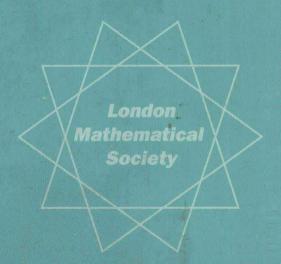
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Edited by lan F. Blake, Gadiel Seroussi, and Nigel P. Smart



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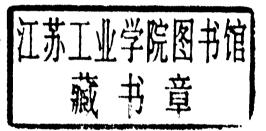
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- 182
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- 227
- 228
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- 281
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Preface

It is now more than five years since we started working on the book *Elliptic Curves in Cryptography* and more than four years since it was published. We therefore thought it was time to update the book since a lot has happened in the intervening years. However, it soon became apparent that a simple update would not be sufficient since so much has been developed in this area. We therefore decided to develop a second volume by inviting leading experts to discuss issues which have arisen.

Highlights in the intervening years which we cover in this volume include:

Provable Security. There has been considerable work in the last few years on proving various practical encryption and signature schemes secure. In this new volume we will examine the proofs for the ECDSA signature scheme and the ECIES encryption scheme.

Side-Channel Analysis. The use of power and timing analysis against cryptographic tokens, such as smart cards, is particularly relevant to elliptic curves since elliptic curves are meant to be particularly suited to the constrained environment of smart cards. We shall describe what side-channel analysis is and how one can use properties of elliptic curves to defend against it.

Point Counting. In 1999 the only method for computing the group order of an elliptic curve was the Schoof-Elkies-Atkin algorithm. However, for curves over fields of small characteristic we now have the far more efficient Satoh method, which in characteristic two can be further simplified into the AGM-based method of Mestre. We shall describe these improvements in this book.

Weil Descent. Following a talk by Frey in 1999, there has been considerable work on showing how Weil descent can be used to break certain elliptic curve systems defined over "composite fields" of characteristic two.

Pairing-Based Cryptography. The use of the Weil and Tate pairings was until recently confined to breaking elliptic curve protocols. But since the advent of Joux's tripartite Diffie–Hellman protocol there has been an interest in using pairings on elliptic curves to construct protocols which cannot be implemented in another way. The most spectacular example of this is the

x PREFACE

identity-based encryption algorithm of Boneh and Franklin. We describe not only these protocols but how these pairings can be efficiently implemented.

As one can see once again, the breadth of subjects we cover will be of interest to a wide audience, including mathematicians, computer scientists and engineers. Once again we also do not try to make the entire book relevant to all audiences at once but trust that, whatever your interests, you can find something of relevance within these pages.

The overall style and notation of the first book is retained, and we have tried to ensure that our experts have coordinated what they write to ensure a coherent account across chapters.

Ian Blake Gadiel Seroussi Nigel Smart

Abbreviations and Standard Notation

Abbreviations

The following abbreviations of standard phrases are used throughout the book:

AES	Advanced Encryption Standard
AGM	Arithmetic Geometric Mean
BDH	Bilinear Diffie-Hellman problem
BSGS	Baby Step/Giant Step method
CA	Certification Authority
CCA	Chosen Ciphertext Attack
CDH	Computational Diffie-Hellman problem
$^{\mathrm{CM}}$	Complex Multiplication
CPA	Chosen Plaintext Attack
DBDH	Decision Bilinear Diffie-Hellman problem
DDH	Decision Diffie-Hellman problem
DEM	Data Encapsulation Mechanism
DHAES	Diffie-Hellman Augmented Encryption Scheme
DHIES	Diffie-Hellman Integrated Encryption Scheme
DHP	Diffie-Hellman Problem
DLP	Discrete Logarithm Problem
DPA	Differential Power Analysis
DSA	Digital Signature Algorithm
DSS	Digital Signature Standard
ECDDH	Elliptic Curve Decision Diffie-Hellman problem
ECDH	Elliptic Curve Diffie-Hellman protocol
ECDHP	Elliptic Curve Diffie-Hellman Problem
ECDLP	Elliptic Curve Discrete Logarithm Problem
ECDSA	Elliptic Curve Digital Signature Algorithm
ECIES	Elliptic Curve Integrated Encryption Scheme
ECMQV	Elliptic Curve Menezes-Qu-Vanstone protocol
GHS	Gaudry-Hess-Smart attack
GRH	Generalized Riemann Hypothesis
HCDLP	Hyperelliptic Curve Discrete Logarithm Problem
HIBE	Hierarchical Identity-Based Encryption

IBE Identity-Based Encryption

IBSE Identity-Based Sign and Encryption

ILA Information Leakage Analysis

KDF Key Derivation Function KDS Key Distribution System

KEM Key Encapsulation Mechanism MAC Message Authentication Code

MOV Menezes-Okamoto-Vanstone attack

NIKDS Non-Interactive Key Distribution System

PKI Public Key Infrastructure

RSA Rivest-Shamir-Adleman encryption scheme

SCA Side Channel Analysis

SEA Schoof–Elkies–Atkin algorithm

SHA Secure Hash Algorithm SPA Simple Power Analysis

SSCA Simple Side-Channel Attack

TA Trusted Authority

Standard notation

The following standard notation is used throughout the book, often without further definition. Other notation is defined locally near its first use.

Basic Notation	
$\mathbb{Z},\mathbb{Q},\mathbb{R},\mathbb{C}$	integers, rationals, reals and complex numbers
$\mathbb{Z}_{>k}$	integers greater than k ; similarly for \geq , $<$, \leq
$\mathbb{Z}/n\mathbb{Z}$	integers modulo n
#S	cardinality of the set S
gcd(f,g), lcm(f,g)	GCD, LCM of f and g
$\deg(f)$	degree of a polynomial f
$\phi_{ m Eul}$	Euler totient function
$\left(\frac{\cdot}{p}\right)$	Legendre symbol
$\log_b x$	logarithm to base b of x ; natural log if b omitted
O(f(n))	function $g(n)$ such that $ g(n) \le c f(n) $ for some constant $c > 0$ and all sufficiently large n
o(f(n))	function $g(n)$ such that $\lim_{n\to\infty} (g(n)/f(n)) = 0$
\mathbb{P}^n	projective space

Group/Field Theoretic Notation

\mathbb{F}_q	nnite neld with q elements
K^*, K^+, \overline{K}	for a field K , the multiplicative group, additive group
	and algebraic closure, respectively
char(K)	characteristic of K
$\langle g \rangle$	cyclic group generated by g
$\operatorname{ord}(g)$	order of an element g in a group
$\operatorname{Aut}(G)$	automorphism group of G
$\mathbb{Z}_p,\mathbb{Q}_p$	p-adic integers and numbers, respectively
$\operatorname{Tr}_{q p}(x)$	trace of $x \in \mathbb{F}_q$ over \mathbb{F}_p , $q = p^n$
$oldsymbol{\mu}_n$	nth roots of unity
$N_{L/K}$	norm map

Function Field Notation deg(D) degree of a divisor

408(2)	degree of a drive
(f)	divisor of a function
f(D)	function evaluated at a divisor
~	equivalence of divisors
$\operatorname{ord}_P(f)$	multiplicity of a function at a point

Galois Theory Notation

Gal(K/F)	Galois group of K over F
$\sigma(P)$	Galois conjugation of point P by σ
f^{σ}	Galois conjugation of coefficients of function f by σ

Curve Theoretic Notation elliptic curve (equation) coordinates of the point P (x_P, y_P) the x-coordinate of the point Px(P)y(P)the y-coordinate of the point Pgroup of K-rational points on EE(K)multiplication-by-m map applied to the point P[m]Pgroup of m-torsion points on the elliptic curve EE[m] $\operatorname{End}(E)$ endormorphism ring of E0 point at infinity (on an elliptic curve) Weierstraß 'pay' function 0 Frobenius map φ $\langle P, Q \rangle_n$ Tate pairing of P and QWeil pairing of P and Q $e_n(P,Q)$ e(P,Q)pairing of P and Q $\hat{e}(P,Q)$ modified pairing of P and QTr(P)trace map \mathcal{T} trace zero subgroup

Authors

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Contents

Preface	page ix
Abbreviations and Standard Notation	xi
Authors	xv
Part 1. Protocols	
Chapter I. Elliptic Curve Based Protocols	
N.P. Smart	3
I.1. Introduction	3
I.2. ECDSA	4
I.3. ECDH/ECMQV	8
I.4. ECIES	12
I.5. Other Considerations	18
Chapter II. On the Provable Security of ECDSA	
D. Brown	21
II.1. Introduction	21
II.2. Definitions and Conditions	23
II.3. Provable Security Results	32
II.4. Proof Sketches	33
II.5. Further Discussion	36
Chapter III. Proofs of Security for ECIES	
A.W. Dent	41
III.1. Definitions and Preliminaries	42
III.2. Security Proofs for ECIES	50
III.3. Other Attacks Against ECIES	58
III.4. ECIES-KEM	61

vi Contents

Part 2. Implementation Techniques	
Chapter IV. Side-Channel Analysis	
E. Oswald	69
IV.1. Cryptographic Hardware	70
IV.2. Active Attacks	71
IV.3. Passive Attacks	72
IV.4. Simple SCA Attacks on Point Multiplications	77
IV.5. Differential SCA Attacks on Point Multiplications	84
Chapter V. Defences Against Side-Channel Analysis	
M. Joye	87
V.1. Introduction	87
V.2. Indistinguishable Point Addition Formulæ	88
V.3. Regular Point Multiplication Algorithms	93
V.4. Base-Point Randomization Techniques	97
V.5. Multiplier Randomization Techniques	98
V.6. Preventing Side-Channel Analysis	100
Part 3. Mathematical Foundations	
Chapter VI. Advances in Point Counting	
F. Vercauteren	103
VI.1. p-adic Fields and Extensions	104
VI.2. Satoh's Algorithm	105
VI.3. Arithmetic Geometric Mean	115
VI.4. Generalized Newton Iteration	121
VI.5. Norm Computation	128
VI.6. Concluding Remarks	132
Chapter VII. Hyperelliptic Curves and the HCDLP	
P. Gaudry	133
VII.1. Generalities on Hyperelliptic Curves	133
VII.2. Algorithms for Computing the Group Law	136
VII.3. Classical Algorithms for HCDLP	140
VII.4. Smooth Divisors	142
VII.5. Index-Calculus Algorithm for Hyperelliptic Curves	144
VII.6. Complexity Analysis VII.7. Practical Considerations	146
	149
Chapter VIII. Weil Descent Attacks	151
F. Hess	151
VIII.1. Introduction – the Weil Descent Methodology VIII.2. The GHS Attack	151
VIII.2. The GHS Attack Using Isogenies	153

Contents	vii
VIII.4. Summary of Practical Implications	
VIII.5. Further Topics	175
Part 4. Pairing Based Techniques	
Chapter IX. Pairings	
S. Galbraith	183
IX.1. Bilinear Pairings	183
IX.2. Divisors and Weil Reciprocity	184
IX.3. Definition of the Tate Pairing	185
IX.4. Properties of the Tate Pairing	187
IX.5. The Tate Pairing over Finite Fields	189
IX.6. The Weil Pairing	191
IX.7. Non-degeneracy, Self-pairings and Distortion Maps	192
IX.8. Computing the Tate Pairing Using Miller's Algorithm	196
IX.9. The MOV/Frey–Rück Attack on the ECDLP	197
IX.10. Supersingular Elliptic Curves	198
IX.11. Applications and Computational Problems from Pairings	201
IX.12. Parameter Sizes and Implementation Considerations	203
IX.13. Suitable Supersingular Elliptic Curves	204
IX.14. Efficient Computation of the Tate Pairing	205
IX.15. Using Ordinary Curves	208
Appendix: Proof of Weil Reciprocity	212
Chapter X. Cryptography from Pairings	
K.G. Paterson	215
X.1. Introduction	215
X.2. Key Distribution Schemes	218
X.3. Identity-Based Encryption	221
X.4. Signature Schemes	228
X.5. Hierarchical Identity-Based Cryptography and Related Topics	235
X.6. More Key Agreement Protocols	240
X.7. Applications and Infrastructures	242
X.8. Concluding Remarks	250
Bibliography	253
Summary of Major LNCS Proceedings	271
Author Index	273
Subject Index	
Subject index	277

Part 1

Protocols