and applies to infinite as well as finite systems. However, some specific features of systems with infinite numbers of degrees of freedom are deliberately left out, such as the thermodynamic limit, critical phenomena, and the elimination of ultraviolet divergences.

The book is divided into four parts. Part I introduces the basic mathematical tools: second quantization with special emphasis on coherent states, canonical transformations required to diagonalize quadratic hamiltonians, Wick theorems and the resulting diagram expansions, and oscillator models of finite systems. The basic tools are then applied in parts II-IV, which provide independent but complementary descriptions of the dynamics of many-particle systems. Part II presents mean field approximations, which play an essential role in the description of finite systems. We emphasize the problem of broken symmetries resulting from the mean field approximations, as well as the associated collective motions. Quantization of collective modes is also discussed in terms of recently developed path-integral methods. Part III is a review of perturbation theory in terms of both time-dependent Feynman diagrams and time-independent Goldstone diagrams. Self-consistent schemes are formulated in terms of symmetry-conserving approximations. In part IV we discuss variational methods based on correlated wavefunctions, including spin correlations. The approximation schemes are formulated for fermions and bosons at zero and at nonzero temperature.

The book has been written after several years of lecturing on many-body problems. It has been conceived as a graduate-level course. The material is presented in a self-contained form, and sufficient detail is given to allow a reader with only an elementary knowledge of quantum mechanics to rederive for himself all the results. Each chapter is followed by problems

(190 altogether). These problems have been designed to help the reader practice the theory, as well as to expand on specific points raised in the text.

The book has also been conceived as a research tool and a reference for the working physicist. It provides the background required to understand the more specialized literature, and it includes some of the most recent developments. We have not included physical applications, which would have of necessity been superficially described in a book of this nature. Applications and further developments of the theory can be found in the references given at the end of each chapter. We preferentially cite reviews, where further references can be found. We also cite those papers to which our presentation is most related. Finally, we cite some recent applications. We have made no effort to retrace the historical development of the field, nor have we attempted to make an exhaustive bibliography. We apologize to all those who may justly feel that their work has not been adequately recognized.

The book is admittedly influenced by the prevailing style at the Service de Physique Théorique in Saclay. We express special thanks to Roger Balian and Michel Gaudin for continuous discussions and generous help. We have greatly benefited from the teachings of Bernard Jancovici, Cyrano de Dominicis, Claude Itzykson, and the late Claude Bloch. We owe to numerous physicists more than can be reckoned from the text. Throughout the years we have been deeply influenced by contacts with Manoj Banerjee, Gordon Baym, Georges Bertsch, Aage Bohr, Gerry Brown, Carl Levinson, Ben Mottelson, Phil Siemens, and Igal Talmi. We wish to thank Paul Bonche, Edouard Brezin, David Brink, Stefano Fantoni, Daniel Gogny, Itzhak Kelson, Claude Mahaux, Gene Marshalek, John Negele, Henri Orland, John Owen, Fabre de la Ripelle, Mannque Rho, Hartmut Schulz, Roger Smith, Dominique Vautherin, Felix Villars, and John Zabolitzky for discussions which have led us to a better understanding of specific aspects of the material presented in this book. During the five years needed to complete this work, one of us (J.-P.B.) spent two years at the University of Illinois at Urbana-Champaign. He would like to thank the Physics Department of this university for its stimulating atmosphere and warm hospitality.

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Contents

Preface	xvii
I BASIC TOOLS	3
1 Second Quantization	3
1.1 States of N -Particle Systems	3
1.2 Fock Space: Creation and Destruction Operators	7
1.3 Identity of Particles and the Pauli Principle	10
1.4 Representation of Symmetric Operators in Fock Space	12
1.5 Independent-Particle States	16
1.6 Coherent states as Eigenstates of Creation and Destruction Operators	18
1.6a Boson Coherent States and the Bargmann Representation	18
1.6b Fermion Coherent States and Grassmann Algebra	21
1.6c Coherent States for Systems with Several Degrees of Freedom	25
1.7 Gaussian Integrals	27
Problems	29
References	32
2 Canonical Transformations	34
2.1 Canonical Transformations	34
2.2 Unitary Canonical Transformations	38
2.3 Special Bogolubov Transformation and Quasispin for Fermion Operators	41
2.4 Special Bogolubov Transformation and Quasispin for Boson Operators	42
2.5 Particle-Hole Conjugation	44
2.6 Thouless Parametrization of a Slater Determinant	45
2.7 Canonical Form of the Generalized Density Matrix	47
2.7a Bosons	50
2.7b Fermions	52
Problems	54
References	57

3	Diagonalization of Quadratic Hamiltonians	58
3.1	Hermitian Quadratic Form of Creation and Destruction Operators	58
3.2	Diagonalization of a Quadratic Hamiltonian of Boson Operators	59
3.3	Diagonalization of a Quadratic Hamiltonian of Fermion Operators	68
3.4	Quadratic Hamiltonian with Conserved Charge	70
3.5	Illustrative Examples	74
3.5a	Normal Modes of a Scalar Field	74
3.5b	Fermions Described by a Dirac Hamiltonian	76
	Problems	80
	References	82
4	Wick Theorems	83
4.1	Preliminary Lemmas	83
4.2	Time Dependent Creation and Destruction Operators	85
4.3	Normal Product, T -product, and Contractions of Creation and Destruction Operators	86
4.4	Wick's Theorem	88
4.5	Matrix Elements between Coherent States	90
4.6	Matrix Elements between Nonorthogonal Slater Determinants	93
4.7	Wick's Theorem for Ensemble Averages	97
	Problems	103
	References	104
5	Diagram Expansions	105
5.1	Faithful Representation of the Expectation Value of a T -Exponential (Labeled Feynman Diagrams)	105
5.2	Proof of the Sign Rule	113
5.3	Unlabeled Diagrams and Symmetry Factors	115
5.4	Connected Diagrams and the Exponentiation Property	117
5.5	Rules for Unlabeled Feynman Diagrams	118
5.6	Symmetrized and Antisymmetrized Matrix Elements and the Hugenholtz Representation	119

5.7	Representation of the Exponential of an Operator in Terms of Time-Independent Diagrams	121
5.8	Rules for Time-Independent Diagrams	124
5.9	Diagrams Involving Local Operators (Yvon-Mayer Diagrams)	125
5.10	Rules for Yvon-Mayer Diagrams	128
	Problems	130
	References	133
6	Oscillator Models	135
6.1	Summary of Harmonic-Oscillator Properties	135
6.1a	One-Dimensional Oscillator	135
6.1b	Three-Dimensional Oscillator	137
6.1c	Two-Dimensional Oscillator	139
6.1d	Moshinsky Coefficients	140
6.2	Oscillator Frequency of a Spherical System	141
6.3	Oscillator Frequencies of a Deformed System	142
6.4	Rotating Oscillator	146
6.5	Particles Interacting with Harmonic Two-Body Forces: Center of Mass Motion	151
6.6	Giant Dipole Resonance	154
6.7	The d -Dimensional Isotropic Oscillator and Hyperspherical Coordinates	156
6.8	Time-Dependent Oscillator	160
	Problems	165
	References	169
II	SELF-CONSISTENT FIELDS	171
7	Static Mean Field Approximations	173
7.1	Variational Principles for Systems in Equilibrium	173
7.1a	Ritz Variational Principle	173
7.1b	Variational Principle for Systems Described by a Statistical Density Matrix	174
7.2	Hartree-Fock Approximation for a Fermion System	177
7.2a	Single-Particle Density Matrix	177
7.2b	Hartree-Fock Equations	178

7.3	Mean Field Approximations for a Boson System	181
7.3a	Boson Condensate Approximation	181
7.3b	Boson Coherent State Approximation	183
7.4	Self-Consistent Pairing Fields of Fermion Systems at Zero Temperature	185
7.4a	Representation of a Quasiparticle Vacuum by a Generalized Density Matrix	186
7.4b	Hartree-Fock-Bogolubov Equations	187
7.4c	Gap Equations: Illustrative Example	191
7.5	Hartree-Fock and Hartree-Fock-Bogolubov Approximations of the Partition Function (Fermions)	193
7.5a	Relation between the Partition Function and the Density of States	193
7.5b	Hartree-Fock Approximation of the Partition Function	194
7.5c	Hartree-Fock-Bogolubov Approximation of the Partition Function	197
7.6	Hartree-Bogolubov Approximation of the Partition Function of a System of Interacting Bosons	199
7.7	Density-Dependent Effective Interactions	204
7.8	Energies of the Hartree-Fock Orbitals and Particle Separation Energies	205
	Problems	208
	References	215
8	Symmetries and Collective Motion Associated with Broken Symmetries	217
8.1	Unitary and Antiunitary Transformations	217
8.2	Symmetries of the Hamiltonian and Broken Symmetries	219
8.2a	Symmetries of the Hamiltonian	219
8.2b	Example of Dynamical Symmetry: the Group $U(3)$ and the Quadrupole-Quadrupole Interaction	220
8.2c	Broken Symmetries and Deformed States	223
8.3	Symmetries and Transformations of Single-Particle States	224
8.3a	Parity and Reflection in a Plane	225
8.3b	Time Reversal	226

8.3c	Space Translation	227
8.3d	Rotation	228
8.3e	Time Translation	230
8.3f	Rotation in Isospin Space	230
8.3g	Rotation in Fock Space Generated by the Particle Number Operator	231
8.4	Self-Consistent and Broken Symmetries of the Hartree-Fock Field	231
8.5	Self-Consistent and Broken Symmetries in the Presence of Pairing Fields	233
8.6	Broken Symmetries in Finite Systems	236
8.6a	Parity as an Example of a Discrete Broken Symmetry	238
8.6b	Deformation in Fock Space and the Nonconservation of Particle Number	239
8.6c	Illustrative Example: A Coherent State as a Trial Wavefunction	246
8.6d	Broken Symmetries in Infinite Systems	250
8.7	Collective Rotation of Deformed Systems	252
8.7a	Angular Momentum Operators in Laboratory and Intrinsic Frames	252
8.7b	Variational Principle and Angular Momentum Projection	255
8.7c	Equation of Motion for Collective Rotation	257
8.7d	Cranking Approximation	259
	Problems	261
	References	267
9	Time-Dependent Self-Consistent Fields	269
9.1	Principles of Stationary Action for the Time Evolution of Quantum Systems	269
9.1a	Variational Principle for Transition Amplitudes	271
9.1b	A Variational Principle for the Time Evolution of a System Described by a State Vector	273
9.1c	Variational Principle for the Time Evolution of a System Described by a Density Operator	275
9.2	Time-Dependent Hartree-Fock Approximation for Fermions	277

9.3	Time-Dependent Mean Field Approximations for Bosons	278
9.4	Hartree-Fock Approximation for Transition Amplitudes	280
9.5	Time-Dependent Hartree-Fock-Bogolubov Approximation for Fermions	283
9.6	Constants of Motion	285
	9.6a Conservation of Energy	288
	9.6b Time-Reversal Invariance	289
9.7	Equation of Continuity	290
9.8	Time-Dependent Mean Field Equations in a Moving Frame	291
9.9	Time-Dependent Mean Field Equations as Classical Equations of Motion	294
	9.9a Time-Dependent Hartree-Fock Equations	296
	9.9b Time-Dependent Hartree-Fock-Bogolubov Equations	298
	Problems	302
	References	307
10	Small-Amplitude Vibrations	309
10.1	Linear Response to a Time-Dependent External Field	309
10.2	Small-Amplitude Approximation of the Time-Dependent Hartree-Fock-Bogolubov Equations	313
10.3	Vibrations of a Normal System	318
	10.3a Normal Vibrations	320
	10.3b Pairing Vibrations	322
10.4	Local Stability Conditions	324
10.5	Sum Rules	326
10.6	Spurious States and the Associated Collective Motion	330
	10.6a Broken Translational Symmetry	331
	10.6b Broken Rotational Symmetry	335
	10.6c Broken Symmetry Due to Nonconservation of Particle Number	336
10.7	Small-Amplitude Vibrations of a Boson System	337
10.8	Linear Response at Nonzero Temperature	339

Problems	341
References	350
11 Quantization of Time-Dependent Self-Consistent Fields	351
11.1 Generalized Coherent States	351
11.1a Coherent States for the Rotation Group	353
11.1b Closure Relations over Slater Determinants	353
11.1c Closure Relations over Slater Determinants using Boson Coherent States	355
11.2 The Schrödinger Equation in the Coherent State Representation	359
11.3 The Schrödinger Equation in the Small-Amplitude Approximation	361
11.4 The Method of Boson Images	368
11.4a Mappings between the Fermion Hilbert Space and a Boson Hilbert Space	368
11.4b Classical Limit of the Boson Equations of Motion	372
11.4c Corrections to the Small-Amplitude Approximation	374
11.5 Path Integrals over Generalized Coherent States	377
11.5a Path Integral for the Transition Amplitude between Two Generalized Coherent States	378
11.5b Path Integral for the Trace of the Evolution Operator	380
11.5c Path Integral over Slater Determinants	381
11.6 Path Integrals over Ordinary Coherent States	382
11.7 Path Integrals over Time-Dependent Mean Fields	385
11.8 Semiclassical Approximations	387
11.8a Stationary Phase Approximation of the Path Integral	387
11.8b Periodic Solutions of the Time-Dependent Hartree-Fock Equations	389
11.8c Quantization of Collective Motion Associated with a Broken Symmetry	393
Problems	394
References	400

III	PERTURBATION THEORY	403
12	Perturbation Theory and Feynman Diagrams	405
12.1	Some Properties of the Evolution Operator	405
12.2	Generating Functionals	407
12.3	Perturbative Expansion of the Evolution Operator	411
12.4	Expansion of the Generating Functional in Terms of Feynman Diagrams	412
12.5	Single-Particle Propagators	417
12.6	Feynman Diagrams in the Energy Representation	422
12.7	Rules for the Calculation of the Free Energy, Expectation Values of Operators, and Green's Functions	426
12.7a	Free Energy	426
12.7b	Expectation Values of Operators	427
12.7c	Green's Functions	428
12.8	Choice of the Unperturbed Hamiltonian H_0	429
12.8a	Unperturbed Hamiltonian and Symmetry Breaking	429
12.8b	Unperturbed Hamiltonian for a Boson System	431
12.9	Illustrative Examples	435
12.9a	Free Energy of a Fermion System up to Second Order	435
12.9b	Expectation Value of a One-Body Fermion Operator up to First Order	437
12.9c	Free Energy of a Boson System up to Second Order	438
12.10	The Generating Functional as a Path Integral	440
12.10a	Path Integral over Ordinary Coherent States	440
12.10b	Path Integral over Time-Dependent Mean Fields	443
	Problems	448
	References	450
13	Time-Independent Goldstone Diagrams and the ϵ^s Method	452
13.1	Expansion of the Resolvent of the Hamiltonian	452
13.2	Rules for Calculation of Goldstone Diagrams	457
13.3	Ground-State Expectation Values of Operators	457

13.4	Illustrative Examples	458
13.4a	Energy of a Fermion System up to Second Order	458
13.4b	Energy of a Boson System up to Second Order	459
13.4c	Ground-State Expectation Value of a One-Body Operator in a System of Interacting Bosons and Fermions	461
13.5	Expansion of the Ground-State Wavefunction of a Fermion System	463
13.6	The ϵ^S method	466
13.6a	Basic Equations	466
13.6b	Second-Order Energy	473
13.6c	Pairing Ladder Diagrams	474
13.6d	Normal-Mode Ring Diagrams	475
13.6e	Low-Density Hole-Line Expansion	476
	Problems	478
	References	480
14	One-Particle Green's Function and the Optical Potential	481
14.1	Definition; Equation of Motion and Boundary Conditions	481
14.2	Spectral Decomposition and Analytic Properties	484
14.3	The Mass Operator and Dyson's Equation	488
14.4	Analytic Properties of the Mass Operator	492
14.5	Poles and Residues of the One-Particle Green's Function	495
14.6	Illustrative Example	497
14.7	Transition Matrix for a Binary Collision	501
14.8	Relation between the Optical Potential and the Mass Operator of the Green's Function	506
14.9	The Optical Potential in the Illustrative Example	507
	Problems	509
	References	511
15	Four-Point Green's Functions	513
15.1	Definitions: Channels	513
15.1a	Particle-Hole Channels	515
15.1b	Particle-Particle Channels	516

15.2	Two-Particle Green's Function	517
15.3	Particle-Hole Green's Function	519
15.4	Transition Amplitudes between States of Systems of $A \pm 1$ Particles	521
15.5	Perturbation Expansion of Four-Point Functions	522
	15.5a Interaction Operator Γ	523
	15.5b Particle-Particle Interaction Γ^{12}	525
	15.5c Particle-Hole Interactions Γ^{13} and Γ^{14}	527
15.6	Parquet Diagrams	530
15.7	Self-Consistent Theory of the Particle-Hole Interaction	533
15.8	Self-Consistent Theory of the Particle-Particle Interaction	536
15.9	Summation of Ring Diagrams	538
15.10	Summation of Ladder Diagrams	541
15.11	Renormalization of a One-Body Operator by Coupling to Normal Vibrations	543
	Problems	545
	References	547
16	Renormalizations	548
16.1	Renormalization of Field Amplitudes	548
	16.1a Renormalization of n -Point Functions	548
	16.1b Renormalization of the Generating Functional W	551
	16.1c Legendre Transform of the Generating Functional	553
16.2	Propagator Renormalization	556
	16.2a Propagator Renormalization of Green's Functions	556
	16.2b Propagator Renormalization of the Generating Functional	557
	16.2c Self-Consistent or Symmetry-Conserving Approximations	561
16.3	Theory of a General Static Mean Field	564
	16.3a Renormalization of Green's Functions by Static Insertions	564
	16.3b Renormalization of the Generating Functional	566
	16.3c Note on the Treatment of Exchange Vertices	567
	16.3d Functional Integral Interpretation	568
16.4	An Example of Vertex Renormalization	569

Problems	571
References	573
IV CORRELATED WAVEFUNCTIONS	575
17 Correlated Boson Wavefunctions	577
17.1 Correlated Boson Wavefunctions	577
17.2 Local Correlation Functions and Their Generating Functional	580
17.3 Expansion in Terms of Yvon-Mayer Diagrams	582
17.4 Point-Irreducible Diagrams and Elimination of the Condensate Orbital in Favor of the Density	585
17.5 Chain Expansion	587
17.6 Hypernetted Chain Expansion	590
17.7 More than Doubly Connected Diagrams and Elimination of the Correlation Factor U in Favor of the Correlation Function g	593
17.8 Calculation of the Energy in Terms of Local Correlation Functions	594
17.8a Jackson-Feenberg Transformation	594
17.8b Calculation of the Energy	595
17.8c Expansion of the Energy in Terms of Yvon-Mayer Diagrams	596
17.9 Euler-Lagrange Equations	598
17.9a Equation for the Single-Particle Density	599
17.9b Equation for the Correlation Function	602
17.10 Long- and Short-Range Correlations in an Infinite Uniform System of Spinless Bosons	603
Problems	606
References	608
18 Correlated Fermion Wavefunctions	610
18.1 Diagrammatic Expansion of the Generating Functional (Local Correlation Factor)	610
18.2 Properties of the Non-Hermitian Density Matrix	612
18.3 Single-Particle Density and Two-Body Correlation Function	614

18.4	Point-Irreducible Diagrams and Elimination of Articulation Points	618
18.5	Chain Equations	621
18.6	Diagrammatic Expansion of the Energy	624
18.7	Euler-Lagrange Equations	626
18.8	Spin Correlations	629
18.9	Rules for Yvon-Mayer Diagrams Involving at Most Second-Order Spin Loops	634
18.10	Point Irreducibility and Vertex Corrections	635
18.11	Two-Particle Correlation Functions	637
18.12	The Energy in the Presence of Spin Correlations	639
	Problems	641
	References	647
	Index	649