C. Carraro and D. Sartore
(editors)

Developments of Control Theory for Economic Analysis

Advanced Studies in Theoretical and Applied Econometrics 7

Kluwer Academic Publishers

Developments of Control Theory for Economic Analysis

edited by

Carlo Carraro and Domenico Sartore
(University of Venice)

1987
KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LANCASTER

Distributors

for the United States and Canada: Kluwer Academic Publishers, P.O. Box 358. Accord Station, Hingham, MA 02018-0358, USA
for the UK and Ireland: Kluwer Academic Publishers, MTP Press Limited. Falcon House, Queen Square, Lancaster LA1 1RN, UK
for all other countries: Kluwer Academic Publishers Group, Distribution Center. P.O. Box 322, 3300 AH Dordrecht, The Netherlands

Library of Congress Cataloging in Publication Data

Developments of control theory for economic analysis.

(Advanced studies in theoretical and applied econometrics ; v. 7)
"Proceedings of the Conference on "Economic Policy and Control Theory" which was held at the University of Venice (Italy) on 27 January-1 February 1985"--Pref.
1. Economic policy--Econometric models--Congresses.
2. Control theory--Econometric models--Congresses.
3. Macroeconomics--Econometric models--Congresses.
I. Carraro, Carlo. II. Sartore, Domenico. III. Conference on "Economic Policy and Control Theory" (1985 : University of Venice) IV. Series.
HD73.D48 1986 338.9'00724 86-8662

ISBN 90-247-3345-6 (this volume)
ISBN 90-247-2622-0 (series)

PRINTED IN THE NETHERLANDS

DEVELOPMENTS OF CONTROL THEORY FOR ECONOMIC ANALYSIS

ADVANCED STUDIES IN THEORETICAL AND APPLIED ECONOMETRICS
VOLUME 7

For a complete list of volumes in this series see final page of this volume.

PREFACE

Giovanni Castellani
Rector of the University of Venice

This book contains the Proceedings of the Conference on "Economic Policy and Control Theory" which was held at the University of Venice (Italy) on 27 January–1 February 1985.

The goal of the Conference was to survey the main developments of control theory in economics, by emphasizing particularly new achievements in the analysis of dynamic economic models by control methods.

The development of control theory is strictly related to the development of science and technology in the last forty years. Control theory was indeed applied mainly in engineering, and only in the sixties economists started using control methods for analysing economic problems, even if some preliminary economic applications of calculus of variations, from which control theory was then developed, date back to the twenties.

Applications of control theory in economics also had to solve new, complicated, problems, like those encountered in optimal growth models, or like the determination of the appropriate intertemporal social welfare function, of the policy horizon and the relative final state of the system, of the appropriate discount factor. Furthermore, the uncertainty characterizing economic models had to be taken into account, thus giving rise to the development of stochastic control theory in economics.

The Conference not only tried to survey the state of art of control theory in economics, but also provided original contributions on the mathematical aspects of control theory and on their macroeconomic applications, on new solution algorithms of nonlinear control problems and on the relative computer software. In particular, the development of new software for performing reliable applications of control methods in economics has to be considered

a relevant achievement for its implications on a wider utilization of control theory.

This book contains therefore the most important results presented at the Venice symposium and can be considered a useful tool both for applied and theoretical economists.

As Rector of the University of Venice, I would like to thank the authors of the articles published in this volume and all the persons who attended the Conference, the editors and the publisher of the book and the Centro Nazionale delle Ricerche whose financial contribution is gratefully acknowledged. I also thank the Comune di Venezia, IBM, the Banco S. Marco, the Bank of Italy and the Comitato per gli Studi e la Progammazione whose financial support made possible the organization of the Conference.

INTRODUCTION

Carlo Carraro and Domenico Sartore

This book contains a selection of the papers presented at the symposium on "Economic Policy and Control Theory" which was held in Venice (Italy) on 27 January–1 February 1985 under the auspices of the University of Venice.

The choice of the topics of the symposium was guided by three principles. First, significant results and advances in the area have taken place in the past ten years and the area is subject to continued extensive research effort at present. Second, some emphasis was placed on theoretical developments, in particular inasmuch as they have contributed improvements to current research in more applied areas of control theory and economic policy, or are likely to do so in the future. Third, the advantage of using sophisticated methodologies, like control theory, for policy analysis was greatly emphasized.

Therefore, several sessions of the symposium were devoted to topics like adaptive control, strategic (game theoretic) control, risk and uncertainty in policy analysis, applied control with particular emphasis on rational expectations models and econometric techniques for control. Furthermore, in order to understand and define future directions of research, some papers were devoted to surveying the state of art of control methods in economics under different viewpoints.

The goal of the symposium was then to review the developments of control theory for economic analysis in the last decade and to propose new theoretical and applied contributions aimed at generalizing and improving the methodology upon which economic analysis should be based.

Since a large number of papers was presented at the conference, it was not possible to publish the complete proceedings of the symposium and a careful selection had to be made. Therefore, all

C. Carraro and D. Sartore (eds). Developments of Control Theory for Economic Analysis

the papers went through a refereeing process and were then screened by following two criteria: the quality of the results contained in the paper and its importance for future developments of research in the area. The four sections of the book reflect this choice and emphasize the editors' viewpoints on different aspects of control theory and economic policy.

The first applications of control theory in economics highlighted the usefulness of this methodology for determining optimal economic policy (see Chow, 1975, for example). Subsequently, control theory has been used for studying the properties of actual and optimal policies by analysing the structure of the policy reaction function and of the econometric model describing the underlying economic system (e.g. Chow, 1981; Preston–Pagan, 1982). When uncertainty was introduced into the policymaker's perception of the economic model or into the welfare function that the policymaker wants to maximizes, control theory developed appropriate tools for dealing with those mathematical and economic problems, making use of sophisticated numerical techniques and computer software (a good example is Kendrick, 1981).

The first section of the book is aimed at surveying all these aspects of control theory and their relationships with policy analysis by presenting the main features of control and controllability theory, optimal decision making under certainty and uncertainty conditions and software for optimal control models. The evolution of economic applications of control methods is also considered and the importance of optimal control for providing better understanding of behaviour, relationships and effects of economic variables is emphasized.

The survey papers included in the first section also defines some directions for future research. In particular, there are suggestions for generalizing control methods both from the theoretical and the computational viewpoint by solving control problems where either the model or the welfare function is assumed to be nonlinear in order to capture the actual features of the economic system and the actual behaviour of the policymaker.

The development of control theory in economics is indeed characterized by an increasing demand for new analytical tools in order to increase reliability and plausibility of control experiments. For years, economists have simply adapted to their own problems mathematical methods derived from other disciplines, in particular

engineering. However, the new features introduced into economic models (e.g. rational expectations), the new hypothesis on the information available to economic agents (incomplete information models) and on the behaviour of the policymaker (risk aversion, strategic planning), have forced economists to derive control methods explicitly conceived for solving the above problems.

Therefore, the essays included into the second section of the book are devoted to determining optimal economic policy when:

(1) the policymaker has time-varying targets;
(2) the policymaker wants to minimize not only the mean value of his loss function, but also its variability. The importance of this second goal is measured by a risk aversion parameter.
(3) there is not enough a priori knowledge about the functional form of the policymaker's loss;
(4) there is uncertainty about the parameters of the model;
(5) the model is large and efficient computational methods are needed.

Another relevant problem is emphasized in Chow's survey article: standard control methods are based on the assumption that the policymaker determines optimal policy by maximizing his welfare function given the economic system, without facing one or more economic agents who rationally anticipate his decisions and maximize their own welfare given those expectations. In other words, the behavioural assumptions underlying standard control methods are not satisfactory and should be revised in order to capture the strategic aspects of policy decisions.

Furthermore, poor assumptions about the expectations of the economic agents hide important methodological and economic problems like:

(1) the time inconsistency of optimal control strategies and the consequent sub-optimality of actual policy decisions (see Kydland-Prescott, 1977);
(2) the possible variability of the parameters of the model as a consequence of the reaction of economic agents to policy-maker's decisions (see Lucas, 1976);
(3) the credibility of the announced control strategy;
(4) the inclusion between the policymaker's targets of non-economic entities like the policymaker's reputation;
(5) the importance of a correct specification of the information available to the policymaker and the economic agents;

Finally, strategic assumptions have to be introduced when the policymaker has to determine his optimal decision without knowing which of several rival models is the true model.

The importance of the previous remarks led us to introduce into the book a section completely devoted to strategic aspects of economic policy and to game-theoretic solutions of policy problems. The third section of the book includes indeed articles which try to solve the above theoretical problems by making use of game theory results. In particular, time-consistency and credibility problems are examined and new solutions are proposed. Furthermore, macro and micro models analysed by game-theoretic methods are presented in order to emphasize the greater insight on economic problems that can be achieved by using game theory.

The last area that has to be covered is the applications of control methods to relevant economic and econometric problems. Applications of optimal control are indeed particularly difficult when the structure of the model is complicated or when new theoretical aspects like expectations or uncertainty are introduced into the model. Therefore, section four presents articles which apply control methods to dynamic macro and micro models, and determine the economic agents' optimal decisions under general hypotheses on the economic system. Important normative conclusions are then derived. Furthermore, the econometric implications of control methods are explored.

Needless to say, each area considered in the four sections of the book is broad, and some problems were probably left uncovered. However, we believe to have included into this book the most relevant developments of control theory for economic analysis and the amount of papers presented at the symposium is the true support to our beliefs.

In this Introduction we try to provide a guide to the contents of the book by taking up briefly some of the major issues considered by the authors. In doing so we have relied on the discussion at the symposium, and we try to point out at least some matters on which further research would be valuable.

Given the nature of the material, this Introduction is divided into four sub-sections with the same title as the four sections of the book. A final sub-section will present our interpretation of some open problems and topics for future research.

1. Introduction to Control Theory: Methods and Algorithms

The first paper of this section contains Gergory Chow's survey of the development of stochastic control theory in macroeconomic policy analysis. The development is separated into three periods. The first is pre-1970 when the major ideas of policy analysis and of optimization were formed. The second is the early and middle 1970's when formal stochastic control theory was rapidly developed for and applied to the study of macroeconomic policy. The third period, beginning in the late 1970's, was stimulated by the introduction of the idea of rational expectations in economic analysis and is characterized by the development of new analytical tools for studying the effects of given macroeconomic policy rules on economic targets. Gregory Chow's survey is also able to point out topics where the results available at present are not satisfactory so that more research effort would valuable.

Roberto Conti's article presents old and new results on controllability theory by providing a unifying framework to the theory of policy effectiveness from a strictly mathematical viewpoint. Both continuous and discrete time models are considered. Necessary and sufficient conditons for point and path controllability are provided and new definitions of system controllability are suggested.

Conti's mathematical approach is translated into economic terms by Petit's article where the theory of policy effectiveness is reviewed from Tinbergen's seminal work to the most recent results. In particular the relationship between Tinbergen's condition for static controllability and dynamic controllability conditions is explored.

The last paper of the first section of the book is David Kendrick's review of available software for optimal control models. The paper contrasts the software of the past with the software of the present and outlines the likely form of the software of the future. In so doing it chronicles the evolution of input and output from numbers to equations to graphics. The evolution of software will be reviewed both for nonlinear models and for linear-quadratic problems. The emphasis in put on deterministic models since the evolution of stochastic software closely parallels that for deterministic models.

Therefore, the four papers of this section cover all the recent developments of control theory in economics both from a theoretical and a computational viewpoint. They also clarify hidden problems

and point out the necessity of new achievements which will be more extensively taken up in the second section of the book.

2. Recent Developments of Control Theory: Objective Function Specification

The contributions of this section are mainly theoretical and provide some interesting generalizations of control methods for economic analysis. In particular, attention is paid to the specification of the policymaker's objective function and functional forms more general and plausible than the quadratic form are proposed. At the same time, the simplicity and analytical elegance of the feedback control solution is often preserved.

The first paper contains the description of a new method for solving control problems. Gruber's paper describes indeed interactive vector optimization methods which are shown to avoid the necessity of explicitly specifying a scalar-valued objective function. This approach has important advantages in comparison with control theoretical decision models in which an explicit objective function must be specified. The advantages are particularly relevant for applied economists whose main interest is not the structure of the feedback policy rule but the optimal values of the policy variables given the specified econometric models. Gruber's paper also describes Rosinger's algorithm for interactive vector optimization by emphasizing those aspects that are likely to be important in applied work.

Brandsma's paper is instead aimed at increasing the robustness of economic policies by using optimisation techniques which try to reduce the impact of uncertainty on policy decisions. Unfortunately, certainty equivalence applied to a quadratic welfare function subject to linear constraints implies decisions which are invariant to the magnitude of the risk undertaken by the policymaker. So, while the importance of risk reduction is widely recognised, economists have seldom been able to compute empirical risk sensitive decisions for the multivariable dynamic control problem which they face in practice. Brandsma's paper is a first attempt in this direction since it provides decision rules which minimizes both the mean and the variance of the policymaker's loss function. The paper provides not only a unifying framework for risk sensitive decision problems but also numerical comparison of different solution methods.

The third paper considers a different generalization of the control problem. In the linear quadratic control model the target values are assumed exogenous. In contrast, Amman and Jager determine the optimal policy rule under the assumption that the target values are a function of the outcome or state variables of the decision model. In this way, the specified welfare (loss) function with time-varying target values can provide a more realistic description of the actual policymakers's behavior. An application to exchange rate policy underline the relevance of the extension of optimal control methods provided by Amman and Jager. The two authors revise indeed Pindyck's algorithm in order to allow for time-varying endogenous target values and then compute the optimal economic policy for a model of the Dutch economy under an exchange rate system characterized by a crawling peg.

In the fourth essay of the section, Eppers and Leserer consider the probabilistic aspects of multi-period planning by comparing the common backward perturbation analysis with forward programming methods. The paper shows the usefulness of forward programming in combination with backward programming by studying the information structure of multi-period planning. The proposed algorithm is a two-stage procedure: first an adaptive feedback optimization is done by looking at past information growth and then adaptive feedforward optimization is done by looking at future uncertainty reduction. This two-stage two-system approach can be shown to be very useful for constructing flexible economic strategies. Some numerical experiments emphasize the characteristics of the algorithm. Eppers and Leserer's paper can be considered an important contribution to adaptive stochastic control theory.

Finally, Montesano's paper faces a strictly theoretical problem by providing a solution to a general intertemporal choice model under uncertainty. In particular, attention is paid to some strong hypotheses that are usually implicitly assumed for the objective function in order to use dynamic optimization methods. A coherent specification of the objective function for intertemporal choice problems under uncertainty is then derived.

All the previous papers provide interesting discussions and new proposals on the specification of the objective function to be optimized by the policymaker. As an explicit feedback solution to the control problem often does not exist when the objective function is not quadratic, it is important to study the properties of

available numerical optimization methods. This task is accomplished by Calzolari and Panattoni whose paper is aimed at generalising the present knowledge on algorithms commonly used in nonlinear applied control. When applying control methods to economic models, the optimization problem is often solved numerically by using methods based on the computation of the Hessian matrix (e.g. Newton method). This paper therefore presents several Montecarlo experiments that have been performed in order to better understand the behaviour of the Hessian and of two of its approximations currently used and quoted in the literature. The results show that this type of optimization procedure (gradient method) converges rapidly when the values of the coefficients are close to the optimum, while the use of a suitable approximation to the Hessian strongly improves the algorithm efficiency far from the optimum and hence its robustness to the choice of the initial values. Furthermore, Calzolari and Panattoni show that the Hessian behaves better in an interval around the optimum which narrower than commonly thought. Even if the numerical experiments are performed in the case of likelihood maximization for FIML estimation of econometric models, the results seem to be relevant for more general optimization problems.

3. Recent Developments of Control Theory: A Game Theoretic Approach

A large part of economic theory is awesomely dependent on game theory, borrowing from it the conceptual apparatus for the analysis of various economic problems. The goal of this section is to show that game theory is also a useful tool for analysing problems related to policymaking both at a macro and micro level. The importance of game theory is due to its ability to facilitate comprehension of different phenomena by preserving at the same time simplicity and mathematical elegance. This is also shown by the papers of this section which are able to solve difficult problems like the optimal policy time inconsistency problem or the rival models problem by using a game-theoretic approach.

The first paper presents Berc Rustem's results on optimal policies with rival models. The problem to be solved is the following: in general, more than one model is claimed to represent the economic

system. Each model is justified by a different theory and may represent a different regime of the economy. How can the policymaker use the information provided by all the rival models? Assuming that all the models are correctly specified, a policymaker may utilize simultaneously more than one of these models to assess their combined effects or may minimize the effect of the most adverse bounded outcome due to uncertainty in a given model. Rustem's paper explores alternative solutions to the rival models problem and determines a robust policy rule by using min-max solutions to the policymaker's optimization problem. This approach involves the simultaneous minimization of the welfare function and its sensitivity to the sources of uncertainty in the model. Furthermore, a Pareto optimal characterization of policy formulation with rival models is given and an algorithm is presented for solving the optimization problem with pooled models. Min-max representations of the pooling problem are formulate both as discrete and as continuous min-max problems. Nash abd Stackelberg strategies are also discussed when multiple models are used by rival agencies within a game theoretic framework.

Hughes–Hallett contribution concerns instead the determination of optimal policy when two policymakers act simultaneously and interdependently on the economic system. Two countries are indeed considered and the optimal strategy of each country is determined by computing the Nash solution of the dynamic game between the two policymakers. It is thus possible to know how much the two economies would gain if their policymakers cooperated and which strategy is optimal under different assumption on the strategic behaviour of the policymakers. Furthermore, the solution techniques utilized in the paper guarantees the time consistency of the optimal strategy.

The time consistency problem is also the main topic of Carlo Carraro's paper which analyzes a new solution of the game between the policymaker and economic agents, the Closed Loop Stackelberg solution, in order to determine the optimal time consistent economic policy. The main features of the Closed Loop Stackelberg solution are examined and it is shown that only if a certain degree of uncertainty is introduced into the model can the Closed Loop Stackelberg solution be made credible. Furthermore, it is shown that the Closed Loop Stackelberg solution of the game can be used to determine the optimal announced economic policy and conditions

are derived for this announcement to be credible and time-consistent. In particular, it is shown that the policymaker can induce the economic agents to behave as if they were acting in the policymaker's interest. The effectiveness of the optimal time consistent strategy is also explored.

The last two papers of the section are microeconomic applications of game theory results which show the importance of using the game-theoretic approach for economic analysis. The first paper, written by Fanchon, Rifkin and Sengupta, presents a dynamic model of price leadership for a dominant firm, where output rather than price is the major decision variable. The relationship between uncertainty, risk and price stability is investigated and a simulation shows the optimal trajectories resulting from various initial states. Conrad's paper analyses instead a duopoly model and determines optimal price and quality strategies under asymmetric information with respect to quality. Since the model is dynamic and information is incomplete, the problem of credibility and reputation immediately arises and is solved under diverse assumptions on the information structure of the players.

4. Economic and Econometric Analysis by Control Methods

The three papers of this section provide other applications of control methods to different aspects of economic theory. Their importance is given by the new insight that can be achieved by looking at the economic problem through control methods.

In particular, the first essay, written by Charles Tapiero, provides a system approach to insurance companies management. The system approach in an insurance setting is broadly stated and applications to a stock and to a mutual insurance firm are used to highlight the potential benefits of this approach to insurance firm's management.

Analogously, the second essay, written by James Gapinski, computes the optimal feedback policy rule for a policymaker facing a dynamic economic system and aiming at stabilizing the business cycle. In particular, the paper studies how the nature of capital influences the effectiveness of feedback control policy.

Both articles carefully describes the structure of the model and then show how control theory can determine the optimal behaviour