ANALYTIC GEOMETRY

Analytic Geometry

FIFTH EDITION

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Preface

One major difference between the present edition and the fourth edition of this book is in the treatment of polar coordinates. Here polar coordinates are introduced in Chapter 2 and used in various later chapters. I have attempted to convey to the student the idea that polar coordinates are a tool, not an isolated topic, in analytic geometry.

The two chapters on calculus have been omitted. Simple differentiations naturally appear in problems involving tangents and are studied in Chapter 10. The basic concepts of curve tracing appear in Chapter 3, but a detailed treatment of the sketching of curves of degree greater than two is delayed until after the conics have been studied. The space devoted to solid geometry has been slightly enlarged.

New topics in the fifth edition are: the distance formula in polar coordinates, circles of Appolonius, radical axis, common chord, tangents to a conic from an external point, chord of contact, the shape of certain higher plane curves, parametric equations of lines, circles, conics, the method of least squares, parametric equations of lines in space, generation of surfaces of revolution.

Certain chapters may be omitted entirely, or in part, to make a shorter course. The chapters on tangents and normals to conics, families of curves, curve fitting are self contained. The chapters on parametric equations, trigonometric functions, exponentials and logarithms can also be omitted, if so stringent a cut is necessary. Attention is called to the fact that a shorter book, Love's *Elements of Analytic Geometry* also published by The Macmillan Company, was prepared for a short course in this subject.

I wish to thank Professor Fred Brafman of Wayne University for an independent check on material in the text and on most of the answers to exercises, Professor Ralph L. Shively, Oppenheim Professor of Mathematics at Manchester College, for an independent reading of the proofs, and Professor Donat K. Kazarinoff of the University of Michigan for various useful suggestions.

Earl D. Rainville

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CHAPTER 1 *Rectangular Coordinates*

1. Introduction. It is common practice to divide geometry into two kinds, synthetic and analytic. Synthetic geometry, usually first studied in high school, employs the straight edge and compass as its basic tools. Elementary analytic geometry, the subject of this course, uses algebra (equations, formulas, and their algebraic manipulation) as its main tool.

A fundamental goal of plane analytic geometry is the investigation of interesting and useful properties of configurations involving points, straight lines, and curves other than straight lines. Not only does the use of algebra contribute to the study of geometry, but geometric interpretation of algebraic equations and manipulations results in a fuller comprehension of many phases of algebra.

Analytic geometry can be developed from a system of axioms and definitions as is usually done with synthetic geometry. In this course, however, we do not attempt a complete separation of the analytic from the synthetic. We shall use freely a few theorems and concepts from earlier courses taken by the student.

2. Directed line segments. When a line segment is measured in a definite sense *from* one endpoint *to* the other, the segment is said to be *directed*. If the terminal points are A and B, we speak of the segment AB or the segment BA according as the sense is from A to B or from B to A.

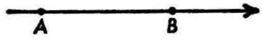

Figure 1

If one sense is chosen as positive, then the opposite sense is negative: thus

$$AB = -BA, \qquad \text{or} \qquad AB + BA = 0.$$

If C is any third point of the straight line through A and B,

Figure 2

then for all possible positions of A, B, and C we have

(1) $$AB + BC = AC,$$

(2) $$AB + BC + CA = 0.$$

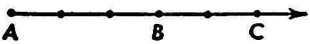

Figure 3

Figure 4

For example, in Fig. 3, with the positive direction to the right,

$$AB = 3, \qquad BC = 2, \qquad AC = 5, \qquad CA = -5;$$

in Fig. 4, $AB = -1$, $BC = 5$, $AC = 4$, $CA = -4$.

Two directed segments lying in the same line or in parallel lines are said to be *equal* if they have the same length and are measured in the same sense.

In ordinary affairs we think of distance as a directed or undirected quantity, according to circumstances. Say that we drive 5 miles, then have to return to the starting point for repairs. As regards gasoline consumption, we have traveled (undirected segments) 10 miles; the net advance toward our destination (directed segments) is zero.

3. Position of a point on a surface. If a point lies on a given surface, two magnitudes, or "coordinates," are necessary to determine its position, each coordinate being measured in a definite sense. Thus we may say that one town is 10 miles east and 8 miles south of another. Note that without the directions — east and south — the coordinates would be ambiguous and therefore useless.