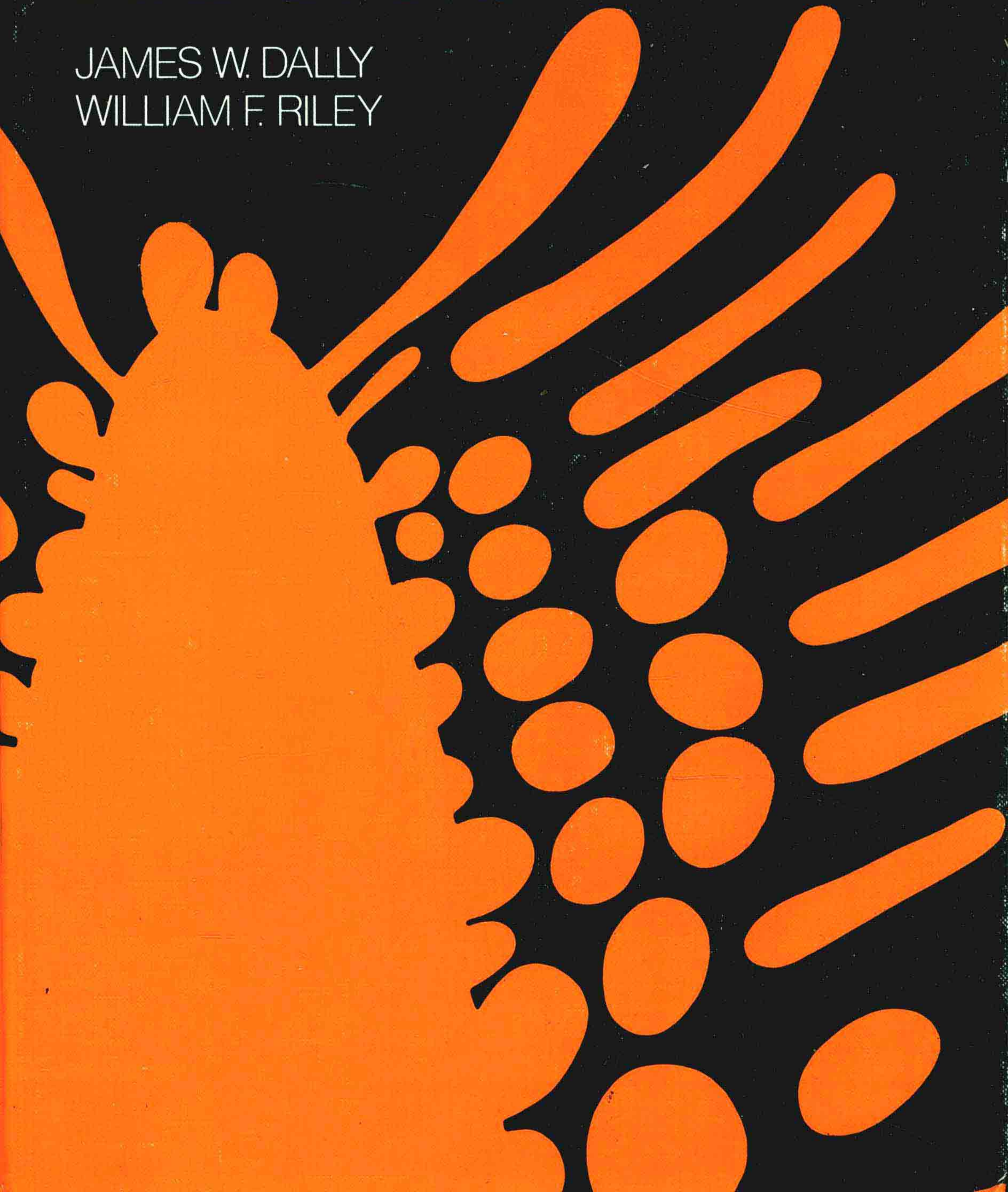
EXPERIMENTAL STRESS ANALYSIS

JAMES W. DALLY
WILLIAM F. RILEY



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Second Edition

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PREFACE

This book has been written for upper-division undergraduate students or graduate students beginning to study experimental stress analysis. The material has been divided into four separate parts:

Part 1. Elementary Elasticity, three chapters
Part 2. Brittle-Coating Methods, one chapter
Part 3. Strain-Measurement, six chapters
Part 4. Optical Methods, six chapters

Each part of the book is essentially independent so that the instructor can be quite flexible in selection of the course content. For instance, a two- or three-credit course on strain gages can be offered by using two chapters of Part 1 and all of Part 3. Parts 1 and 4 can be combined to provide a thorough three- or four-credit course on photoelasticity and moiré. Selected chapters from all four parts can be organized to introduce the broader field of experimental stress analysis. A complete detailed treatment of the subject matter covered here supplemented with laboratory exercises on brittle coatings, strain gages, photoelasticity, and moiré will require six- to eight-credit hours

The essential feature of the text is its completeness in introducing the entire range of experimental methods to the student. A reasonably deep coverage is presented of the theory necessary to understand experimental stress analysis and of the four primary methods employed: brittle coatings, strain gages, photoelasticity, and moiré. While the primary emphasis is placed on the theory of experimental stress analysis, the important experimental techniques associated with each of the four major methods are covered in sufficient detail to permit the student to begin laboratory work with a firm understanding of experimental procedures. Many exercises designed to support and extend the treatment and to show the application of the theory have been placed at the end of all of the chapters.

Laboratory exercises have not been included, since the laboratory work will depend so strongly on local conditions such as the equipment and supplies available, the interests of the instructor and students, and the local industrial problems of current interest. It is believed that the instructor is best qualified to specify the associated laboratory exercises on the basis of interest, equipment, supplies, and time available for this important supplement to the course.

This second edition incorporates extensive revisions which reflect the changes in experimental methods that have occurred in the past 13 years. With brittle coatings, material covering the new nontoxic, nonflammable coatings has been introduced. With electrical strain gages, the changes in the text indicate the almost exclusive use of foil gages today, the marked improvements in instrumentation associated with dynamic recording, and the new data acquisition systems which are being employed to process the experimental data. The text, in part, accounts for the very significant advances made in the past decade in optical methods of experimental stress analysis. The basic chapter on properties of light has been completely revised to account for the behavior of coherent light, and the coverage on both moiré and birefringent coatings has been expanded to a complete chapter on each method. In spite of these extensive revisions, space limitations and publishing costs did prevail and it was necessary to delete coverage of several important topics including statistics, holography, failure theories, fracture mechanics, and nondestructive testing.

It is anticipated that the instructor will, in certain instances, treat these topics by using his own lecture notes or recent papers published in the technical journals. While the text is not in any sense a treatise, it does contain most of the fundamental material necessary to present a complete and practical course on the theory of experimental stress analysis.

The material presented here has been assembled by both authors over a period of 25 years. Courses have been developed on Experimental Stress Analysis, Photoelasticity, and Photomechanics at Illinois Institute of Technology, Cornell University, Iowa State University, and the University of Maryland. The material has been shown to be interesting and comprehensible by the students participating in these courses. The mathematics employed in the treatment can easily be understood by senior undergraduates. Cartesian notation and/or vector notation has been used to enhance understanding of the field equations. A great deal of effort was devoted to the selection and preparation of the illustrations employed. These illustrations complement the text and should aid appreciably in presenting the material to the student.

The contribution of many hundreds of investigators working in the field of experimental mechanics should be acknowledged. This book represents a summary of the state of the art in a field which is continually being advanced by the combined efforts of many patient investigators. In particular, we should like to thank Dr. A. J. Durelli, our mentor for many years. Also, we should acknowledge excellent illustrations provided by M. M. Leven, D. Post, C. E. Taylor, and the many suppliers of commercial equipment. Thanks are also due to Professors Robert Mark of Princeton University, and C. W. Smith of Virginia Polytechnic Institute and State University for their careful review of the manuscript.

James W. Dally
William F. Riley

CONTENTS

PART FOUR OPTICAL METHODS OF STRESS ANALYSIS

LIST OF SYMBOLS

a	amplitude of a light wave
A	area
b	aperture width
B	strength of a magnetic field
B_L	lateral boundary
c	relative stress optical coefficient
c	velocity of light in a vacuum
c_1, c_2	stress optical coefficients
C	capacitance
C	galvanometer constant
C_c	coating coefficient of sensitivity
C_s	specimen coefficient of sensitivity
C_ν	Poisson's ratio mismatch correction factor
d	degree of damping
D	damping coefficient
D	diameter
D	volume dilatation $(\epsilon_1 + \epsilon_2 + \epsilon_3)$
D_0	fluid damping constant
D_0	fog density of a photographic film
e	electron charge
E	electromotive force or voltage
E	exposure
E	magnitude of a light vector
E	modulus of elasticity

$\mathbf{E}$	potential gradient
E^c	modulus of elasticity of a coating
E_m	back electromotive force
E_0	exposure inertia of a photographic film
E^s	modulus of elasticity of a specimen
E^*	modulus of elasticity of a calibrating beam
f	focal length of a lens
f	frequency
f_ϵ	material strain-fringe value
f_σ	material stress-fringe value
F	force
F_{CB}	bending correction factor
F_{CR}	reinforcing correction factor
F_x, F_y, F_z	cartesian components of the body-force intensity
F_r, F_θ, F_z	polar components of the body-force intensity
g	gravitational constant
G	shear modulus of elasticity
G	torsional spring constant
h	thickness
$\mathbf{i}$	current density
I	current
I	intensity of light
I	moment of inertia
I_e	intensity of emerging light
I_g	gage current
I_G	galvanometer current
I_i	intensity of incident light
I_r	intensity of reflected light
I_1, I_2, I_3	first, second, and third invariants of stress
J	polar moment of inertia
J_1, J_2, J_3	first, second, and third invariants of strain
k	dielectric constant
K	brittle-coating strength ratio
K	bulk modulus
K	optical strain coefficient
K	strength of a light source
K_t	transverse-sensitivity factor
K_T	compressibility constant
l	length
l_g	gage length
L	length
$\mathscr{L}$	loss factor
M	magnification factor
n	index of refraction
n	integer

n_0	index of refraction in an unstressed medium
n_1, n_2, n_3	index of refraction along the principal directions
n_1, n_2, n_3	principal directions
N	cycles of relative retardation
N	fringe order
N	number of calibration values
N	number of charge carriers
N	number of cycles
p	pitch of a moire grating
p	pressure
P	force, applied load
P	power
P_D	power density
P_g	power dissipated by a gage
q	resistance ratio
Q	figure of merit
r	resistance ratio
R	radius
R	reflection coefficient
R	resistance
R_b	ballast resistor
R_e	equivalent resistance
R_g	gage resistance
R_G	galvanometer resistance
R_M	measuring-circuit resistance
R_p	parallel resistor
R_s	series resistor
R_x	external resistance
s	distance
s_1, s_2	curvilinear coordinates along an isostatic
S	sensitivity index
S	standard deviation
S_a	axial strain sensitivity of a gage
S_A	strain sensitivity of a material
S_c	circuit sensitivity
S_{CG}	galvanometer-circuit sensitivity
S_g	strain sensitivity of a gage, gage factor
S_{pb}	parallel-balance-circuit sensitivity
S_{sc}	strain sensitivity of a semiconductor material
S_t	gage sensitivity to time
S_t	transverse strain sensitivity of a gage
S_T	gage sensitivity to temperature
S_ϵ	strain sensitivity
S_θ	galvanometer sensitivity
S_σ	stress sensitivity

t	time
T	period
T	temperature
T	torque
T	transmission coefficient
$\mathbf{T}_n$	resultant stress
T_{nx}, T_{ny}, T_{nz}	cartesian components of the resultant stress
T_x, T_y, T_z	cartesian components of the surface tractions
u, v, w	cartesian components of displacement
u_r, u_θ, u_z	polar components of displacement
V	voltage
V	volume
w	width
x, y, z	cartesian coordinates
α	angle of incidence
$\alpha, \beta, \theta, \phi$	angles
α	coefficient of thermal expansion
α_p	polarizing angle
β	angle of reflection
β	coefficient of thermal expansion
γ	angle of refraction
γ	shear strain component
γ	temperature coefficient of resistivity
$\gamma_{r\theta}$	shear strain component in polar coordinates
$\gamma_{xy} = \gamma_{yx}$, $\gamma_{yz} = \gamma_{zy}$, $\gamma_{zx} = \gamma_{xz}$	cartesian shear strain components
δ	displacement
δ	linear phase difference
Δ	relative phase difference
Δ	relative retardation
ΔR_T	resistance change due to temperature
ΔR_ϵ	resistance change due to strain
ϵ	normal strain
ϵ_a	axial strain
ϵ_c	calibration strain
ϵ_n	normal strain component
$\epsilon_r, \epsilon_\theta, \epsilon_z$	normal strain components in polar coordinates
ϵ_t	transverse strain
ϵ_{t^*}	threshold strain for a brittle coating under a uniaxial state of stress
$\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}$	normal strain components in cartesian coordinates
$\epsilon_1, \epsilon_2, \epsilon_3$	principal normal strains
$\epsilon_1^c, \epsilon_2^c$	principal normal strains in a coating
$\epsilon_1^s, \epsilon_2^s$	principal normal strains in a specimen
η	nonlinear term

η_1, η_2	material viscosities
θ	angular deflection
θ_s	steady-state deflection of a galvanometer
λ	Lame's constant
λ	wavelength
μ	mobility of charge carriers
μ	shear modulus
ν	Poisson's ratio
ν^c	Poisson's ratio of a coating
ν^s	Poisson's ratio of a specimen
ν^*	Poisson's ratio of a calibrating beam
ξ	wave number
π	piezoresistive proportionality constant
ρ	radius of curvature
ρ	resistivity coefficient
ρ	specific resistance
σ	normal stress component
σ_n	normal component of the resultant stress
$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$	normal stress components in polar coordinates
σ_{uc}^c	ultimate compressive strength of a brittle coating
σ_{ut}^c	ultimate tensile strength of a brittle coating
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	normal stress components in cartesian coordinates
$\sigma_1, \sigma_2, \sigma_3$	principal normal stresses
σ_1^c, σ_2^c	principal normal stresses in a coating
σ_1^s, σ_2^s	principal normal stresses in a specimen
σ'_1, σ'_2	secondary principal stresses
σ^*	normal stress in a calibrating beam
τ	shear stress component
τ_n	shear stress component of the resultant stress
$\tau_{r\theta}$	shear stress component in polar components
$\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{xz}$	shear stress components in cartesian coordinates
ϕ	Airy's stress function
ω	angular frequency
Ω	body-force function

PART ONE

ELEMENTARY ELASTICITY