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Managing Editors: M. Beckmann and W. Krelle

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Dynamic Games and Applications in Economics

Edited by T. Başar

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PREFACE

This volume contains eleven articles which deal with different aspects of dynamic and differential game theory and its applications in economic modeling and decision making. All but one of these were presented as invited papers in special sessions I organized at the 7th Annual Conference on Economic Dynamics and Control in London, England, during the period June 26-28, 1985.

The first article, which comprises Chapter 1, provides a general introduction to the topic of dynamic and differential game theory, discusses various noncooperative equilibrium solution concepts, including Nash, Stackelberg, and Consistent Conjectural Variations equilibria, and a number of issues such as feedback and time-consistency. The second chapter deals with the role of information in Nash equilibria and the role of leadership in Stackelberg problems. A special type of a Stackelberg problem is the one in which one dominant player (leader) acquires dynamic information involving the actions of the others (followers), and constructs policies (so-called incentives) which enforce a certain type of behavior on the followers; Chapter 3 deals with such a class of problems and presents some new theoretical results on the existence of affine incentive policies. The topic of Chapter 4 is the computation of equilibria in discounted stochastic dynamic games. Here, for problems with finite state and decision spaces, existing algorithms are reviewed, with a comparative study of their speeds of convergence, and a new algorithm for the computation of nonzero-sum game equilibria is presented.

Chapter 5 of the volume illustrates the open-loop Stackelberg equilibrium solution by means of specific economic examples arising in a regional investment allocation problem. The study leads to interesting economic interpretations, and a number of theoretical questions. Chapter 6 provides an in-depth survey on the applications of dynamic game theory to macroeconomics, covering primarily areas such as economic growth and income distribution, macroeconomic stabilization, modeling of interaction between government and private sector, international policy coordination, and conflicts among sectors of an economy. Chapter 7 deals with the analysis of a number of issues in international policy making, by applying the framework of dynamic game theory to a dynamic inflationary model of two independent economies.

In this model, the objective of each policy maker is taken to reflect a trade-off between the rate of inflation and unemployment in his country and the equilibrium is studied under feedback Nash, feedback Stackelberg and feedback consistent conjectural variations type of behavior, comparatively and numerically.

Chapter 8 is a comprehensive survey on optimal dynamic pricing in oligopolistic markets, and it provides a critical evaluation of the role of differential game theory in such applications. Chapter 9 deals with a specific model of dynamic advertising and pricing in an oligopoly, and obtains open-loop Nash equilibrium solutions. Chapter 10 surveys some dynamic game theory models of fishery management, and discusses the noncooperative feedback Nash and the cooperative Nash bargaining solutions in this context. The final chapter, Chapter 11, is devoted to a differential game theoretic modeling and analysis of the problem of common property exploitation under random hazards of extinction. The focus is on long run behavior, stationary policies, and a comparison of noncooperative and cooperative solutions, using a differential game model involving a jump process.

Even though this volume is not exhaustive in the choice of topics on economic applications of dynamic games, it does cover a rather broad spectrum in both theory and applications, and should therefore be useful to researchers in mathematical economics, operations research, systems and control, and differential games. I would like to thank the authors for their contributions both to this volume and to the invited sessions on "Dynamic Game Theory and Applications in Economics" at the 7th Annual Conference on Economic Dynamics and Control in London. I am sure every chapter in this volume has benefited from the stimulating discussions conducted during and after the paper presentations at the Conference.

My sincere thanks also go to Ms. Dixie Murphy for the secretarial undertaking and the editorial assistance during all phases of this project.

October 1985 Tamer Başar
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TABLE OF CONTENTS

A TUTORIAL ON DYNAMIC AND DIFFERENTIAL GAMES

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Abstract

A general formulation of dynamic and differential games is given, which includes both discrete and continuous time problems as well as deterministic and stochastic games. Solution concepts are introduced in two categories, depending on whether the dynamic game is defined in normal or extensive form. For the former, we present the Nash, Stackelberg and Consistent Conjectural Variations (CCV) equilibria, with considerable discussion devoted to the CCV solution, including comparisons with other more specific definitions found in the literature. For games in extensive form, we discuss the feedback solution concepts, and elaborate on the time consistency issue, which is currently of major interest in the economics literature. The chapter concludes with a discussion which puts into proper perspective the topics and contributions of the ten papers to follow, and their relationships with each other.

1. Introduction and Main Ingredients

Dynamic game theory provides a framework for a quantitative modeling and analysis of the interactions of economic agents among themselves and with the (uncertain) environment, and sets the appropriate mathematical tools for arriving at "optimal" decisions under varying behavioral stipulations. The basic ingredients of such a quantitative theory are the following:

(1) The number of economic agents (synonymously, decision makers or players): $(1,2,\ldots,n) \triangleq \mathbf{N}$, where $\mathbf{N}$ will be called the *player set*, and each agent will be denoted generically by i.

(2) The *time interval* on which the decision process is defined: $\Xi = [0,t_f]$. This could be a discrete interval, which corresponds to the situation when decisions are made at discrete instants of time: $0 = t_0,t_1,t_2,\ldots,t_f$; or it could be a continuous interval, which corresponds to the case when decisions are made throughout

the time interval $[0,t_f]$. The former characterizes, along with other ingredients, a "discrete-time (dynamic) game" (synonymously, "difference game"), whereas the latter leads to a "continuous-time (dynamic) game" also known as a "differential game."

(3) *Decision* (control) *variable* for each agent: $u_i \in \mathbb{U}_i$, $i \in \mathbb{N}$, where $\mathbb{U}_i$ is the decision space of agent i. In a dynamic game, u_i depends on the time variable t, $u_i = u_i(t)$, and for each $t \in \Xi$, $u_i(t) \in U_i$, where U_i is also called the decision or action space; this is the set in which the decision variable takes values for each fixed $t \in \Xi$.

(4) *Disturbance variable*, $w \in \mathbb{D}$, which is not under the control of the agents, but whose probabilistic description is common knowledge to all agents. For each $t \in \Xi$, we will let $w(t) \in W$, where W is some appropriate set.

(5) *Information structure*, $\eta = (\eta_1, \ldots, \eta_n)$, which is an appropriate mapping defined on the product space $(\underset{i \in \mathbb{N}}{X} \mathbb{U}_i) X \mathbb{D}$, satisfying some causality requirements. Here $\eta_i(t)$ represents the precise information acquired by agent i on $(u_1, \ldots, u_n, w)$ by the time point $t \in \Xi$.

(6) *Policy* (strategy) *variables*, $\gamma_i \in \Gamma_i$, $i \in \mathbb{N}$, where Γ_i is known as the policy space of agent i. It is the collection of all appropriately chosen mappings $\gamma_i : \eta \to u_i$.

(7) *Objective* (loss) *functionals*, L_i, $i \in \mathbb{N}$, which map $(\underset{i \in \mathbb{N}}{X} \mathbb{U}_i) X \mathbb{D}$ into the real line; L_i stands for the loss (or minus payoff) accrued to agent i as a result of actions taken by all the agents, and as a function of the realized value of $w \in \mathbb{D}$. These loss functionals can be transformed into cost functionals defined on the product policy space $\underset{i \in \mathbb{N}}{X} \Gamma_i$, using the relationship

$$J_i(\gamma_1, \ldots, \gamma_n) = \underset{w}{E}\{L_i(\gamma_1(\eta_1(u,w)), \ldots, \gamma_n(\eta_n(u,w)), w)\}$$

where the expectation is taken with respect to the statistics of w, defined on $\mathbb{D}$. The functionals J_i thus generated are also called objective functionals, and they correspond to the so-called *strategic* (synonymously, *normal*) form of the dynamic game.

Even though the above seven ingredients provide a complete description of an n-agent stochastic dynamic game (other than the solution concept), it has been common practice (for several reasons) to introduce an intermediate variable x, called the *state variable*, which carries some aggregate information concerning the evolution of the decision process. Such a (stochastic) variable would be defined by

$$x(t) = T_t(x(\tau), u_1(\tau), \ldots, u_n(\tau); w(\tau); \tau < t)$$

$$t > 0 \quad , \quad x(0) = x_0$$

where T is some causal (nonanticipative) mapping from $(\underset{i \in \mathbb{N}}{X} \mathbb{U}_i) X \bar{\mathbb{X}} X \mathbb{D}$ into $\bar{\mathbb{X}}$, where the latter is the so-called "state space" where the state x belongs. The variable x_0 is the initial state, which may be taken, in this general formulation, as a component of w. For each fixed t, $x(t) \in X$, where X may also be called the state space. Associated with x, is another (stochastic) variable $y_i \in \mathbb{Y}_i$, $i \in \mathbb{N}$, which is the measurement (observation) process of agent i; it is given by

$$y_i(t) = S_{it}(x(\tau), u_1(\tau), \ldots, u_n(\tau); w(\tau); \tau \leq t) \quad , \quad i \in \mathbb{N}$$

where $S_i : (\underset{i \in \mathbb{N}}{X} \mathbb{U}_i) X \bar{\mathbb{X}} X \mathbb{D} \to \bar{\mathbb{X}}$ is some nonanticipatory mapping for each $i \in \mathbb{N}$. Let us denote by Y_i the set in which $y_i(t)$ takes values. Now, the information acquired by agent i can be expressed in terms of the measurement process $y = (y_1, \ldots, y_n)$:

$$\eta_i(t) = C_{it}(y(\tau); \tau \leq t)$$

where C_i is some appropriate nonanticipative mapping. The intermediate variable x introduced above generally admits a physical or economic interpretation, and thus it becomes more appropriate to define the loss functionals, say $\tilde{L}_i$, $i \in \mathbb{N}$, on $(\underset{i \in \mathbb{N}}{X} \mathbb{U}_i) X \bar{\mathbb{X}} X \mathbb{D}$; however, since x can in turn be expressed in terms of (u,w), this can again be transformed into functionals L_i, $i \in \mathbb{N}$, defined on the original space. The *strategic* (normal) *form* of the dynamic game, which involves only the strategy spaces $\Gamma_1, \ldots, \Gamma_n$ and the functionals $J_1, \ldots, J_n$, therefore remains intact, regardless of whether $\tilde{L}_i$ or L_i are chosen as loss functionals of the so-called *extensive form* of the game.

Two special cases of this general formulation are Differential and Difference Games with Perfect State Information (PSI):

<u>Differential Game with PSI</u>

$$\Xi = [0, t_f]$$

$$T_t(\alpha(\tau); \tau < t) = \int_0^t \alpha(\tau) d\tau$$

$$S_{it}(x(\tau), u_1(\tau), \ldots, u_n(\tau); \tau \leq t) = x(t)$$

$$L_i(\alpha) = \int_0^{t_f} f_\tau^i(\alpha(\tau)) d\tau, \text{ for some } f^i.$$

Difference Game with PSI

$$\Xi = \{0,1,2,\ldots,t_f\}$$

$$T_t(\alpha(\tau),\tau < t) = \alpha(t-1)$$

$$S_{it}(x(\tau),u_1(\tau),\ldots,u_n(\tau);w(\tau);\tau \leq t) = x(t)$$

$$L_i(\alpha) = \sum_{t\in\Xi} f_t^i(\alpha(t))$$

In each case the probability measure on $\mathbb{D}$ associated with the random variable w is taken to be one-point, this being common information to all agents. Hence, here we are in the realm of so-called *deterministic games*, for which three types of information structures (IS) have been prevelant in the literature during the last twenty or so years. These are:

1. $C_{it}(y(\tau);\tau \leq t) = x(t)$: closed-loop no-memory or feedback IS.
2. $C_{it}(y(\tau);\tau \leq t) = \{x(\tau),\tau \leq t\}$: closed-loop IS (this incorporates memory).
3. $C_{it}(y(\tau);\tau \leq t) = x_0$: open-loop IS.

Various extensions of these are possible in the case of stochastic dynamic games (where w is a "nonsingular" random variable or process): For stochastic games with perfect state measurements, the above three IS's are still valid, and arise frequently in applications. Other infor mation structures involve explicitly the noisy measurements y_i, $i\in\mathbb{N}$, and are classified according to whether the measurements are shared with other agents or not. Some examples are:

4. $C_{it}(y(\tau);\tau \leq t) = \{y(\tau),\tau \leq t\}$: closed-loop IS with memory and total sharing of measurements.
5. $C_{it}(y(\tau);\tau \leq t) = y(t)$: noisy feedback IS with total sharing of measurements.
6. $C_{it}(y(\tau);\tau \leq t) = y_i(t)$: noisy feedback IS with no sharing of measurements.
7. $C_{it}(y(\tau);\tau \leq t) = \{y_i(\tau),\tau \leq t;y_j(\tau),\tau \leq t-\varepsilon,j\in\mathbb{N},j\neq i\}$: closed-loop IS with memory and sharing of measurements with a delay of ε time units.

2. Solution Concepts

There are two categories of solution concepts applicable to dynamic games; one of these uses the normal form of the game whereas the other one uses the extensive form. Recall that the normal (strategic) form suppresses all the informational aspects of the game and is characterized completely by policy spaces and cost functionals:

$$(\gamma_1, \gamma_2, \ldots, \gamma_n) \in \Gamma_1 \mathrm{X} \ldots \mathrm{X} \Gamma_n$$

$$J_i(\gamma_1, \ldots, \gamma_n) \quad , \quad i \in N \quad .$$

Hence, the solution concepts to be introduced for this form are valid irrespective of whether the game is static or dynamic, or whether it is deterministic or stochastic. We have basically three kinds of non-cooperative equilibrium solutions in the first category, as delineated below:

Solution Category 1: Normal Form Description

1. NASH EQUILIBRIUM: $(\gamma_1^*, \ldots, \gamma_n^*) \triangleq \gamma^* \in \underset{i \in N}{\mathrm{X}} \Gamma_i$

The policy n-tuple γ^* is in *Nash equilibrium* if (and only if)

$$\gamma_i^* = \arg \min_{\gamma_i \in \Gamma_i} J_i(\gamma^{i*}, \gamma_i)$$

where

$$(\gamma^{i*}, \gamma_i) \triangleq (\gamma_1^*, \ldots, \gamma_{i-1}^*, \gamma_i, \gamma_{i+1}^*, \ldots, \gamma_n^*)$$

A Nash equilibrium solution is <u>stable</u> if it can be obtained through an iterative procedure (in policy space) and regardless of what initial choice starts the iteration. One such recursion is

$$\begin{cases} \gamma_i^* = \lim_{k \to \infty} \gamma_i^{(k)} \quad , \quad \forall \gamma_i^{(0)} \in \Gamma_i \quad , \quad i \in N \\ \gamma_i^{(k+1)} = \arg \min_{\gamma_i \in \Gamma_i} J_i(\gamma_1^{(k)}, \ldots, \gamma_{i-1}^{(k)}, \gamma_i, \gamma_{i+1}^{(k)}, \ldots, \gamma_n^{(k)}) \end{cases}$$

which corresponds to the scheme whereby agents update their policies simultaneously, using the most recent policies of other agents. Yet another scheme is

$$\begin{cases} \gamma_i^* = \lim_{k \to \infty} \gamma_i^{(k)} \quad , \quad \forall \gamma_i^{(0)} \in \Gamma_i \quad , \quad i \in N \\ \gamma_i^{(k+1)} = \arg \min_{\gamma_i \in \Gamma_i} J_i(\gamma_1^{(k+1)}, \ldots, \gamma_{i-1}^{(k+1)}, \gamma_i, \gamma_{i+1}^{(k)}, \ldots, \gamma_n^{(k)}) \end{cases}$$

where the agents update their policies in a predetermined order, using the most recent available information.

Remark 1: A Nash equilibrium solution is <u>unique</u> if it is stable accord-