



# *System Identification for Self-Adaptive Control*

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*To Hazel*

# *Preface*

This book is intended mainly to convey to the practical engineer the idea of using a self-adaptive control system in some particular application. The book is not aimed towards the mathematician or a high powered research application, but rather an attempt has been made to keep the concepts and the associated mathematics down to a practical level. In this way the practicing engineer or newly graduated student may be able to understand the concepts and problems associated with self-adaptive control with the minimum of effort.

The material presented in this book concerns most of the major parts that compromise the theory and design of self-adaptive control systems, with special emphasis on the identification problem. That is, the task of characterizing a particular system and its performance so that a self-improving loop may be closed around it, is considered one of the most important of the problems concerned, and so the major portion of this book dwells upon this subject.

The main identification scheme described is one that has evolved mainly in the past few years, and is presently being accepted as current practice especially in the Process Industries. This method is the method of Random Signal Testing and five of the eleven chapters are dedicated to this particular topic.

The book itself may be conveniently broken down into five parts. The first part, which is essentially the first chapter, describes the basic concepts involved in the design and application of the self-adaptive control system, highlighting the main problem areas.

The second part, which consists of chapters 2, 3, 4, 5, and 6 deals with the method of Random Signal Testing, and discusses in detail the advantages of using pseudo-random noise as excitation signals in the identification of processes, a step which must necessarily be the first in the design of a completely self-adaptive control system.

Part three, which comprises of chapters 7 through 9 inclusive—discusses the philosophy of automatic identification systems and describes in detail various schemes of achieving this goal. The methods presented are applicable to the identification of both multi input/output systems and multiparameter systems.

Part four consists of Chapter 10 alone, and describes perhaps the most important aspect in the design of a self-adaptive control system—the figure of merit and index of performance. Without such a definition of system performance quality, the optimum cannot be defined, and so the whole concept of self-adaptive control is defeated.

Chapter 11 comprises the fifth and last part of this book and describes various techniques of closing the self-adaptive loop once the identification problem has been solved and the figure of merit defined. A complete, closed loop, self-adaptive control system may thus be designed and become a practical engineering tool rather than an interesting research project.

An appendix is also included which is intended to tie in with Chapters 3 and 5, and lists in table form some of the more vital parameters that are associated with binary maximum length sequences.

Throughout the book—emphasis has been placed upon the use of a digital computer as the main control component. Since the trend during the past few years has been towards the increasing use of computers to control processes (e.g. direct digital control systems) it is suggested that the digital computer is the ideal tool for mechanizing the rather complex self-adaptive control system. Consequently, most of the design techniques described are oriented such that solutions by digital techniques are implied.

In presenting the material in this book, great care has been taken to minimize the background information expected from the reader, whether graduate student, or practicing engineer. The reader should encounter no difficulty in reading the subject matter if he is familiar with the elementary principles of Laplace and Fourier transforms, linear system theory, and basic continuous time control system theory.

During the process of writing this book, the author received many valuable suggestions from colleagues, associates and friends. They are too numerous to mention and thank individually, but special thanks are due to the staff of the Central Instrument Research Laboratory of I.C.I. Ltd., the Autonomics Division of N.P.L. and the many colleagues at Bell Aerospace. Finally the author owes a special debt to Hazel Davies and Kathy Fonte for their encouragement and assistance in all phases of the preparation of this book, and without whom it would never have been completed.

W. D. T. DAVIES

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## CHAPTER 1

# *The Concept of Self-Adaptive Control Systems*

### 1.1. The Definition of Adaptive Control Systems

There does not seem to be a generally accepted definition of a self-adaptive control system although several have been suggested. The most general definition suggested to date is probably Truxal's<sup>1</sup>: 'An adaptive system is any physical system which has been designed with an adaptive viewpoint'. Starting from this definition, one may argue that a system which is designed to have at all times a very small sensitivity to variations in plant characteristics so that the complex, overall system remains unchanged, is in a sense an adaptive system. This in turn implies that an adaptive control system is basically a feedback control system which automatically achieves a desired response in the presence of extreme changes in the controlled system's parameters and/or major external disturbances.

This viewpoint leads to the rather specific definition that will be used in the book, namely: 'A self-adaptive control system is one that provides a means of continuously measuring the system's performance in relation to a given criterion or Figure of Merit, and a means of automatically modifying the system's adjustable parameters by closed loop action so that the Figure of Merit may be satisfied'.<sup>2</sup>

This definition implies that a self-adaptive control system must react or adapt itself to changes in its environment. For this purpose, 'environment' can in turn be defined as that set of conditions which should ordinarily be taken into account by the system's designer. This set will obviously include such elements as the nature of the system's input signals, the noise against which the system should discriminate, and the values of the various factors upon which the parameters of the system may be dependent.

Such a system must therefore automatically measure the characteristics of the outputs of the control system, and of the process under control, and on the basis of these measurements, adjust the overall system towards some previously defined optimum conditions or characteristics. Such a system is illustrated in block diagram form in Figure 1.1. It should be

noted here that (a) the adaptive loop must be closed by comparison of the actual performance with the desired standard—this excludes any simple programmed or scheduled system, and (b) the signal computed by the adaptive controller must be used for parameter adjustments—thus eliminating such systems as simple integral control, etc.

It may therefore be seen that such a definition demands that three main functions be performed. These are—Identification—Decision—Modification. In any particular application it may be difficult to separate those

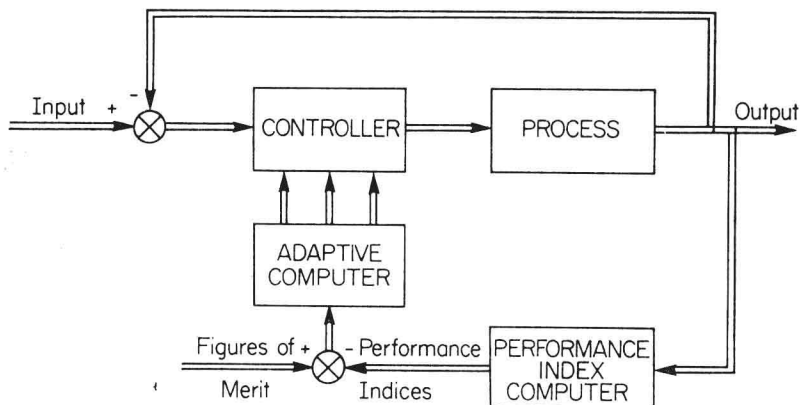


FIG. 1.1. A Typical Self-Adaptive System

parts of the overall system which are associated with each of these functions, especially with the first two, however all three are necessary for adaptive action.

The main part of this book will deal with the first of these requirements, i.e. Identification. This is the process by which the system is characterized, and is one of the limiting factors of today's adaptive technology. An engineer or system designer can carefully consider all the available information about a system and even define that system by its impulse response or transfer function; however identification generally implies, in addition to this, the formulation of an Index of Performance by which a quality level of system performance may be established. Quite often an adaptive control system also involves some special means of monitoring the system inputs and/or outputs, and a means of randomly or otherwise disturbing the system or the parameters so that deductions about the system performance can be made.

Optimal control systems may, based on the above definition of adaptive control systems, be considered as a subclass of adaptive control systems, a class in which the Index of Performance can be measured directly. This

interrelation between adaptive and optimal control may best be illustrated by considering as an example the control system for a complex chemical process. Since a primary requirement for such a system may be maximum throughput, it is necessary to choose certain desired control settings based on *a priori* data. It is therefore necessary to incorporate an optimal control system to maximize throughput. However, this quiescent operating point is a function of the particular chemical components used. Since the properties of these may change with time e.g. catalyst decay, the overall process dynamics will slowly change, and the control system must also utilize an adaptive control system to maintain satisfactory dynamic control.

An optimal control system would thus depend on some *a priori* information, or assumption, based upon experience, to determine an optimum operating condition relative to this assumption.

### 1.2. Why use an Adaptive Control System?

An adaptive control system is required when a conventional controller cannot work. For example, a controller might be required to compensate for such things as:

- (a) Changes in the process transfer function, either in order, or in parameter value, with changes in its environment, e.g. an aircraft's transfer function changes with air speed, height, air pressure, etc.
- (b) Changes in the system itself, e.g. wear, mechanical failure, etc.
- (c) Changes in the nature of the inputs and disturbances to which the system is subject.

Other examples of such variations include the parameters of a continuous chemical process which may change as functions of ambient temperature, or as functions of rates of inflow or outflow, etc. In a paper mill, the inertia of the winding reel changes as the paper is wound on, and the motor torque required to maintain constant tension in the web changes with reel diameter. In a missile, the mass and the centre of gravity change as the fuel is consumed. In a supersonic aircraft, the aerodynamic parameters vary widely as the plane climbs from sea level to say 50,000 ft.

Obviously then, under any of these conditions, a conventional feedback control system is no longer indifferent to variations, and system performance soon degrades to beyond acceptable limits. Early attempts to overcome these problems involved the design of nonlinear control systems, which in the simplest case merely program a deliberate adjustment of some system quantity, usually loop gain, to compensate for a change in system parameters caused by a change in environment. These have often been very successful, but such a 'fixed' compensator can only be adequate over

a restricted range of operating conditions. Outside this range the compensation must be changed, i.e. the system must be adapted to cope with the new conditions.

In these cases therefore, the system is required to automatically change its own compensation, a requirement leading to a self-adaptive control system.

### 1.3. The Classification of Adaptive Control Systems

There is a clear implication in the definition of adaptation and adaptive control systems given above, that the characteristics of at least one component of the system vary, sometimes appreciably with changes in the environment. It is therefore sometimes convenient to regard systems that are designed to have an inherently low sensitivity to changes in plant parameters as *passive* adaptive systems.

*Active* adaptive control systems may therefore be defined as requiring a measurement of the performance of the system (according to some criterion) and the modification of the system, on the basis of this measurement, either so as to render the performance of the complete system insensitive to changes in environment, or so as to improve the performance, or both.

A conventional feedback control system which is designed to furnish an output which follows an input command with a minimum of steady-state error, may therefore be considered as a subclass of adaptive control systems which could be called *static* adaptive systems. Taking this definition, a *dynamic* adaptive control system may be defined as a feedback control system which exhibits essentially the same dynamic response under the influence of changing system parameters. This classification enables a feedback control system to be defined as being self-adaptive if, and only if, it exhibits dynamic adaptability.

It thus follows that in addition to the components required for operation in a fixed environment, an active, dynamic adaptive control system must contain the following characteristics:

(a) The definition of an optimum operating condition or Figure of Merit, i.e. a standard of performance with which the performance of the actual system is compared.

(b) A means of measuring the appropriate characteristics of the actual performance of the system in the form of an Index of Performance for comparison with the desired response or performance.

(c) A means of determining, on the basis of this comparison, a modification of the system that produces a decrease in the difference between the Performance Index and the Figure of Merit.



(d) A means of effecting this modification. In general, this requires the altering of the values of one or more parameters of the system and/or the altering of the structure of the system by the introduction of one or more new components, and/or the removal of one or more of the components originally present.

If one accepts this concept of dynamic adaptivity, then those systems that qualify as adaptive control systems under the above definition can be further classified according to the mechanism of adaptation employed. Some of these categories are:

1. *Passive Adaptive Systems*—These systems are initially designed to give a satisfactory performance for wide variations in the environment and for extreme changes in the controlled system's parameters, with no modification of the controller. It is immediately noticed that these systems do not conform to the definition of adaptive control systems as given earlier, but are included for the sake of completeness.

2. *Input Signal Adaptation*—An input adaptive system measures one or more characteristics of the input signal, such as signal-to-noise ratio, frequency spectrum, r.m.s. value, etc., and modifies the basic control system in accordance with changes in these characteristics. In this type of system no sensing of the system response or output signal is necessary, in which case the resulting adaptive system is essentially open loop, unless of course in the case where the resulting mean square error is monitored and used to determine the final adjustment.

3. *Plant Adaptive Control Systems*—This type of system adjusts its own parameters to compensate for transfer function changes in the controlled plant. That is, the basic control system is modified either in accordance with measurements of the dependent variables of the system, in which case it could be called plant-variables adaptive, or in accordance with measurements of its transfer function, when the term plant-characteristics adaptive could apply. A particular system which comes under this classification is the model reference adaptive system, which is discussed in more detail later.

4. *Parameter Adaptive Systems*—These systems achieve adaptation by the direct adjustment of the system gains, time constants, or other loop parameters. Obviously from the definition given above, it may be seen that all self-adaptive control systems come into this classification, however it is essential to define such a classification so as to distinguish between direct parameter adaptation and what may be defined as—

5. *Input Signal Shaping Adaptation*—This is achieved by generating an input control signal that is more capable of producing the desired system response than is the direct input signal, when the actuator's transfer