Ellis Horwood Series in **MECHANICAL ENGINEERING**

VIBRATIONS AND CONTROL SYSTEMS

C. F. Beards

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ELLIS HORWOOD SERIES IN MECHANICAL ENGINEERING

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Preface

The demands made on many present day systems are so severe, that the analysis and assessment of the dynamic performance is now an essential and very important part of system design. Dynamic analysis is performed so that the system response to the expected excitation can be predicted, and modifications made as necessary. It is also an essential technique to apply to existing dynamic systems, when considering the effects of modifications and searching for performance improvement.

There is, therefore, a need for all practising designers, engineers, and scientists, as well as students, to have a good understanding of the analysis methods for predicting the vibration response of a system, and methods for determining control system performance. It is also essential to be able to understand, and contribute to, published and quoted data in this field.

There is great benefit to be gained by studying vibration analysis and control systems dynamics together, and in having this information in a single text, because the analyses of the dynamics of control systems and the vibration of elastic systems are closely linked. This is because in many cases the same equations of motion occur in the control system as in the vibrating system, and thus the techniques and results developed in the analysis of one system may be used in another. This has been successfully demonstrated in my earlier book, *Vibration analysis and control system dynamics*, from which a number of examples and presentation techniques have been employed in this more comprehensive and enhanced text.

Excellent advanced specialised texts on vibration analysis and on control system dynamics are available, and some are referred to later, but they require advanced mathematical knowledge and understanding of dynamic systems, and often refer to idealised systems rather than to mathematical models of real systems. This text links basic dynamic analysis with these advanced texts so that it gives an introduction to advanced and specialised analysis methods, and describes how system parameters can be changed to achieve a desired dynamic performance. The mathematical modelling and analysis of real systems is also emphasised.

The book is intended to give practising engineers and scientists, as well as students of engineering and science to first degree level, a thorough understanding of the principles involved in the analysis of vibrations and control systems, and to provide a sound theoretical basis for further study. More than fifty worked examples have been included in achieving this, together with over one hundred and fifty problems for the reader to try.

C. F. Beards, June 1988

General Notation

a	damping factor, dimension, displacement.
b	circular frequency (rad/s), dimension, port coefficient.
c	coefficient of viscous damping, velocity of propagation of stress wave.
c_c	coefficient of critical viscous damping $= 2\sqrt{mk}$.
c_d	equivalent viscous damping coefficient for dry friction damping $=$ $4F_d/\pi\omega X$.
c_H	equivalent viscous damping coefficient for hysteretic damping $=$ $\eta k/\omega$.
d	diameter.
f	frequency (Hz), exciting force.
f_s	Strouhal frequency (Hz).
g	acceleration constant.
h	height, thickness.
j	$\sqrt{-1}$.
k	linear spring stiffness, beam shear constant, gain factor.
k_T	torsional spring stiffness.
k^*	complex stiffness $= k(1 + j\eta)$.
l	length.
m	mass.
q	generalised coordinate.

r	radius.
s	Laplace operator $= a + jb$.
t	time.
u	displacement.
v	velocity,
	deflection.
x	displacement.
y	displacement.
z	displacement.
A	amplitude,
	constant,
	cross-sectional area.
B	constant.
$C_{1,2,3,4}$	constants.
D	flexural rigidity $= Eh^3/12\,(1-\nu^2)$,
	hydraulic mean diameter,
	derivative w.r.t. time.
E	modulus of elasticity.
E'	in-phase, or storage modulus.
E''	quadrature, or loss modulus.
E^*	complex modulus $= E' + jE''$.
F	exciting force amplitude.
F_d	coloumb (dry) friction force $= \mu N$.
F_T	transmitted force.
G	centre of mass,
	modulus of rigidity,
	gain factor.
I	mass moment of inertia.
J	second moment of area,
	moment of inertia.
K	stiffness,
	gain factor.
L	length.
$\mathcal{L}$	Laplace transform.
M	mass,
	moment,
	mobility.
N	applied normal force,
	gear ratio.
P	force.
Q	factor of damping,
	flow rate.
Q_i	generalised external force.
R	radius of curvature.
$[S]$	system matrix.
T	kinetic energy,
	tension,
	time constant.

T_R	transmissibility = F_T/F.
V	potential energy,
	speed.
X	amplitude of motion.
$\{X\}$	column matrix.
X_S	static deflection = F/k.
X/X_S	dynamic magnification factor.
Z	impedance.
α	coefficient,
	influence coefficient,
	phase angle,
	receptance.
β	coefficient,
	receptance.
γ	coefficient,
	receptance.
δ	deflection.
ϵ	short time,
	strain.
ϵ_o	strain amplitude.
η	loss factor = E''/E'.
ζ	damping ratio = c/c_c.
θ	angular displacement,
	slope.
λ	matrix eigenvalue,
	$[\rho A\omega^2/EI]^{1/4}$.
μ	coefficient of friction,
	mass ratio = m/M.
ν	Poisson's ratio,
	circular exciting frequency (rad/s).
ρ	material density.
σ	stress.
σ_o	stress amplitude.
τ	period of vibration = $1/f$.
τ_d	period of dry friction damped vibration.
τ_v	period of viscous damped vibration.
ϕ	phase angle,
	function of time,
	angular displacement.
ψ	phase angle.
ω	undamped circular frequency (rad/s).
ω_d	dry friction damped circular frequency.
ω_v	viscous damped circular frequency = $\omega\sqrt{(1-\zeta^2)}$.
Λ	logarithmic decrement = $\ln X_1/X_{11}$.
Φ	transfer function.
Ω	natural circular frequency (rad/s).

CHAPTER 1

Introduction

The vibration which occurs in most machines, structures, and dynamic systems is undesirable, not only because of the resulting unpleasant motions, the noise, and the dynamic stresses which may lead to fatigue and failure of the structure or machine, but also because of the energy losses and the reduction in performance which accompany the vibrations.

Until early this century, machines and structures usually had very high mass and damping, because heavy beams, timbers, castings, and stonework were used in their construction. Since the vibration excitation sources were often small in magnitude, the dynamic response of these highly damped machines was low. However, with the development of strong lightweight materials, increased knowledge of material properties and structural loading, and improved analysis and design techniques, the mass of machines and structures built to fulfil a particular function has decreased. Furthermore, the efficiency and speed of machinery have increased so that the vibration exciting forces are higher, and dynamic systems often contain high energy sources which can create intense vibration problems. This process of increasing excitation with reducing machine mass and damping has continued at an increasing rate to the present day, when few, if any, machines can be designed without carrying out the necessary vibration analysis, if their dynamic performance is to be acceptable. The demands made on machinery, structures, and dynamic systems are also increasing, so that the dynamic performance requirements are always rising.

There have been very many cases of systems failing or not meeting performance targets because of resonance, fatigue, or excessive vibration of one component or another. Because of the very serious effects which unwanted vibrations can have on dynamic systems, it is essential that vibration analysis be carried out as an inherent part of their design, when necessary modifications can most easily be made to eliminate vibration, or at least to reduce it as much as possible. However, it must be recognised that it may sometimes be necessary to reduce the vibration of an existing machine, either because of inadequate initial design, or by a change in function of the machine, or by a change in environmental conditions or performance requirements. Therefore techniques for the analysis of vibration in dynamic systems should be applicable to existing systems as well as to those in the design stage: it is the solution to the vibration problem which may be different, depending on whether or not the system already exists.

The demands made on automatic control systems are also increasing. Systems are becoming larger and more complex, whilst improved performance criteria, such as reduced response time and error, are demanded. Whatever the duty of the system, from the control of factory heating levels to satellite tracking, or from engine fuel control to controlling sheet thickness in a steel rolling mill, there is continual effort to improve performance whilst making the system cheaper, more efficient, and more compact. These developments have been greatly aided in recent years by the wide availability of microprocessors. Accurate and relevant analysis of control system dynamics is necessary in order to determine the response of new system designs, as well as to predict the effects of proposed modifications on the response of an existing system, or to determine the modifications necessary to enable a system to give the required response.

There are two reasons why it is desirable to study vibration analysis and the dynamics of control systems together as dynamic analysis. Firstly, because control systems can then be considered in relation to mechanical engineering using mechanical analogies, rather than as a specialised and isolated aspect of electrical engineering, and secondly, because the basic equations governing the behaviour of vibration and control systems are the same: different emphasis is placed on the different forms of the solution available, but they are all dynamic systems. Each analysis system benefits from the techniques developed in the other.

Dynamic analysis can be carried out most conveniently by adopting the following three stage approach:

Stage I. Devise a mathematical or physical model of the system to be analysed.

Stage II. From the model, write the equations of motion.

Stage III. Evaluate the system response to relevant specific excitation.

These stages will now be discussed in greater detail.

Stage I. The mathematical model

Although it may be possible to analyse the complete dynamic system being considered, this often leads to a very complicated analysis, and the production of much unwanted information. A simplified mathematical model of the system is therefore usually sought which will, when analysed, produce the desired information as economically as possible and with acceptable accuracy. The derivation of a simple mathematical model to represent the dynamics of a real system is not easy, if the model is to give useful and realistic information.

However, to model any real system a number of simplifying assumptions can often be made. For example, a distributed mass may be considered as a lumped mass, or the effect of damping in the system may be ignored particularly if only resonance frequencies are needed or the dynamic response required at frequencies well away from a resonance, or a non-linear spring may be considered linear over a limited range of extension, or certain elements and forces may be ignored completely if their effect is likely to be small. Furthermore, the directions of motion of the mass elements are usually restrained to those of immediate interest to the analyst.

Thus the model is usually a compromise between a simple representation which is easy to analyse but may not be very accurate, and a complicated but more realistic model which is difficult to analyse but gives more useful results. Consider for example, the analysis of the vibration of the front wheel of a motor car. Fig. 1.1

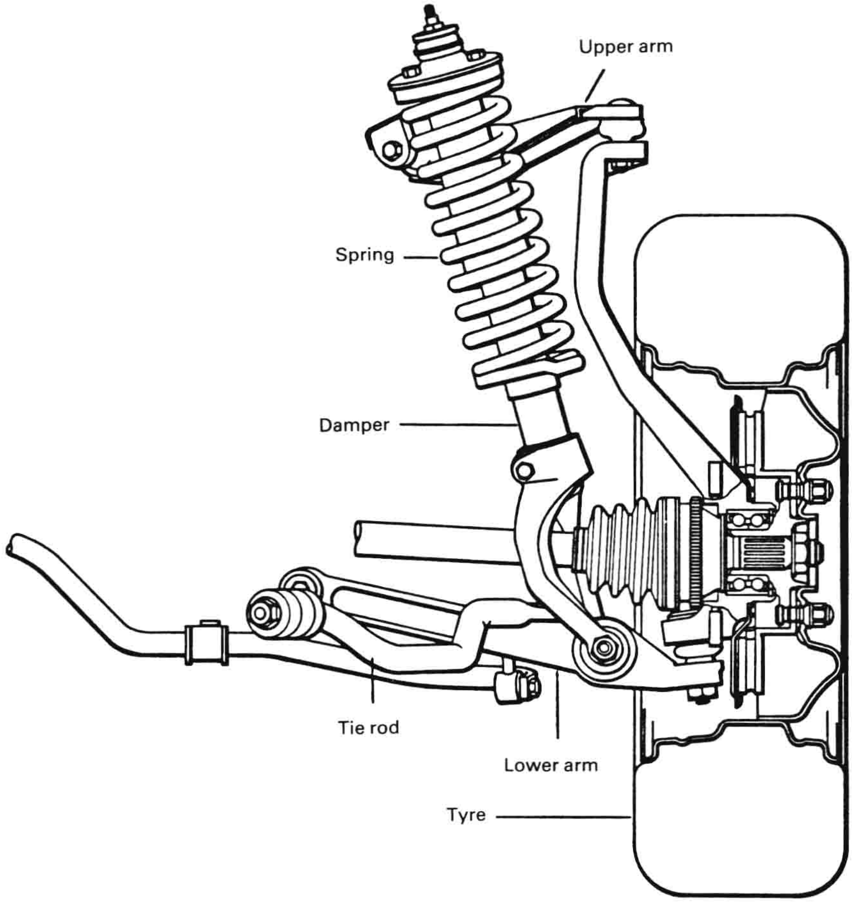

Fig. 1.1 – Rover 800 front suspension.
(By courtesy of Rover Group)

shows a typical suspension system. As the car travels over a rough road surface, the wheel moves up and down, following the contours of the road. This movement is transmitted to the upper and lower arms, which pivot about their inner mountings, causing the coil spring to compress and extend. The action of the spring isolates the body from the movement of the wheel, with the shock absorber or damper absorbing vibration and sudden shocks. The tie rod controls longitudinal movement of the suspension unit.

Fig. 1.2(a) is a very simple model of this same system, which considers translational motion in a vertical direction only: this model is not going to give much useful information, although it is easy to analyse. The model shown in Fig. 1.2(b) is capable of producing some meaningful results at the cost of increased labour in the

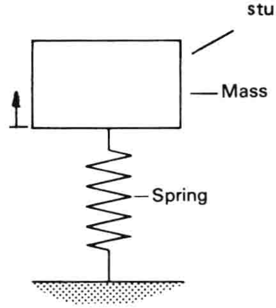

Fig. 1.2(a) – Simplest model – Motion in a vertical direction only can be analysed.

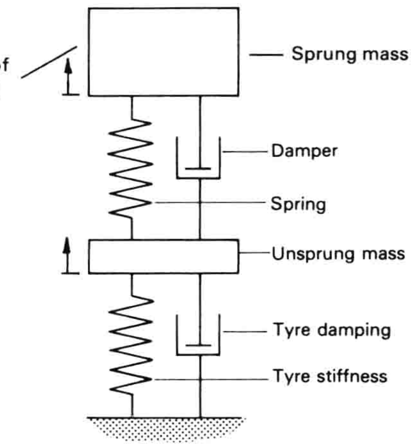

Fig. 1.2(b) – Motion in a vertical direction only can be analysed.

analysis, but the analysis is still confined to motion in a vertical direction only. A more refined model, shown in Fig. 1.2(c), shows the whole car considered, translational and rotational motion of the car body being allowed.

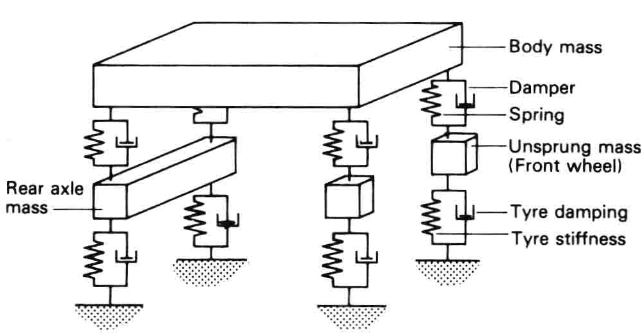

Fig. 1.2(c) – Motion in a vertical direction, roll, and pitch can be analysed.

If the modelling of the car body by a rigid mass is not acceptable, a finite element analysis may prove useful. This technique would allow the body to be represented by a number of mass elements.

The vibration of a machine tool such as a lathe, can be analysed by modelling the machine structure by the two degree of freedom system shown in Fig. 1.3. In the simplest analysis the bed can be considered to be a rigid body with mass and inertia, and the headstock and tailstock are each modelled by lumped masses. The bed is supported by springs at each end as shown. Such a model would be useful for determining the lowest or fundamental natural frequency of vibration. A refinement to this model, which may be essential in some designs of machine where the bed cannot be considered rigid, is to consider the bed to be a flexible beam with lumped masses attached as before.

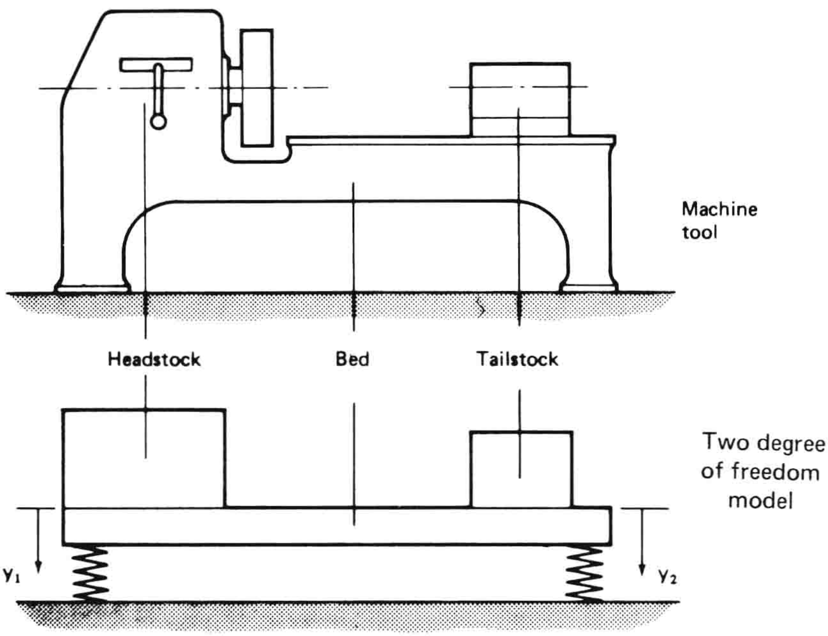

Fig. 1.3 – Machine tool vibration analysis model.

A block diagram model is usually used in the analysis of control systems. For example, a system used for controlling the rotation and position of a turntable about a vertical axis is shown in Fig. 1.4. The turntable can be used for mounting a telescope or gun, or if it forms part of a machine tool it can be used for mounting a workpiece for machining. Fig. 1.5 shows the block diagram model used in the analysis.

It can be seen that the feedback loop enables the input and output positions to be compared, and the error signal, if any, is used to activate the motor and hence rotate the turntable until the error signal is zero; that is, the actual position and the desired position are the same.