Advances in
CONTROL SYSTEMS
Theory and Applications

Volume 5

ADVANCES IN

CONTROL SYSTEMS

THEORY AND APPLICATIONS

Edited by

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DEPARTMENT OF ENGINEERING
UNIVERSITY OF CALIFORNIA
LOS ANGELES, CALIFORNIA

VOLUME 5 1967

ACADEMIC PRESS New York and London

ACADEMIC PRESS INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS INC. (LONDON) LTD.
Berkeley Square House, London W.1

LIBRARY OF CONGRESS CATALOG CARD NUMBER: 64–8027

PRINTED IN THE UNITED STATES OF AMERICA

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Preface

The fifth volume of *Advances in Control Systems* continues in the purpose of this serial publication to bring together diverse information on important progress in the field of control and systems theory and applications, as achieved and discussed by leading contributors.

The problem of the optimal control of a system for which accurate knowledge of its dynamic characteristics exists has received a great deal of attention over the past few years. An aspect of the control problem which has received far less attention is that of optimizing system performance in the absence of any a priori knowledge of the system or plant dynamic characteristics. If the dynamics of the system are unknown and vary from time to time in an unpredictable manner, it is evident that some type of identification scheme must be incorporated into the system operation in order to achieve and maintain optimal performance in any meaningful sense. The techniques which have been proposed to cope with this problem, often referred to as an adaptive or adaptive optimal control problem, usually require some a priori information of the plant dynamic characteristics (e.g., the order and form of the differential equations may be known though some of the coefficients may be unknown). Such techniques are useful when the order and form of the system's differential equations are known. On the other hand, there exist many practical situations in which the dynamic characteristics of the system are too complex to permit a representation in any reasonably simple form. A method for optimizing in some sense the system performance which does not require the complete identification of the system dynamics and which does not presume a knowledge of the order or form of the system differential equations is clearly desirable in such cases. The first contribution in this volume by A. E. Pearson deals with this problem and presents some rather basic techniques for it, many of which are original with Pearson.

Fundamental necessary and sufficient conditions in the calculus of variations, basic to the optimal control problem, have been under investigation by mathematicians for many decades, in fact, for hundreds of years. Algorithms for the solution of the optimization problem have been under investigation for many years, but it is only in the last five or ten years that this extremely important area has received intensive effort. There are several fundamental approaches to algorithms for the solution of the optimization problem or what we may also refer to as the trajectory optimization or reference control input problem, and several of these have been treated in earlier volumes of this series. The contribution by D. K. Scharmack presents one of the most important efforts to date on the initial value iteration algorithmic approach to the solution of the optimization A further notable feature of this contribution is the applicatic approach to several substantive illustrative problems.

One of the basic problems in control and systems theory in gen determination of the set of states that can be reached at time T gi scribed class of admissible control functions and an initial state at initial time for a nonlinear system. This problem is treate contribution by D. R. Snow, and it is referred to there as the p determining the T-reachable region. A related problem, also t Snow, is the determination of the T-controllable region describ In developing the results presented in this contribution Snow has many of the classical results of the calculus of variations and H Jacobi theory to optimal control problems through the use of Carat unifying approach. Although some of these extensions have been in the literature during the past three or four years, they are here in a unified form for the first time.

The contribution by J. R. Fisher presents a number of rath mental results in optimal nonlinear filtering. The differential of a system which, when driven by a noise-corrupted measureme will generate either the conditional probability density or the c characteristic function of the state vector of a nonlinear system cor in time are derived. It is shown that certain general classes of ide problems are results of this theory. It is also shown that the opt linear, non-Gaussian prediction problem is simply a two-stage a of the theory developed in this contribution. A rather comp survey of earlier work and contributions is presented here also.

An important application area of control and systems the optimal control of nuclear reactors. The contribution by D. M presents a rather comprehensive treatment of some of the fu techniques possible here. A mathematical model of the system trolled is developed. Methods of estimating the effects of spatia on system stability are presented. Analytical design techniques feedback control systems are presented. Questions of controll distributed parameter systems are examined here also.

This volume closes with a contribution by J. McIntyre and B. P on optimal control with bounds on the state variables. There practical instances of where such problems occur. There have be ous published results in the literature exploring various aspe important problem. This contribution reviews many of these r presents an over-all view of the status of the techniques in this fi

July, 1967 C. T.

Contents of Previous Volumes

Contents

Adaptive Optimal Steady State Control of Nonlinear Systems

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I. Introduction

Optimization problems in the theory of control systems have received considerable attention in recent years. The mathematical techniques which have been applied to optimal control problems depend upon an accurate knowledge of the plant dynamic characteristics. Given the differential equations of the plant to be controlled, a performance functional to be minimized over a certain set of control functions, and various constraints pertinent to the problem at hand, the mathematical techniques facilitate the determination of the necessary conditions for optimal system performance (*1–3*).

An aspect of the control problem which has received far less attention is that of optimizing the system performance in the absence of any *a priori* knowledge of the plant dynamic characteristics. If the plant dynamics are unknown and vary from time to time in an unpredictable manner, it is evident that some type of identification scheme must be incorporated into the system operation in order to achieve and maintain optimal performance in any meaningful sense. The techniques which have been proposed to cope with this problem, often referred to as an adaptive or adaptive-optimal control problem, usually require some *a priori* information of the plant dynamic characteristics; e.g., the order and form of the differential equations may be known though some of the coefficients may be unknown [see (*4*) for a review of adaptive control techniques]. Such techniques are useful when the order and form of the plant differential equations are known. On the other hand, there exist many practical situations in which the plant dynamic characteristics are too complex to permit a representation in any reasonably simple form. A method for optimizing (in some sense) the system performance which does not require the complete identification of the plant dynamics and which does not presume a knowledge of the order or form of the plant differential equations is clearly desirable in such cases.

An adaptive optimal control scheme which does not rely upon *a priori* knowledge of the order or form of the plant differential equations is the Draper and Li extremum or peak holding controller (*5*, *6*). This approach depends upon the existence of a plant output variable, or collection of variables, possessing an unknown maximum (or minimum) which may be slowly varying in time. The performance criterion in this case is to maintain the plant output as close to the extremum value as possible. The peak holding controller applies a slowly changing input signal (slow compared to the longest time constant of the plant) in a fixed direction until it is observed that the pertinent plant output variable has passed through its extremal value, whereupon the controller switches the direction of the input signal to force the plant variable back through its extremum. Assuming that the transient properties of the plant (the plant dynamic characteristics aside from the unknown extremal value) remain reasonably fixed in time, the peak holding controller continually adapts its operation to follow the slowly varying extremum of the output variable.

In 1961, Kulikowski (*7*) introduced another approach to the particular adaptive optimal control problem considered by Draper and Li. In order to remove the assumption that the transient properties of the plant remain essentially fixed, Kulikowski proposed alternating periods of identification with periods of optimization in determining the form of the input signal. This is in contrast with the peak holding controller in which the saw-toothed nature of the input signal is specified in advance. Kulikowski introduced a greater degree of mathematical formalism into the problem by focusing attention upon minimizing the performance functional

$$P(\mathbf{u}) = \lambda \int_0^T u^2(t)\,dt - \int_0^T y(t)\,dt \tag{1}$$

where $y(t)$ is the output variable which possesses an unknown maximum value and $u(t)$ is the input to the plant. (The notation $\mathbf{z}$ is used to indicate the time function segment $\mathbf{z} = \{(t, z(t)) | 0 < t < T\}$.) The motivation for specifying the functional (1) stems from the desire to maximize the entire transient behavior of the plant over a fixed interval $(0, T)$ and, at the same time, to minimize the cost for control,

$$\int_0^T u^2(t)\,dt$$

as weighted by the constant $\lambda > 0$.

In brief, Kulikowski's approach involves carrying out a certain type of identification at each step in the construction of a sequence of input functions $\{\mathbf{u}_n\}$, $n = 1, 2, \ldots$, which under appropriate conditions will converge to an optimum input function $\mathbf{u}^*$ satisfying $P(\mathbf{u}^*) \leqslant P(\mathbf{u})$. The relationship between the plant input and output functions is denoted symbolically by an operator A, $y(t) = A(\mathbf{u})$, $0 < t < T$, which maps elements $\mathbf{u}$ from a space of input functions $\mathscr{U}$ into elements $\mathbf{y}$ belonging to a space of response functions $\mathscr{Y}$. The amount and type of identification which is to be performed with respect to each element $\mathbf{u}_n$ is based upon the information needed to compute $\mathbf{u}_{n+1}$ in an iterative minimization of (1). Kulikowski showed that the identification at each step involves the measurement of output elements $A(\mathbf{u}_n + \epsilon\mathbf{v})$ corresponding to various known input elements of the form $\mathbf{u}_n + \epsilon\mathbf{v}$ where ϵ is a small parameter.

Although the assumption was made in (*7*) that the plant operator A possess certain symmetry properties which are rarely upheld in practice, this assumption was removed in subsequent work (*8–10*). One of the most important results of Kulikowski's original paper concerns the identification requirement that output elements $\mathbf{y} = A(\mathbf{u}_n + \epsilon\mathbf{v})$ be measured as time function segments rather than requiring explicit or detailed knowledge of the operator A. Thus, in practice, the dynamic characteristics of the plant may be very complex since there is no need to assume linearity or any specific form of plant differential equations. The only assumptions needed concerning the plant dynamics are that they vary slowly in time relative to the time spent in constructing the sequence $\{\mathbf{u}_n\}$ and that the plant operator A possess a sufficient degree of smoothness to guarantee the existence of the gradient of the functional $P(\mathbf{u})$.

In subsequent papers (*8–10*) Kulikowski expanded the above approach and paved the way for further investigation (*11–14*). Pearson and Sarachik (*15*) later showed that the memory of the plant influenced the mathematical formulation of Kulikowski's approach. It was shown that this influence could be taken into account by requiring that the plant be in the proper steady state operation before measuring output elements $A(\mathbf{u}_n + \epsilon\mathbf{v})$ during the identification procedure. A somewhat different approach to basically the same class of adaptive optimal control problems has been reported by Zaborszky and Humphrey (*16*).

This chapter extends the work which has been reported on Kulikowski's approach to adaptive optimal control problems. The formulation has been modified to include optimizing the total amount of input accumulation,

$$\int_{-\infty}^{t_0} u(t)\,dt$$

in addition to optimizing the form of the periodic input signal, in the case of plants possessing infinite memory, i.e., plants possessing pure integrators within their structure. The class of adaptive problems has been extended to include the optimization of system performance when the desired response of the system is a periodic function of time. The formulation of the adaptive optimal control problem is presented in Section II in cognizance of the practical considerations for identification, Section IV, and the computational aspects of achieving optimum system operation, Section III. Sections II–IV are concerned only with single input-single output plants; however, the extension to the general case of multivariable nonlinear plants is indicated in the final section.

II. Formulation

In general terms the problem of concern here is the optimization of the steady state performance of a plant whose dynamic characteristics are unknown and slowly varying with time. No precise meaning will be attached to the phrase "slowly varying with time" although it will be clear that the plant dynamic characteristics must remain essentially fixed during the time required to carry out the identification and computations for the adaptive optimizing procedure. Emphasis will be placed upon determining a periodic input signal such that the plant output is forced into a steady state periodic signal and the resulting over-all system performance is optimum. The meaning of optimum steady state system performance is to be interpreted as the minimization of a performance index which evaluates the system performance over one period of steady state operation.

A. Statement of the Problem

It is assumed that there is a performance or cost functional of the general form

$$P = \int_{\substack{\text{one}\\\text{period}}} G[u(t), y(t), y_d(t)]\,dt \tag{2}$$

which serves to evaluate the output behavior of the plant and to assess the cost of operating the plant over one period of steady state operation. The plant input and output variables are denoted by $u(t)$ and $y(t)$, respectively, and $y_d(t)$ is a desired output. The function $G(u, y, y_d)$ is assumed to be twice differentiable in each of its arguments.

The desired output of the plant, $y_d(t)$, is assumed to be either a constant or a periodic function of time with period T_d. The object is to find a control function $\mathbf{u}^* = \{[t, u^*(t)] | 0 < t < T\}$ in a space of admissible control functions $\mathscr{U}$, where T is a submultiple of T_d ($T_d = \alpha T$, α = an integer), such that a periodic input signal constructed from suitable repetitions of the element $\mathbf{u}^*$ forces the plant into its optimum steady state operation. The choice of the integer α and the value of $T = 1/\alpha T_d$ depends upon the energy storage properties of the plant as will be discussed in Part B of this section. In addition to finding the best control element $\mathbf{u}^* \in \mathscr{U}$ to be used in forming a periodic input signal, it is necessary to determine the optimum level of input accumulation,

$$s^* = \int_{-\infty}^{t_0} u(\tau)\, d\tau,$$

which is present at the start of each period of steady state operation in the case of plants possessing pure integrators within their structure.

The problem of optimizing the steady state performance of a plant with unknown dynamic characteristics will be approached utilizing a step-by-step optimization and identification procedure. Mathematically, the problem will be viewed in terms of constructing a sequence of elements $\{\mathbf{u}_n\}$, $n = 1, 2, \ldots$, in the case of finite memory plants, or a sequence of ordered pairs $\{\mathbf{u}_n, s_n\}$ in the case of plants possessing infinite memory, such that steady state optimal performance is achieved in the limit as $n \to \infty$. The space of functions $\mathscr{U}$ from which the elements $\mathbf{u}_n$ are drawn in forming a periodic input signal is assumed to be an unbounded space of square integrable functions defined on the interval $(0, T)$. Physically this assumption means that there is sufficient fuel, energy, or power to accomplish the control objectives. Amplitude constraints, such as would be caused by a valve or saturating amplifier, can be included in this formulation if the device responsible for the saturation can be approximated by a twice differentiable function, e.g., $u = \tan^{-1} ku'$. The saturating device can be included as part of the nonlinear plant operator with the input to the device a member of an unbounded space.

The identification of certain essential aspects of the plant dynamic characteristics is to be carried out with respect to each element $\mathbf{u}_n$, or pair $(\mathbf{u}_n, s_n)$, in order to compute the succeeding element $\mathbf{u}_{n+1}$, or pair $(\mathbf{u}_{n+1}, s_{n+1})$, in an iterative minimization of the functional (2). The dynamic characteristics of the plant in steady state operation will be represented symbolically by an operator A which in the case of finite memory plants maps elements $\mathbf{u}$ from the space $\mathscr{U}$ into corresponding output elements $\mathbf{y}$ in a space of output functions $\mathscr{Y}$. In the case of plants possessing infinite memory, the steady state operator A maps ordered pairs of elements $(\mathbf{u}, s)$ from the product space $\mathscr{U} \times \mathscr{R}$ (where $\mathscr{R}$ denotes the set of real numbers) into a function space $\mathscr{Y}$ of response time function segments $\mathbf{y}$. Although it is not necessary to assume any specific knowlege about the plant dynamic characteristics, it is assumed that the plant possesses a finite settling time and that it can be forced into a steady state operation with respect to an arbitrary input element $\mathbf{u} \in \mathscr{U}$ which comprises a suitable periodic input signal.

B. Evaluation of System Performance

The first question to be considered in formulating the adaptive optimal steady state control problem is the manner in which an arbitrary input element $\mathbf{u} \in \mathscr{U}$ should be used in forming a periodic input signal such that the plant is forced into the proper steady state operation. This question is equivalent to establishing a basis for evaluating the system performance in steady state operation which guarantees that the periodic output of the plant is due only to the input element $\mathbf{u}$ in question. It is clear that a distinction must be made between plants which possess a finite memory, referred to here as type-zero plants, and plants possessing pure integrators within their structure which are capable of storing energy indefinitely. In either case it is necessary to assume that the plant possesses a finite settling time and that the output of the plant can be forced into a periodic signal of the same fundamental period as the input, for an arbitrary element $\mathbf{u} \in \mathscr{U}$ to be used in forming the periodic input signal.

1. Type-Zero Plants

If the plant does not possess any pure integrators within its structure which are capable of storing energy indefinitely, it is clear that simple repetitions of an input element $\mathbf{u}$ over successive time intervals of length T will eventually force the output into a periodic signal of the same fundamental period. After such steady state operation has been achieved, the output $\mathbf{y}$ measured over one period in phase with the input can be considered as the image element under a map A of the input element $\mathbf{u}$,

$$\mathbf{y} = A(\mathbf{u}) \tag{3}$$

where A is the steady state plant operator (see Fig. 1). The steady state performance of the system is appropriately evaluated with respect to an arbitrary input element $\mathbf{u} \in \mathscr{U}$ via the functional

$$P(\mathbf{u}) = \int_0^T G(\mathbf{u}, \mathbf{y}, \mathbf{y}_d)\, dt \tag{4}$$

where T is chosen such that

$$T = T_d$$

and $\mathbf{y} = A(\mathbf{u})$ is measured after steady state operation has been achieved.

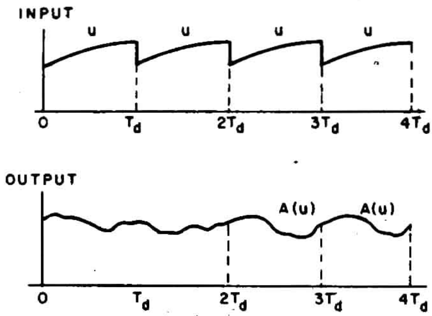

FIG. 1. Procedure for establishing steady state operation of type-zero plants.

If the desired output y_d is a constant ($T_d = \infty$), it will be assumed that T is chosen independently of the optimization procedure. It is natural in this case that T should be chosen larger than the anticipated plant settling time if the latter is known.

There are other ways of constructing a periodic input signal from an arbitrary element $\mathbf{u}$ which would suffice to ensure that the ensuing periodic output of a type-zero plant depends only on the element $\mathbf{u}$. Consider, for example, choosing the length T of the time function segment $\mathbf{u}$ according to

$$T = \tfrac{1}{2}T_d \tag{5}$$

and forming a periodic input signal of period T_d such that the sequence

$$u(t) = \begin{cases} \mathbf{u} & \text{for} \quad 0 < t < \tfrac{1}{2}T_d \\ \mathbf{0} & \text{for} \quad \tfrac{1}{2}T_d < t < T \end{cases} \tag{6}$$

is repeated over each period (see Fig. 2). The resulting steady state output can be represented by the two time function segments $\mathbf{y}_1$ and $\mathbf{y}_2$, each of length T, such that the steady state plant operator is characterized by the two operators A_1 and A_2 as defined by

$$\mathbf{y}_1 = A_1(\mathbf{u}), \qquad \mathbf{y}_2 = A_2(\mathbf{u}) \tag{7}$$