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PREFACE

These are the proceedings of the SECOND SIEGEN TOPOLOGY SYMPOSIUM which was held at the University of Siegen, July 27 - August 1, 1987. There was a rich program of plenary lectures and special sessions centering around differential topology and especially around linking phenomena in arbitrary dimensions. Some of these activities are reflected in this collection of research papers.

I would like to thank everyone who contributed to the success of the symposium. In particular, I am most grateful to M. Kervaire and W. Neumann for their help in planning the scientific program. Also there was very dedicated assistance from many members of Siegen University and especially from the young topologists U. Kaiser and P. Mrozik as well as from Mrs. M. Sprenger.

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Siegen, Spring 1988

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C O N T E N T S

Linking phenomena and 3-dimensional topology

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Immersions and vector bundle morphisms

Manifolds and algebraic topology

ON THE UNIVERSAL GROUP OF THE BORROMEAN RINGS

Hugh M. Hilden *

Maria Teresa Lozano **

José Maria Montesinos **

1. Introduction.

Let U be a discrete group of isometries of hyperbolic 3-space, H^3. We say that U is universal if it has the following property: If M^3 is any closed oriented 3-manifold, then there is a finite index subgroup, $G(M^3)$, of U such that M^3 is the orbit space of the action of $G(M^3)$ on H^3.

In an earlier paper [1], the authors demonstrated the existence of universal groups by giving an explicit example. The purpose of this paper is to improve upon the results of that paper and to give applications of universal groups by proving several theorems about 3-manifolds. Perhaps our most surprising result is the following:

THEOREM: Let π be the fundamental group of a compact orientable 3-manifold. Then π acts as a fixed point free group of isometries of a complete hyperbolic 3-manifold. The orbit space is a compact hyperbolic 3-manifold with a tesselation by regular hyperbolic dodecahedra.

2. Redefinition of the universal group U.

It is well known that the group of hyperbolic isometries of H^3 is isomorphic to $PSL_2(\mathbb{C})$. In [2], an explicit formula is given for the action of a matrix on the Poincaré model of H^3. In our paper, [1], we showed that a group U generated by three matrices in $PSL_2(\mathbb{C})$

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was universal. It turns out that certain changes in the proof given there lead to matrices that are significantly simpler. Here, we repeat part of that proof, but then diverge from it, to get explicit representations of new matrix generators. The "new" group we obtain, which we continue to call U, is conjugate to the old one.

We begin, as in [1], by constructing the regular Euclidean dodecahedron whose intersection with the positive octant is depicted in Figure 1. The value of t is $\frac{1}{2}(\sqrt{5}-1)$ and $a = 1 - t$. The rest of the dodecahedron is obtained from the reflections in the $x-y$, $y-z$, and $x-z$ planes. The reader can verify that this really is a regular Euclidean dodecahedron using only vector calculus. He can also verify that two planes containing adjacent faces of the dodecahedron intersect the sphere of radius $R = \sqrt{1+t}$, centered at the origin, in a pair of circles that intersect each other at right angles. The details of this calculation are done in [1] and again require only vector calculus.

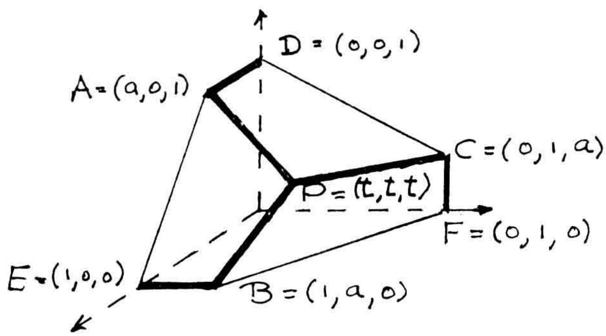

Figure 1

We view the open ball of radius R as the Klein model for hyperbolic 3-space. The dihedral angle between two intersecting planes in the Klein model is the same as the Euclidean angle between their bounding circles. Thus all dihedral angles of our dodecahedron are right angles. Also, Euclidean rotations and reflections about axes and planes through the origin are hyperbolic isometries in the Klein model. Thus the dodecahedron we have constructed is a regular hyperbolic dodecahedron with dihedral angles right angles. Now we describe an isomorphism from the Klein to the Poincaré model and find the image of the directed lines $\overrightarrow{AD}$, $\overrightarrow{CF}$ and $\overrightarrow{BE}$ under this isomorphism.

Let S be the bounding sphere of the Klein model and let C be the bounding complex plane of the Poincaré upper half space model, including the point at ∞. We define a map $T : P \to P'$ from S to C by projection from the point (0,0,R). (See Figure 2; the point (0,0,R) goes to ∞). The map T is 1 - 1 and onto. Since each Klein line intersects S in two points and each Poincaré line intersects C in two points, T induces a map from Klein lines to Poincaré lines. We can get a map from Klein points to Poincaré points by choosing any

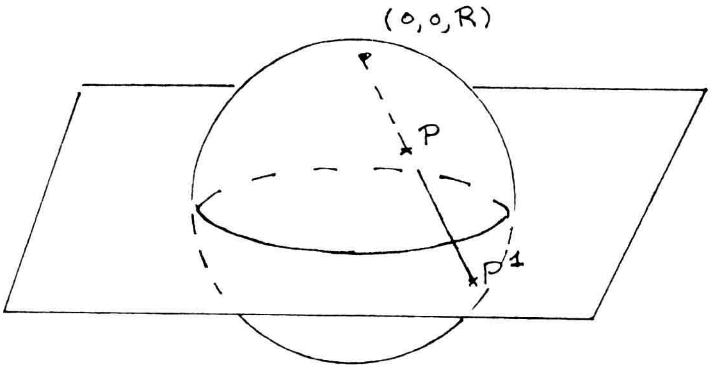

Figure 2

two lines through a point, finding the image of the lines and mapping the original point to the intersection of the images of the two lines. This map is well defined and an isomorphism of the two models.

The lines $\overrightarrow{AD}$, $\overrightarrow{BE}$, and $\overrightarrow{CF}$ intersect the sphere S in the ordered point pairs ((b,0,1),(-b,0,1)), respectively, where $R^2 = 1 + b^2$. Since also $R^2 = 1 + t$, and $t^2 + t = 1$, it follows that $Rb = 1$ and $R^4 = 1 + R^2$. Using these last equalities and easy similar triangle arguments, we see that the three ordered point pairs project to $(1/(R-1), -1/(R-1))$, $(1+ib, 1-ib)$ and $(i(R^2+1), i(R^2-1))$ respectively.

In [1] we explained why H^3 has a tesselation by regular hyperbolic dodecahedra of the type we have just constructed, and we showed that a universal group was generated by three 90° rotations, with axes taken from each of three pairs of opposite edges of any one dodecahedron. (For example, the axes AD, BE, and CF of the dodecahedron in Figure 1.) Thus any one dodecahedron can serve as fundamental domain. The orbit space is S^3 and is obtained by identifying pairs of pentagons that are adjacent along an axis of rotation. (Refer again to Figure 1). The Borromean rings are the image of the three axes of rotations.

In the sequel, we will need more precision in identifying our matrices. First of all, we orient the three edges of the dodecahedron in Figure 1 with directions $\overrightarrow{AD}$, $\overrightarrow{BE}$, and $\overrightarrow{CF}$. This orientation is preserved by the 120° rotation about the line through the origin and (1,1,1) in Figure 1. We will consider various embeddings of this dodecahedron in the Poincaré upper half space model of H^3. We will use the letters A, B and C to denote the positive (using the right hand rule) 90° rotations about the oriented axes corresponding to $\overrightarrow{AD}$, $\overrightarrow{BE}$ and $\overrightarrow{CF}$, respectively. The orientation of the three axes induces the usual orientation of the Borromean rings with its three-fold symmetry. This is depicted in Figure 3 with the components labelled a, b and c corresponding to the axes of rotation of A, B, and C.

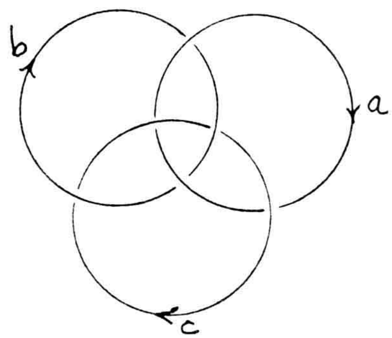

Figure 3

On the other hand, considering the branched covering $H^3 \to H^3/U = S^3$, the orientation of the Borromean rings induces an orientation in every axis of rotation of U. Thus we shall speak of 90° rotations as being either "positive" or "negative".

By the preceding conventions, we may choose as generators of our universal group U, the isometries A_1, B_1 and C_1 that are the positive 90° rotations about the directed lines $(1/(R-1), -1/(R-1))$ $(1+ib, 1-ib)$ and $(i(R^2+1), i(R^2-1))$ respectively.

The matrices that represent these rotations, which we also denote by A_1, B_1, and C_1 are:

$$A_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i/(R-1) \\ -i(R-1) & 1 \end{bmatrix}$$

$$B_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-R & R^3 \\ -R & 1+R \end{bmatrix}$$

$$C_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1-iR^2 & -R^2 \\ -1 & 1+iR^2 \end{bmatrix}$$

These matrices have been computed in the following way. The Möbius transformation, $w = \frac{z - z_0}{z - z_1}$, sends z_0 to 0, z_1 to ∞, and correponds to a matrix T. The matrices T, T^{-1}, and the matrix $R(0,\infty)$ representing positive 90° rotation about the directed axis $(0,\infty)$ are:

$$T = \begin{bmatrix} 1 & -z_0 \\ 1 & -z_1 \end{bmatrix}, \quad T^{-1} = \frac{1}{(z_0 - z_1)} \begin{bmatrix} -z_1 & z_0 \\ -1 & 1 \end{bmatrix}, \quad R(0,\infty) = \begin{bmatrix} \omega & 0 \\ 0 & \overline{\omega} \end{bmatrix}$$

where $\omega = \sqrt{2}(\frac{1}{2} + \frac{1}{2}i)$.

It follows that the matrix $R(z_0,z_1)$, representing positive 90° rotation about the directed axis (z_0,z_1) is given by $T^{-1}R(0,\infty)T$. Using $\omega^2 = i$, we compute that:

$$R(z_0,z_1) = \frac{\overline{\omega}}{(z_0 - z_1)} \begin{bmatrix} z_0 - iz_1 & -z_0z_1(1-i) \\ 1-i & -z_1 + iz_0 \end{bmatrix}$$

The matrices A_1, B_1, and C_1 were obtained by substituting in this formula, making heavy use of the algebraic relation $R^2(R-1)(R+1) = R^4 - R^2 = 1$.

But the entries of the matrices A_1, B_1, and C_1 are not algebraic integers. Let $P = DEF$ where $\lambda^2 = R$, $\mu = \frac{1}{2}\sqrt{2}(1 + R + R^2)$, so that $1/\mu = \frac{1}{2}\sqrt{2}(1 - 2R + R^3)$, and D, E and F are:

$$D = \begin{bmatrix} \lambda & 0 \\ 0 & 1/\lambda \end{bmatrix}, \quad E = \begin{bmatrix} 1 & \frac{1}{2}\sqrt{2} \\ 0 & \frac{1}{2}\sqrt{2} \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}$$

Conjugating the matrices A_1, B_1, and C_1 by P is equivalent to choosing a different embedding of our dodecahedron in the Poincaré-

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upper half space model. The matrices $X = P^{-1}X_1P$ where $X = A,B,C$ constitute new generators for our universal group U, which will remain fixed for the rest of the paper. The matrices A,B,C were computed directly and are given below. We found the matrices D,E,F by following ideas in [3].

$$A = \frac{1}{2}\sqrt{2}\begin{bmatrix} 1 - iR + iR^2 & -iR - iR^2 - iR^3 \\ 1 - 2iR^2 + iR^3 & 1 + iR - iR^2 \end{bmatrix}$$

$$B = \frac{1}{2}\sqrt{2}\begin{bmatrix} 1 - R + R^2 & 1 - R + R^2 \\ -R - R^2 + R^3 & 1 + R - R^2 \end{bmatrix}$$

$$C = \frac{1}{2}\sqrt{2}\begin{bmatrix} 1 + R - iR^2 & -i - 2iR^2 - iR^3 \\ -1 - R + R^2 & 1 - R + iR^2 \end{bmatrix}$$

We can verify directly that all the entries of A, B, and C are algebraic integers. For example if x is the 1,2 entry of B then $2x^2 = (1-R+R^2)^2 = 1+R^2+R^4-2R+2R^2-2R^3 = 2-2R+4R^2-2R^3$, so that $x^2 = 1-R+2R^2-R^3$. Since R is an algebraic integer, so is x^2, and so is x. We summarize the preceding remarks and computations in a theorem. (Recall $R = \sqrt{1+t} = \sqrt{1/t}$.)

<u>THEOREM 1.</u> The universal group U is a subgroup of $PSL_2(\hat{A})$ where $\hat{A}$ is the ring of algebraic integers of the field $Q(\sqrt{2}, i, \sqrt{t})$.

This theorem improves upon theorem 5.1 of [1] by getting rid of $\sqrt{3}$, thus reducing dimension over Q from 32 to 16. Next we compute a presentation for U.

<u>THEOREM 2.</u> A presentation for the universal group U is

$$< A,B,C;\ (\bar{C}A\bar{C}\bar{A})B = B(\bar{C}A\bar{C}\bar{A}),\ (\bar{A}BA\bar{B})C = C(\bar{A}BA\bar{B}),\ (\bar{B}CB\bar{C})A = A(\bar{B}CB\bar{C}),$$
$$I = A^4 = B^4 = C^4 >$$

<u>PROOF.</u> If we remove the Borromean rings from S^3 and all the axes of rotations in U from H^3, then we have a regular covering space $p : (H^3 - \text{axes}) \to (S^3 - \text{Borromean rings})$. It follows from covering space theory that $U \cong \pi_1(S^3 - \text{rings})$ modulo $p_*\pi_1(H^3 - \text{axes})$.

The usual Wirtinger presentation for the Borromean rings, obtained from Figure 3, is $\langle a,b,c;\ (\bar{c}a\bar{c}\bar{a})b = b(\bar{c}a\bar{c}\bar{a}),\ (\bar{a}b\bar{a}\bar{b})c = c(\bar{a}b\bar{a}\bar{b}),$ $(\bar{b}c\bar{b}\bar{c})a = a(\bar{b}c\bar{b}\bar{c})\rangle$ where a, b, and c are "positive" meridians about each of the three components. Also, it follows from the definition of the isomorphism $\pi_1(S^3 - \text{rings})/p_*\pi_1(H^3 - \text{axes}) \cong U$, that positive meridians lift to positive rotations. Choosing the base point for $\pi_1(H^3 - \text{axes})$, in the embedded dodecahedron fundamental domain, the meridians a, b, and c lift to the rotations A, B, and C respectively. The group $\pi_1(H^3 - \text{axes})$ is generated by meridians and, because the branching is all of order four, meridians in $\pi_1(H^3 - \text{axes})$ project to fourth powers of meridians in $p_*\pi_1(H^3 - \text{axes})$. Since any meridian in $\pi_1(S^3 - \text{rings})$ is conjugate to one of the meridians a, b, or c, it follows that the group $p_*\pi_1(H^3 - \text{axes})$ is the normal closure of a^4, b^4, and c^4. Now it is clear that a presentation for U is obtained by adding the three additional relations to the presentation of the Borromean rings group. □

If we examine the relations in the presentation for U we see that the concept of words of even length in A, B, and C is well defined. Let W be the index two subgroup of U consisting of words of even length.

<u>THEOREM 3.</u> The group W, of words of even length in A, B, C, is a subgroup of $PSL_2(\hat{B})$ where $\hat{B}$ is the ring of algebraic integers of the field $Q(i, \sqrt{t})$.

<u>PROOF.</u> The group W is generated by all products XY where X and Y equal A or B or C. From Theorem 1 any such product has algebraic integer entries. From the expressions for A, B, and C any such product has entries in the field $Q(i, \sqrt{t})$. □

A well known result from Kleinian group theory implies the group U has a finite index subgroup that acts freely on H^3. We identify such a group as the kernel of a homomorphism we now construct.

The Borromean rings are oriented, so we have the natural homomorphism from $\pi_1(S^3 - \text{Borromean rings})$ to $H_1(S^3 - \text{Borromean rings}) \cong Z \oplus Z \oplus Z$, which we compose with reduction modulo four to get a homomorphism onto $Z/4 \oplus Z/4 \oplus Z/4$. Since the presentation of U is obtained from the presentation of $\pi_1(S^3 - \text{Borromean rings})$ by adding the additional relators A^4, B^4, C^4, and these are sent to zero, a homomorphism of U is induced which we call α.

$$\alpha : U \longrightarrow Z/4 \oplus Z/4 \oplus Z/4$$

Composing α with the additive homomorphism from $Z/4 \oplus Z/4 \oplus Z/4$ to $Z/4$ we obtain a homomorphism we call θ.

$$\theta : U \longrightarrow Z/4 \quad .$$

Let N be the index four subgroup, kernel θ.

A positive rotation in U corresponds to a positive meridian in $\pi_1(S^3$ - rings). A positive meridian is sent to (1,0,0), (0,1,0), or (0,0,1) under the homomorphism from homotopy to homology. It follows that the homomorphism θ sends a positive rotation to 1, and sends any rotation to 1, 2, or 3. Consequently kernel θ contains no rotations and the next theorem follows.

<u>THEOREM 4.</u> The universal group U, has an index four subgroup N which acts freely on H^3. Also, U/N is cyclic.

An interesting property of U, unusual for Kleinian groups, is that it has a homomorphism onto a Euclidean crystallographic group.

We shall now describe the Euclidean crystallographic group V, the homomorphism from U onto V and the induced homomorphism from U to the point group of V.

Take a unit cube in E^3 with faces bisected and labelled as in Figure 4, and form a tesselation of E^3 from the integer translations of the cube. The face bisectors fit together to form a family of non intersecting lines. Define V to be the group generated by 180^o rotations in these lines. Then we see that V has the following properties:

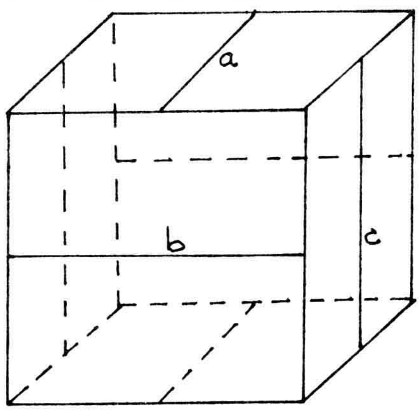


Figure 4

1. The group V is a Euclidean crystallographic group.

2. The translation subgroup T has index eight.

3. The point group $V/T \cong Z/2 \oplus Z/2 \oplus Z/2$.

4. Any one cube (we shall take the one in Figure 4) serves as fundamental domain.

5. The group V is generated by the rotations $\hat{A}, \hat{B}, \hat{C}$ about the axes a, b, c respectively.

6. The map $E^3 \to E^3 / V = S^3$ is a branched covering space map. The image of the axes of rotation is the Borromean rings.

7. In exact analogy with the more difficult case of U, the group V has a presentation $\langle \hat{A}, \hat{B}, \hat{C}$; relators coming from the Borromean rings, $\hat{A}^2, \hat{B}^2, \hat{C}^2 \rangle$.

The presentation of V is exactly the same as the presentation of U except for the power of the last three relators, which is two instead of four. It follows that there is a homomorphism from U onto V defined by sending A, B, C to $\hat{A}, \hat{B}, \hat{C}$ respectively. We call this homomorphism β and we call the homomorphism from U to V/T obtained by composition φ.

$$\beta : U \longrightarrow V$$

$$\varphi : U \longrightarrow Z/2 \oplus Z/2 \oplus Z/2$$

The natural maps $H^3 \to H^3 / \text{kernel } \beta \approx E^3 \to H^3 / U \approx S^3$ are both branched covering space maps, the group of covering transformations for the latter is V.

3. Applications to 3-manifold theory.

Every closed oriented 3-manifold is obtained as the orbit space of the action of a finite index subgroup of U on H^3. This fact enables us to prove several theorems about the structure of 3-manifolds and their fundamental groups.

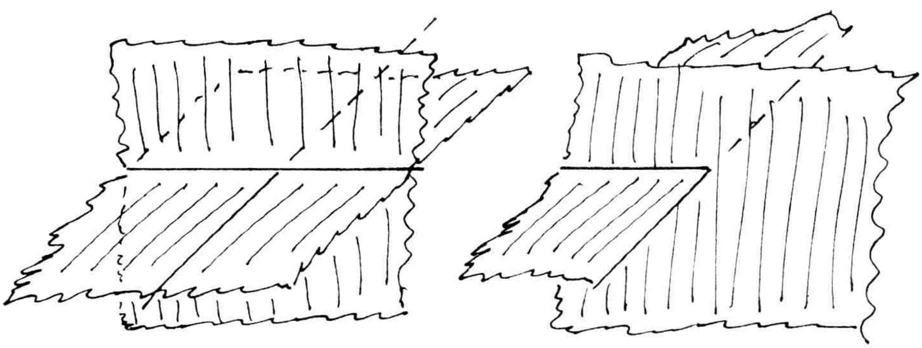
THEOREM 5. Every closed oriented 3-manifold can be "pentagulated"; that is, obtained from a finite set of dodecahedra by pasting along pentagonal faces in pairs.

PROOF. Given M^3 let $G(M^3)$ be a finite index subgroup of U with $M^3 \approx H^3/G(M^3)$. Since a fundamental domain for U is any one dodecahedron, a fundamental domain for $G(M^3)$ is n dodecahedra where n

is the index of $G(M^3)$ in U. The theorem now follows from the definition of fundamental domain. □

The presentation of a 3-manifold given in the previous theorem actually determines a cell decomposition with very special properties. To see this consider the pentagonal faces of the dodecahedra in the tesselation of H^3 left invariant by U. Because the dihedral angles of the dodecahedra are right angles, the pentagonal faces fit together to form hyperbolic planes tesselated by pentagons. All such planes form what we shall call "the family of planes". Any two planes in the family either do not intersect or intersect at right angles. If three planes intersect, they intersect the way the $x-y$, $y-z$ and $x-z$ planes do in E^3. The group U, and its subgroup $G(M^3)$, leave the family of planes invariant, and the 2-skeleton is the orbit space of the family of planes under the action of $G(M^3)$.

Since $G(M^3)$ contains only elements that act freely, or are two-fold or four-fold rotations, the singularity structure is easy to determine from the singularity structure of the family of planes, or locally from the action of a two-fold or four-fold rotation on the family of planes. The singularities in the two skeleton of the cell decomposition of M^3 are of the type in the family of planes or are of the type in Figure 5.



two-fold axis four-fold axis

Figure 5

In fact the 2-skeleton has the structure of an immersed submanifold. We summarize this in our next theorem.

THEOREM 6. Any closed oriented 3-manifold has a cell decomposition whose 2-skeleton is the image of an immersion of a disconnected surface with boundary.

The immersion is in general position. There are no branch points and the double points are as in Figure 5.

There are subgroups of U (such as N) that act freely on H^3 producing special hyperbolic manifolds as orbit spaces. We formalize the special properties of these manifolds in the following definition.

DEFINITION. A 3-manifold is called dodecahedral if it is a complete hyperbolic 3-manifold with a tesselation by regular, right-dihedral angled hyperbolic dodecahedra.

Let X be the manifold H^3/N where N is the index 4-subgroup of U from Theorem 4. Then, since N acts freely on H^3 and preserves the tesselation, X is an example of a dodecahedral manifold. The next theorem indicates why dodecahedral manifolds may be important.

THEOREM 7. Every closed oriented 3-manifold is the orbit space of an orientation preserving $Z/4$ action on a dodecahedral manifold.

PROOF. Given M^3, choose $G(M^3)$ of finite index in U with $H^3/G(M^3) \approx M^3$, and with $G(M^3)$ containing a four-fold rotation. Then kernel θ restricted to $G(M^3)$ equals $N \cap G(M^3)$, where θ was defined in section 2. Since $N \cap G(M^3) \subset N$ which acts freely, it follows that $H^3/G(M^3) \cap N$ is a covering space of X, is compact, and is dodecahedral. Since $G(M^3)/(G(M^3) \cap N) \cong Z/4$, the map $H^3/(G(M^3) \cap N) \to H^3/G(M^3) \approx M^3$ is a four-fold cyclic covering space map. □

Next we prove a theorem about fundamental groups of 3-manifolds and dodecahedral manifolds.

THEOREM 8. Let π be the fundamental group of a compact oriented 3-manifold M^3. Then π is isomorphic to a group of fixed point free, tesselation preserving, isometries of a dodecahedral manifold.

PROOF. Let $G(M^3)$ and N be as in the proof of the preceding theorem. In [1] we showed that $\pi_1(M^3) \cong G(M^3)/TORG(M^3)$ where $TORG(M^3)$ is the subgroup of $G(M^3)$ generated by rotations. (This

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