MODELS AND SENSITIVITY OF CONTROL SYSTEMS

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STUDIES IN AUTOMATION AND CONTROL

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Introduction

From the very beginning of civilization man has purposely been influencing his environment. This influence can be analysed from various viewpoints.

We can consider the philosophical aspects of this influence, characterizing in various ways the purposefulness of that activity. The scientific aspects are related to the development of natural and social sciences which make an intentional influence possible and to that end are, in fact, being developed. The evaluation of the results achieved by this influence can lead to various conclusions; mankind is not always able to predict precisely the results of its activities. Therefore the methodology of a purposeful activity, of decision making and of analysing the possible misjudgements has attracted the interest of many researchers in various branches of science. One of the branches related to those problems is *control theory.*

Control. A purposeful influence of man or of the technical devices constructed by him on the environment, and in particular on other technical devices or processes, is generally called *control.* The branch of science related to control problems is often referred to as *cybernetics.* The most mathematical part of that science is called, though less commonly, *control theory.* The latter has developed its own system of basic concepts and definitions, but relies on the results of modern mathematics.

The concept of control consists of such detailed concepts as controlled process, constraints, control goal and control performance. The *controlled process* or *object* is that part of the environment which is influenced by the control. The use of the word *process* emphasizes the fact that control and its outcome should not relate to a given moment and steady state only, and that we are primarily interested in the dynamic development in a given time interval of the results of control. *Constraints* of control reflect the fact that the process cannot be controlled in an arbitrary way. The *goal* of control is the postulated result of our control action. If that goal is attainable, then it can usually be reached in many ways. Therefore, it is useful to specify the *performance index* of control as the measure of the quality of control, making it possible to choose between various control actions leading to the same goal. The choice of the best control action that ensures the realization of the control goal and minimization or maximization of the performance index under the given constraints is called *control optimization*. The control goal can also be defined indirectly and the definition results only from control optimization, since an optimal control process might be a goal in itself.

Mathematical model. The choice of a control action is based on the available knowledge of the controlled process, its constraints, its goal and performance index. This knowledge is often not precisely recognized. It may just be a part of our intuition or experience. However, if control actions are to be performed automatically by technical devices, without continuous interventions of man, then it is necessary to specify the available knowledge in a form which is precise enough to be transferred to technical devices. More generally, if a decision to be made is sufficiently important, a detailed analysis of the possible decisions and their outcomes or even a decision optimization might be desirable. Also in this case it may prove useful to specify the available and pertinent knowledge in the form of a mathematical model.

The concept of a *model*, though often abused in the popular sense, is in fact fundamental for many sciences, has a basic cognitive and methodological meaning, and provides a cardinal tool of research. Models can be divided into *cognitive models* and *purpose-oriented models* (though every cognitive model obviously has its purpose); they can be also classified into mathematical, physical, etc., models. For control theory and its applications, *mathematical purpose-oriented models* are of primary importance since their form and accuracy can be adapted to the particular problem of control or decision choice. The methodology of model building and of analysing various properties and applications of mathematical models has been developed together with control theory, particularly in the course of the last four decades. Some recent books—as an example, the monograph by Kalman, Arbib and Falb [55]—try to summarize this broad knowledge. However, usually this synthesis is limited to more abstract properties of mathematical models. The models are then referred to, though not very luckily, as systems or dynamic systems, since the word system has in fact a broader sense.

There are many textbooks and monographs dealing with the fundamental types of models in control theory or with the properties of models for given classes of controlled processes. In this book an attempt is made to present the problems related to mathematical models of control processes from a different viewpoint. Starting with the analysis of the concept of a model, special attention is paid to the purpose-oriented form of the model and the methodology of model-building for a given process, thus through an analysis of the properties of various classes of mathematical models and a description of model identification and verification methods we arrive at a general class of abstract mathematical models. This class is slightly larger than that investigated in [55], since it contains both dynamic and static models.

The above problems are discussed in detail in Chapter 1 of the present book. Here, we shall review some basic notions related to the use of mathematical models for control problems, since they are necessary to permit an approach to the problems considered in the later chapters.

A mathematical model of a control problem is, somewhat simplifying, the sum of the pertinent knowledge related to the controlled process, constraints, control goal and performance index, expressed in the form of mathematical relations. There are many types of such models.

A most useful form of a mathematical model is the *structural model*, which classifies *signals* or *variables* in the process to be controlled and indicates types of relations between those variables without specifying the relations precisely. The classification of process variables starts with the classic division into *input variables* which express the influence of man or of the environment on the process, and *output variables* which express the influence of the process on the environment. The goal of control is usually expressed in terms of output variables.

As regards input variables, it is useful to distinguish (Fig. I.1) *control variables* or *controls* which represent the purposeful influence on the process, *disturbing variables* or *disturbances* which represent various other (usually random) influences of the environment on the process and make the achievement of the goal difficult, and *parameters* or *explanatory variables* which can be assumed to be known with a given accuracy—at least in a given interval of time in which the model is investigated and assumed to be valid. Similarly, it is useful to classify further the output variables.

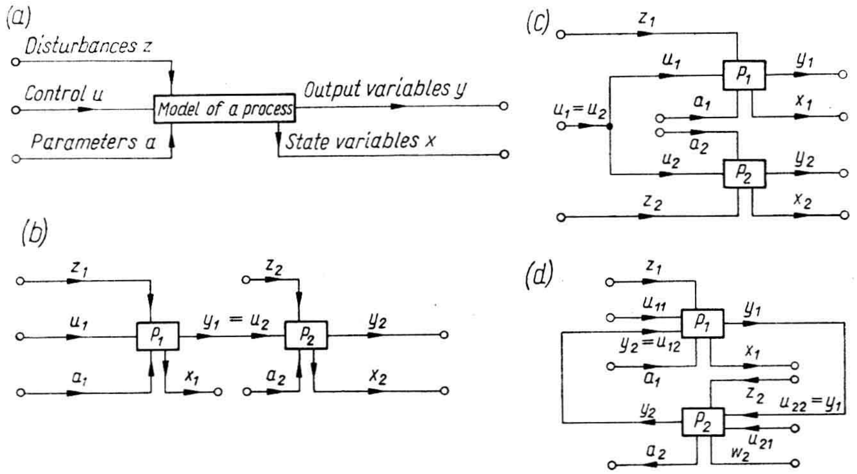

Fig. I.1 Structural models: (a) simplest one, (b) tandem connection, (c) parallel connection, (d) feedback connection.

Processes of a veritably dynamic character are distinguished as possessing some type of memory: the output variables depend not only on the current values of the input variables, but also on their past course. In most types of mathematical models it is possible to represent the whole past of the process by introducing a sufficient number of additional *internal* or *endogenous variables,* called in this case *state variables*†; some of these variables can also be interpreted as output variables. More precisely, the *state of a mathematical model of a process* (briefly, the state of a process) is the smallest set of variables whose values determine the future course of the output variables, if the precise form of the model, its parameters and the future course of other input variables are known. The necessary number of state variables, called the *process dimension,* is usually greater than the number of output variables that can actually be observed in a real process (though of course, the state variables can always be treated as output variables of the process model).

If the analysed process is complicated enough and it is possible to split it into simpler subprocesses with distinct input and output variables, then

† If the concept of state is sufficiently extended, it is possible to provide such a representation of the past for all types of mathematical models. However, the concept of the state is often used only for those models which have a finite dimension, that is, a finite number of state variables. As in the theory of stochastic processes, such models are called Markov-type models.

a structural model comprising the connections between various subprocesses contains important information. There are several basic types of interconnections between subprocesses; some of them are presented in Fig. I.1.

A necessary supplement to the structural model is the information on sets of values of model variables. Depending on the kinds of these sets, we classify the basic types of models. If the sets of values of input and output variables (including state variables) are countable, we speak of a *model discrete in level* or *quantified model.* If, furthermore, these sets are finite then we speak of a *finite model* or a *model with a finite number of states*†. The simplest models of this type are just logical relations between input and output variables. If the sets of values of input and output variables are uncountable (usually they are compact subsets of real axes), we speak of a *model continuous in level.*

Also vital is the information on the set of time instants in which we analyse the behaviour of the process or its model. If this is a countable set, then we are dealing with a *time discrete model* (a difference equation is then the typical form of the input/output relation). If, however, the set is uncountable, then we are dealing with a *model continuous in time* (a differential equation is then the typical form of input/output relation).

Similarly important for the sake of mathematical exactness of model formulation are assumptions about function spaces whose elements are the model variables. For example the control of a time discrete model may be a bounded sequence, or a square summarizable sequence; the control of a model continuous in time may be a square–integrable or bounded or piece-wise constant function.

All the above types of models may be either static or dynamic. In a *static model,* the present values of input variables determine the present values of output variables. In such a model there is no need to introduce the concept of state; the model is zero-dimensional. In a *dynamic model,* the state variables are defined and the model dimension is not zero.

Models which are continuous in level (but not necessarily in time) may be linear, i.e. described by linear relations†† between input and output

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† The finite model should not be confounded with the Markov model. The Markov model has a finite number of indices of state variables, whereas the finite model has additionally a finite number of values of state variables. Technical devices represented by finite models are often called *finite automata.*

†† Strictly speaking, for a model to be linear it is sufficient that the dependence between input, state, and output variables be affine. Addition to the output variable of a component which is independent of input variables does not change the essence of the model.

state variables. A *linear model* is usually the simplest form of a model, and its first approximation which is valid for minor deviations from the nominal working conditions.

A complete model of a control problem contains not only the structural model and a description of its variables but also the exact form of input/output or input/state/output relations, as well as relations which define constraints, the control goal and performance index. In order to obtain a possibly precise model, in the beginning we usually define only the general form of these relations, which depend on parameters with unknown values. When testing the actual process experimentally, and comparing its behaviour with the model, the values of these parameters may be selected to achieve the possibly best representation of the process by the model. This method of fitting a model to a process is called *process identification*. There are many identification methods, suitable for different types of models. In some special cases, a model can be ideally fitted to a single run of a process; but it is impossible to build an ideal model which will truly reflect the process behaviour in all possible situations. This results from the fact that a model is just a model and does not cover the wide scope of phenomena occurring in the process and in the surrounding environment.

An often useful method of model perfection is the use of a *probabilistic model*—a model in which relations between output and input variables are not fully determined, but only described by probability distributions or by stochastic processes†. But a probabilistic model remains a model; to define it fully we must specify the types of probability distributions and the parameters of these distributions, which again leads to identification problems, etc. In some real applications better results are achieved by applying probabilistic models, in others, *deterministic models*††. However,

† Since model parameters are understood as a special type of input variables, we can assume that without loss of generality, the input/output relations are always non-probabilistic, and only the input variables—and, consequently the output variables—may be of a probabilistic nature. A probabilistic model can therefore be understood as one in which at least one input variable is a random variable or a stochastic process.

†† More precisely, nonprobabilistic models, because probabilistic models may also be interpreted from a deterministic point of view. The terms determinism and indeterminism are philosophical concepts, expressing a point of view on the character of nature; the probabilistic and nonprobabilistic methods may be used by researchers with either deterministic or indeterministic points of view (even though indeterminism considers probabilistic models as ultimate, cf. the probabilistic theory of elementary

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both these types of model involve a basic inconsistency related to the concept of mathematical model: the greater exactness of the model is required (when the amount of experimental data is limited) the less confident conclusions can be drawn regarding its form, parameters, and extent of applications.

The inconsistency between the exactness and utility of a mathematical model is one of the principal paradoxes in the history of science, in the process of man learning about nature. The notion "physical, biological or economical law" has been used in two senses. In one sense, it is understood as an objective property of nature, of our environment. Owing to such objective property, the behaviour of our environment can be scientifically examined and is reproducible provided that the corresponding experiments are conducted in the same conditions. In another sense, a law of nature denotes its model, usually a mathematical model, formulated by man. In this sense we speak of Newton's laws, the Einstein laws or the probabilistic model of an atom. And even though such models are of tremendous cognitive significance, it has always been so that their area of application is limited, that more precise models can be constructed, often of a different type, after a suitable amount of experimental data have been gathered and analysed. Thus we are close to the conclusion that it is a methodological error to attach an absolute value to a more-or-less perfect model, which error can only partially be justified by the natural desire of researchers to gain a better knowledge of nature by constructing ever-better models of its objective laws.

Sensitivity analysis. In the applications of control theory the choice of a model, when constructing purpose-oriented models for solving a particular control problem, is to a great extent arbitrary, being the result of compromise between the required exactness and the effort required to collect a greater amount of experimental data and determine a more exact model. Since absolutely precise models are not used (in fact do not exist), the effect of model inaccuracy on decisions based on this model acquires basic significance.

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particles). Nevertheless the term *deterministic model* is still often used in the everyday meaning of *nonprobabilistic* model, and it is in this sense that it will be used in the present book.

This problem is not new and its simpliest variant has been examined in many fields of science. The basic variant may be formulated as follows. A certain decision is considered which is supposed to have a specific effect when applied to a given model. The first question is: how will the effects of this decision change if we apply it to real life and not to a model? This question can only be answered by practical tests which, however, may turn out to be costly and time-consuming. Hence let us pose the next question: how will the results of decision be affected if the model to which this decision applies is changed (usually only slightly)?

Notice that in fact we are dealing here with two models; the first, on which the decision has been based, and the second, for which the influence of model changes on the results of the decision is analysed. These models differ in their purpose and character. The first one represents the best purpose-oriented approximation of our knowledge of the problem at hand and serves for decision making. We shall call it the *basic model.* The other one represents the possible deviations of reality from the first model, more precisely—those deviations which we consider to be crucial and likely to occur; we shall call it the *extended model,* because it is usually more complicated than the basic one.

Consider, by way of example, a problem of tolerance analysis, that is, the examination of the correctness of an engineering design while allowing for possible inaccuracies of the elements. Let the decision, based on the basic model, relate here, for example, to the dimensions of mechanical components; the extended model represents the possible inaccuracies of these dimensions characterized either in a deterministic way by specifying the tolerance limits, or in a statistical way by specifying the random distributions of the deviations or their basic moments.

The need for a fully precise distinction between the two models, the basic and the extended one, is not very apparent when only the first variant of the problem is considered. It might seem that the analysis of one model with changing parameters would be sufficient. In the above example it seems to be immaterial whether an extended model which allows for inaccuracies of dimensions (resulting, e.g., from the manufacturing method) is considered, or if it is assumed that deviations of the same character occur in the basic model (resulting, e.g., from inaccuracies in designer's calculations). Such a principle of relativity of the basic and extended model deviations could be very useful, since it would contribute to a significant simplification of the analysis. However, this principle, though it can be applied in many cases, is not universally true. It may give wrong results, for example, when the basic model has extremal properties (i.e., if deviations of any sign in the model result in decision deviations of one sign only).

The situation becomes even more complicated when we consider models of control problems. One of the basic methods of decreasing the influence of model inaccuracy on the results of control is the use of *feedback* or *closed-loop control systems.* Instead of accepting that control decisions are dependent only on a mathematical model of the problem (the so-called *open-loop control system,* see Fig. I.2a), we can use the mathematical model to select a certain control law, which makes the control of the real process dependent on the results of measurements of output variables in this process (the closed-loop control system, see Fig. I.2b).

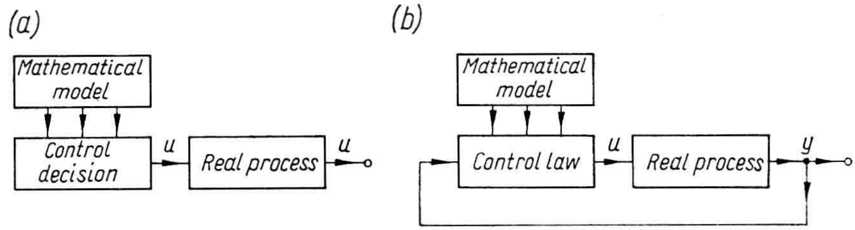

Fig. I.2 Basic structures of control systems: (a) open-loop, (b) closed-loop.

The problem of the effect of model inaccuracy on the results of control can then be formulated as follows: a certain control law is given, which when applied to the given model brings about some expected results. How would these results change, if the control law were applied to a real process and not to the model? If experimental tests are to be preceded by a more detailed analysis, then how do the postulated results of control change when we change the model of the process to which the law is applied?

Of course, the selection of the control law is also a certain decision, so the methodology of analysis does not change fundamentally. Thus for a precise definition of the problem two models are required: the basic one, on which the control law is based, and the extended one, which represents the most likely and significant deviations of the basic model from reality. The control law should be applied to the extended model and the influence of model changes on the control results should be examined.

However, the methodology of analysis changes since cases in which the basic model and the control law resulting from it have extremal properties, are much more frequent. This is the case especially when control optimization is performed, or the optimal control law is defined. Moreover, optimal control systems can have open-loop or closed-loop structures as shown in Fig. I.2, as well as many other structures.

Examination of the influence of model inaccuracy on the outcome of application of the control law resulting from it is called *sensitivity analysis* of the control system. These studies have quite a history. For nonoptimal control systems sensitivity, such investigations were started by Bode [12]; Tomović was the first to devote a monograph to this problem [115]; further, the works of Kokotović and Rutman [57] as well as the others [18] deserve mention. In all these publications, however, distinction between the concepts of a basic model and an extended one is neither methodologically analysed nor consistently pursued; also, conditions under which this distinction is not necessary are not precisely formulated. This approach resulted from the intuitional assumption of relativity of changes in these two models; on the other hand, this assumption is correct in most cases of nonoptimal control system analysis. The problem of optimal control system sensitivity analysis was defined by Bellman [9] and next analysed by Dorato, Pagurek and Witsenhousen [95, 133, 18]. These publications already distinguish two models without which it would be impossible to analyse the optimal control system sensitivity. However, they provide no thorough analysis of the basic properties of these models and their relation to the formulation of the optimal control problem. Pagurek and Witsenhousen have achieved here a mathematically correct result, which, however, presents serious interpretational difficulties. They have proved that the linear approximations of the performance index changes, caused by changes of parameters in open- and closed-loop systems, do not depend on the use of feedback, and therefore are the same for both these systems. From this fact a paradoxical conclusion was drawn that the performance index sensitivity is the same in both the closed- and open-loop optimal control systems. Further works have either accepted this conclusion or confirmed it under more general assumptions and in a different formulation. Kreindler [62], accepting that the performance index sensitivity is identical for closed- and open-loop systems, examined the differences of state trajectory sensitivity in these systems. Kokotović *et al.* [58] have gone further: assuming the possibility of an exact evaluation of parameter changes and of an approximation of the control law by a Taylor series expansion depending on these changes, they have proved that also higher derivatives of performance index changes can be identical for both the closed- and open-loop systems. This trend has been pursued in a number of other works [18].

The present author undertook studies in another direction [121, 122, 128]. While examining the basic conditions of control optimality and fundamental properties of the basic and extended models, it is possible to prove that the Pagurek–Witsenhousen result is a natural outcome of these conditions and properties, and, moreover, that it is true for any optimal control system, not being limited only to the basic variants of open- and closed-loop systems. One should not conclude, however, that this implies identical sensitivity in open- and closed-loop systems. In view of the basic conditions of control optimality, the sensitivity of different systems can only be compared on the basis of second derivatives of the performance index with respect to parameters (even though the first derivatives may be non-zero, as optimization relates to control and not to parameters). Under realistic assumptions the differences in the performance index sensitivity can be very large†.

This trend was followed in many further works [7, 10, 25, 45, 82, 97, 114, 117, 118, 125, 135]. Chapters 2 and 4 of this book dealing with the sensitivity analysis of optimal and nonoptimal control systems constitute a summary and an attempt to synthesize the results of these works.

Chapter 2 deals with model sensitivity and control system sensitivity. Model sensitivity analysis is considered to be the examination of changes of solutions, output variables, or state variables of a mathematical model due to changes of its parameters. Results of such an analysis are useful, if the hypothesis of relativity of the extended and basic model parameter changes is valid in the control system sensitivity analysis. Thus concepts and properties of extended and basic models are also discussed in detail in Chapter 2, the relativity principles, that is, conditions under which control system sensitivity analysis is equivalent to examining the sensitivity of only one model, being examined. Though a part of the detailed results in Chapter 2 has been derived from earlier works, their presentation and

† Seemingly, this statement contradicts the results obtained by Kokotović *et al.* [58] that also higher derivatives of the performance index changes can be identical. However, the results of the quoted authors are based on the assumption that a precise measurement of the parameter changes is possible and that these measurements are used for control purposes. Such an assumption might be subjected to a methodical criticism: had it been acceptable, then an expanded model could always be made equal to the basic one and entire sensitivity analysis would be pointless.

full interpretation is based on the generalization of the concept of a model and on an analysis of the properties of extended and basic models.

Chapter 3 constitutes a preparation to Chapter 4. It deals with optimization problems and methods for the determination of optimal control. This chapter is a synthetic review of an extraordinarily wide range of research results in this field, also taking into account some results of the present author and his collaborators [42, 68, 73, 126, 129]. The systematics of the material is taken from the present author's work [39].

Chapter 4 is dedicated to the sensitivity analysis of optimal control systems. It is a synthesis of research results obtained by the author and his collaborators in this field. It contains a generalized formulation and a detailed discussion of optimal control system sensitivity analysis, the distinction between the extended and basic model analysis, an interpretation of the Pagurek–Witsenhousen paradox, and several examples of differences in performance index sensitivity. Different structures of optimal control systems, conclusions on their applicability, as well as some more detailed problems are discussed in this Chapter.

The present book is basically a monograph addressed to researchers in the field. However, the more fundamental parts of it are formulated in text-book form, to facilitate its use. It may serve as supplementary reading for senior or graduate students of control sciences, computer sciences, operational research, or systems sciences, and also for students of mathematics interested in mathematical control theory. For its full use, a certain mathematical knowledge in the fields of algebra, differential and difference equations, operations calculus, the Laplace, Fourier, and Laurent transforms, fundamentals of the calculus of variation and functional analysis, mathematical statistics and stochastic process theory is required.

Although the book has a fairly theoretical approach, it gives examples of the construction of specific models for industrial processes as well as of the identification and utilization of these models for optimal control purposes, together with sensitivity analysis. It may therefore also be of interest to engineers designing complex systems of industrial control.

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