Communications and Control Engineering

Zdzislaw Bubnicki

Analysis and Decision Making in Uncertain Systems

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With 108 Figures

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Preface

Problems, methods and algorithms of decision making based on an uncertain knowledge now create a large and intensively developing area in the field of knowledge-based decision support systems. The main aim of this book is to present a unified, systematic description of analysis and decision problems in a wide class of uncertain systems described by traditional mathematical models and by relational knowledge representations. A part of the book is devoted to new original ideas introduced and developed by the author: the concept of uncertain variables and the idea of a learning process consisting in knowledge validation and updating. In a certain sense this work may be considered as an extension of the author's monograph *Uncertain Logics, Variables and Systems* (Springer-Verlag, 2002). In this book it has been shown how the different descriptions of uncertainty based on random, uncertain and fuzzy variables may be treated uniformly and applied as tools for general analysis and decision problems, and for specific uncertain systems and problems (dynamical control systems, operation systems, knowledge-based pattern recognition under uncertainty, task allocation in a set of multiprocessors with uncertain execution times, and decision making in an assembly system as an example of an uncertain manufacturing system). The topics and the organization of the text are presented in Chapter 1 (Sects 1.1 and 1.4).

The material presented in the book is self-contained. I hope that the book can be useful for graduate students, researchers and all readers working in the field of control and information science, especially those interested in the problems of uncertain decision support systems and uncertain control systems.

I wish to acknowledge with gratitude the encouragement and help I received from Professor Manfred Thoma, editor of this series. His inspiration and interest have been invaluable in the preparation of the book.

I wish also to express my gratitude to my co-workers at the Institute of Control and Systems Engineering of Wroclaw University of Technology, who assisted in the preparation of the manuscript. My special thanks go to Dr. L. Siwek for the valuable remarks and discussions, and for his work concerning the formatting of the text. Thanks are also due to Dr. D. Orski who assisted in the final phase of preparation of the manuscript.

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Z. Bubnicki

Contents

1 Introduction to Uncertain Systems

1.1 Uncertainty and Uncertain Systems

Uncertainty is one of the main features of complex and intelligent decision making systems. Various approaches, methods and techniques in this field have been developed for several decades, starting with such concepts and tools as adaptation, stochastic optimization and statistical decision theory (see e.g. [2, 3, 68, 79, 80]). The first period of this development was devoted to systems described by traditional mathematical models with unknown parameters. In the past two decades new ideas (such as learning, soft computing, linguistic descriptions and many others) have been developed as a part of modern foundations of knowledge-based Decision Support Systems (DSS) in which the decisions are based on *uncertain knowledge*. Methods and algorithms of decision making under uncertainty are especially important for design of computer control and management systems based on incomplete or imperfect knowledge of a decision plant. Consequently, problems of analysis and decision making in uncertain systems are related to the following fields:

1. General systems theory and engineering.
2. Control and management systems.
3. Information technology (knowledge-based expert systems).

There exists a great variety of definitions and formal models of uncertainties and uncertain systems. The most popular non-probabilistic approaches are based on fuzzy sets theory and related formalisms such as evidence and possibility theory, rough sets theory and fuzzy measures, including a probability measure as a special case (e.g. [4, 7, 9, 64, 65, 67, 69, 71, 74, 75, 78, 81, 83, 84, 96–100, 103, 104]). The different formulations of decision making problems and various proposals for reasoning under uncertainty are adequate for the different formal models of uncertainty. On the other hand, new forms of uncertain knowledge representations require new concepts and methods of information processing: from computing with numbers to granular computing [5, 72] and computing with words [101].

Special approaches have been presented for multiobjective programming and scheduling under uncertainty [91, 92], for uncertain object-oriented databases [63], and for uncertainty in expert systems [89]. A lot of works have been concerned with specific problems of uncertain control systems, including problems of stability and stabilization of uncertain systems and an idea of robust control (e.g. [31, 61, 62, 77, 87, 88]).

In recent years a concept of so-called *uncertain variables* and their applications to analysis and decision problems for a wide class of uncertain systems has been developed [25, 30, 35, 40, 42, 43, 44, 46, 50, 53, 54, 55]. The main aim of this book is to present a unified, comprehensive and compact description of analysis and decision problems in a class of uncertain systems described by traditional mathematical models and by relational knowledge representations. An attempt at a uniform theory of uncertain systems including problems and methods based on different mathematical formalisms may be useful for further research in this large area and for practical applications to the design of knowledge-based decision support systems. The book may be characterized by the following features:

1. The problems and methods are concerned with systems described by traditional mathematical models (with number variables) and by knowledge representations which are treated as an extension of classical functional models. The considerations are then directly related to respective problems and methods in traditional system and control theory.
2. The problems under consideration are formulated for systems with unknown parameters in the known form of the description (*parametric problems*) and for the direct non-deterministic input–output description (*non-parametric problems*). In the first case the unknown parameters are assumed to be values of random or uncertain variables. In the second case the values of input and output variables are assumed to be values of random, uncertain or fuzzy variables.
3. The book presents three new concepts introduced and developed by the author for a wide class of uncertain systems:
   a. Logic-algebraic method for systems with a logical knowledge representation [9 – 14].
   b. Learning process in systems with a relational knowledge representation, consisting in *step by step* knowledge validation and updating (e.g. [18, 22, 25]).
   c. Uncertain variables based on uncertain logics.
4. Special emphasis is placed on uncertain variables as a convenient tool for handling the uncertain systems under consideration. The main part of the book is devoted to the basic theory of uncertain variables and their application in different cases of uncertain systems. One of the main purposes of the book is to present recent developments in this area, a comparison with random and fuzzy variables and the generalization in the form of so-called *soft variables*.
5. Special problems such as pattern recognition and control of a complex of operations under uncertainty are included. Examples concerning the control of manufacturing systems, assembly processes and task distributions in computer systems indicate the possibilities of practical applications of uncertain variables and other approaches to decision making in uncertain systems.

The analysis and decision problems are formulated for input–output plants and two kinds of uncertainty:

1. The plant is non-deterministic, i.e. the output is not determined by the input.

2. The plant is deterministic, but its description (the input–output relationship) is not exactly known.

The different forms of the uncertainty may be used in the description of one plant. For example, the non-deterministic plant may be described by a relation such that the output is not determined by the input (i.e. is not a function of the input). This relation may be considered as a *basic description* of the uncertainty. If the relation contains unknown parameters, their description, e.g. in the form of probability distributions, may be defined as an *additional description* of the uncertainty or the *second-order uncertainty*.

In the wide sense of the word an uncertain system is understood in the book as a system containing any kind and any form of uncertainty in its description. In a narrow sense, an uncertain system is understood as a system with the description based on uncertain variables. In this sense, such names as "random, uncertain and fuzzy knowledge" or "random, uncertain and fuzzy controllers" will be used. Additional remarks will be introduced, if necessary, to avoid misunderstandings. Quite often the name "control" is used in the text instead of decision making for a particular plant. Consequently, the names "control plant, control system, control algorithm, controller" are used instead of "decision plant, decision system, decision algorithm, decision maker", respectively.

1.2 Uncertain Variables

In the traditional case, for a static (memoryless) system described by a function $y = \Phi(u, x)$ where u, y, x are input, output and parameter vectors, respectively, the decision problem may be formulated as follows: to find the decision u^* such that $y = y^*$ (the desirable output value). The decision u^* may be obtained for the known function Φ and the value x. Let us now assume that x is unknown. In the probabilistic approach x is assumed to be a value of a random variable $\tilde{x}$ described by the probability distribution. In the approach based on uncertain variables the unknown parameter x is a value of an uncertain variable $\bar{x}$ for which an expert gives the certainty distribution $h(x) = v(\bar{x} \cong x)$ where v denotes a certainty index of the soft property: "$\bar{x}$ is approximately equal to x" or "x is the approximate value of $\bar{x}$". The certainty distribution evaluates the expert's opinion on approximate values of the uncertain variable. The uncertain variables, related to random variables and fuzzy numbers, are described by the set of values X and their certainty distributions which correspond to probability distributions for the random variables and to membership functions for the fuzzy numbers. To define the uncertain variable, it is necessary to give $h(x)$ and to determine the certainty indexes of the following soft properties:

1. "$\bar{x} \,\tilde{\in}\, D_x$" for $D_x \subset X$, which means "the approximate value of $\bar{x}$ belongs to D_x" or "$\bar{x}$ belongs approximately to D_x".

2. "$\bar{x} \,\tilde{\notin}\, D_x$" = "$\neg(\bar{x} \,\tilde{\in}\, D_x)$", which means "$\bar{x}$ does not belong approximately to D_x".

To determine the certainty indexes for the properties: $\neg(\bar{x} \,\tilde{\in}\, D_x)$, $(\bar{x} \,\tilde{\in}\, D_1) \vee (\bar{x} \,\tilde{\in}\, D_2)$ and $(\bar{x} \,\tilde{\in}\, D_1) \wedge (\bar{x} \,\tilde{\in}\, D_2)$ where $D_1, D_2 \subseteq X$, it is necessary to introduce an *uncertain logic*, which deals with the soft predicates of the type "$\bar{x} \,\tilde{\in}\, D_x$". In Chapter 4 four versions of the uncertain logic have been defined and used for the formulation of the respective versions of the uncertain variable.

For the proper interpretation (semantics) of these formalisms it is convenient to consider $\bar{x} = g(\omega)$ as a value assigned to an element $\omega \in \Omega$ (a universal set). For fixed ω its value $\bar{x}$ is determined and $\bar{x} \in D_x$ is a crisp property. The property $\bar{x} \,\tilde{\in}\, D_x = x \in D_x$ = "the approximate value of $\bar{x}$ belongs to D_x" is a soft property because $\bar{x}$ is unknown and the evaluation of "$\bar{x} \,\tilde{\in}\, D_x$" is based on the evaluation of $\bar{x} \cong x$ for the different $x \in X$ given by an expert. In the first version of the uncertain variable, $v(\bar{x} \,\tilde{\in}\, D_x) \neq v(\bar{x} \,\tilde{\notin}\, \overline{D}_x)$ where $\overline{D}_x = X - D_x$ is the complement of D_x. In the version called the *C*-uncertain variable, $v_c(\bar{x} \,\tilde{\notin}\, D_x) = v_c(\bar{x} \,\tilde{\in}\, \overline{D}_x)$ where v_c is the certainty index in this version

$$v_c(\bar{x} \,\tilde{\in}\, D_x) = \frac{1}{2}[v(\bar{x} \,\tilde{\in}\, D_x) + v(\bar{x} \,\tilde{\notin}\, \overline{D}_x)].$$

The uncertain variable in the first version may be considered as a special case of the possibilistic number with a specific interpretation of $h(x)$ described above. In our approach we use soft properties of the type "P is approximately satisfied" where P is a crisp property, in particular $P =$ "$\bar{x} \in D_x$". It allows us to accept the difference between $\bar{x} \,\tilde{\in}\, D_x$ and $\bar{x} \,\tilde{\notin}\, \overline{D}_x$ in the first version. More details concerning the relations to random variables and fuzzy numbers are given in Chapter 6. Now let us pay attention to the following aspects which will be more clear after the presentation of the formalisms and semantics in Chapter 4:

1. To compare the meanings and practical utilities of different formalisms, it is necessary to take into account their semantics. It is specially important in our approach. The definitions of the uncertain logics and consequently the uncertain variables contain not only the formal description but also their interpretation. In particular, the uncertain logics may be considered as special cases of multi-valued predicate logic with a specific semantics of the predicates. It is worth noting that from the formal point of view the probabilistic measure is a special case of the fuzzy measure and the probability distribution is a special case of the membership function in the formal definition of the fuzzy number when the meaning of the membership function is not described.
2. Even if the uncertain variable in the first version may be formally considered as a very special case of the fuzzy number, for simplicity and unification it is better to introduce it independently (as has been done in the book) and not as a special case

of the much more complicated formalism with different semantics and applications.
3. *Uncertainty* is understood here in the narrow sense of the word, and concerns an incomplete or imperfect knowledge of something which is necessary to solve the problem. In our considerations, it is the knowledge of the parameters in the mathematical description of the system or the knowledge of a form of the input–output relationships, and is related to a fixed expert who gives the description of the uncertainty.
4. In the majority of interpretations the value of the membership function means a *degree of truth* of a soft property determining the fuzzy set. In our approach, "$\bar{x} \in D_x$" and "$x \in D_x$" are crisp properties, the soft property "$\bar{x} \,\tilde{\in}\, D_x$" is introduced because the value of $\bar{x}$ is unknown and $h(x)$ is a *degree of certainty* (or $1-h(x)$ is a degree of uncertainty).

1.3 Basic Deterministic Problems

The problems of analysis and decision making under uncertainty described in the book correspond to the respective problems for deterministic (functional) plants with the known mathematical models. Let us consider a static plant described by a function $y = \Phi(u)$ where $u \in U = R^p$ is the input vector, $y \in Y = R^l$ is the output vector, U and Y are p-dimensional and l-dimensional real number vector spaces, respectively. The function Φ may be presented as a set of functions

$$y^{(i)} = \Phi_i(u^{(1)}, u^{(2)}, ..., u^{(p)}); \quad i = 1, 2, ..., l$$

where $y^{(i)}$ is the i-th component of y and $u^{(j)}$ is the j-th component of u.

Analysis problem: Given the function Φ and the value $u = u^*$, find the corresponding output $y^* = \Phi(u^*)$.

Decision problem: For the given function Φ and the value y^* required by a user, find the decision u^* such that $y = y^*$.

The solution of the problem is reduced to solving the equation $y^* = \Phi(u)$ with respect to u. In general, we may obtain a set of decisions

$$D_u = \{u \in U: \quad \Phi(u) = y^*\}.$$

In particular $D_u = \varnothing$ (an empty set), which means that the solution does not exist. For the plant described by a function $y = \Phi(u, z)$ where z is a vector of external disturbances, the set of solutions $D_u(z)$ depends on z. In the case of a unique solution we obtain $u^* \stackrel{\Delta}{=} \Psi(z)$, i.e. the deterministic decision (control) algorithm in an open-loop decision system when z is measured (Fig. 1.1). For the plant described by the function $y = \Phi(u)$, on the assumption that the equation $\Phi(u) = y^*$ has a unique solution, the decision u^* may be determined by the following recursive algorithm:

$$u_{n+1} = u_n - K[y^* - \Phi(u_n)]; \quad n = 0, 1, \ldots \tag{1.1}$$

where u_n denotes the n-th approximation of u^* and K is a matrix of coefficients. Under some conditions concerning Φ and K, the sequence u_n converges to u^* for any u_0. The algorithm (1.1) may be executed in a closed-loop decision system (Fig. 1.2) where the output $y_n = \Phi(u_n)$ is measured. It is worth noting that to assure the convergence, it is not necessary to know exactly the function Φ. Then feedback is a way to achieve the proper decision u^* for the uncertain plant, i.e. it is one of the possible approaches to decision making under uncertainty.

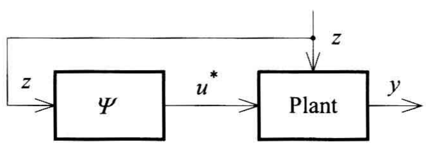

Figure 1.1. Open-loop decision system

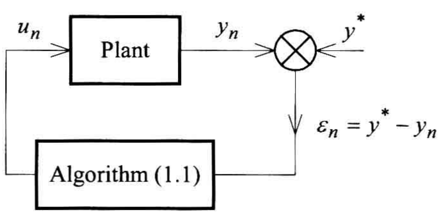

Figure 1.2. Closed-loop decision system

If there are additional constraints and/or the solution of the equation $\Phi(u) = y^*$ does not exist, the decision problem may be formulated as an optimization problem consisting in finding u^* minimizing a quality index $\varphi(y, y^*)$, e.g.

$$\varphi(y, y^*) = (y - y^*)^{\mathrm{T}}(y - y^*)$$

where vectors are presented as one-column matrices and T denotes transposition of a matrix.

The formulations of basic analysis and decision problems may be extended to deterministic dynamical plants. Let us consider a plant described by the equation

$$s_{n+1} = f(s_n, u_n); \quad n = 0, 1, \ldots$$

where s_n is a state vector.

Analysis problem: For the given function *f*, initial state s_0 and the sequence $u_0, u_1, \ldots, u_{N-1}$ one should find the sequence $s_1, s_2, \ldots, s_N$.

One of the possible formulations of a decision problem is the following: for the given function *f*, s_0 and $s_N = s^*$ required by a user, one should determine the sequence of decisions $u_0, u_1, \ldots, u_{N-1}$ such that $s_N = s^*$. The solution exists for sufficiently large N if the plant is controllable. The optimization problem corresponding to the minimization of $\varphi(y, y^*)$ for a static plant may be formulated as follows.

Optimal decision problem: For the given function *f*, state s_0 and a quality index $\varphi(s, s^*)$, one should determine the sequence $u_0, u_1, \ldots, u_{N-1}$ minimizing the global performance index

$$Q_N = \sum_{n=1}^{N} \varphi(s_n, s^*) = \sum_{n=0}^{N-1} \varphi[f(s_n, u_n), s^*].$$

1.4 Structure of the Book

The book consists of two informal parts. The first part containing Chapters 2–7 presents basic analysis and decision problems for static plants. The second part containing Chapters 8–14 concerns dynamical systems and special problems connected with learning and complex systems, pattern recognition and operation systems. The parts are organized as follows.

Chapter 2 presents basic analysis and decision problems for static plants described by relations. A general concept of so-called *determinization*, consisting in replacing an uncertain description by its deterministic representation, is introduced. Two kinds of relational knowledge representation are considered: the knowledge of the plant and the knowledge of the decision making.

Chapter 3 deals with the application of random variables to the description of the uncertainty. In the first part of the chapter, analysis and decision problems are considered for the functional and relational plant with random parameters. The second part is devoted to the respective problems with a non-parametric description of the uncertainty. In this case the knowledge of the plant has a form of conditional