

ELEMENTARY MATHEMATICS

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ЭЛЕМЕНТАРНАЯ МАТЕМАТИКА

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PREFACE

This second edition of *Elementary Mathematics*, which first came out in 1967, has been fundamentally reworked. In accordance with the present division of the course of elementary mathematics, the first part (algebra) and the third part (trigonometry) of the first edition have been combined into one part (arithmetic, algebra, and elementary functions). Quite naturally this required a rehauling of the entire text and could not be done mechanically. The result has been a redistribution of the material, with all functions, except the trigonometric functions, combined into one chapter.

Several fundamental problems of the course of mathematics—these include the method of mathematical induction and reductio ad absurdum proof—are discussed in the introduction.

The present text is designed for the student who wishes to prepare for entrance examinations to technical colleges.

A great loss in our undertaking was the sudden death of the initiator and one of the authors of this book, Mark Ivanovich Skanavi, during the early stages of work on the second edition. His co-authors, Zaitsev and Ryzhkov, completed reworking the chapters devoted to arithmetic and algebra along the lines that he had proposed and of course are fully responsible for the quality of this work.

The arduous job of checking the answers to the exercises was done by Sergei Berkesov to whom we express our heartfelt gratitude.

The Authors

HOW TO USE THE BOOK

This text is a review course of elementary mathematics in the sense that it envisages a reader who has studied the subject but desires to expand, reinforce and systematize his knowledge (for example, for the purpose of getting prepared for the entrance examinations to a higher educational institution). For this reason the authors frequently refer to later chapters in the book, assuming that the reader has at least a rough idea of the subject matter in question. For the sake of convenience the book is divided into articles that are numbered in a single sequence. References are made in the following form: see Art. 184, or simply, Art. 55. Figures are also numbered sequentially throughout the text. Formulas and equations have a double number: for example, (4.5) or (217.3), which stands for the fifth equation of Article 4 or the third formula of Article 217.

The reader who remembers at least some of the material of the school programme can study the text as it stands; however, it is not advisable to leave the study of geometry to the end (ordinarily geometry is studied in parallel with the other branches of mathematics). Here is a suggested variant for studying the present text:

(1) Chapters I-VII; (2) Chapters VIII, XVI and Chapter XVII up to Article 216 inclusive; (3) the remaining chapters of the first part; (4) the remaining portion of the second part.

Students of preparatory courses will want to use this text in the sequence indicated by the syllabus of the given courses.

For the reader who wishes to extend his mathematical knowledge still further (this is important for those entering colleges with expanded mathematical courses) we recommend the following literature.

(1) G. Dorofeev, M. Potapov, N. Rozov, *Elementary Mathematics*, Mir Publishers, Moscow, 1973.

(2) M. Vygodsky, *Mathematical Handbook (Elementary Mathematics)*, Mir Publishers, Moscow, 1975.

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