DORAISWAMI RAMKRISHNA

NEAL R. AMUNDSON

LINEAR OPERATOR METHODS in CHEMICAL ENGINEERING

with Applications to Transport and Chemical Reaction Systems

PRENTICE-HALL INTERNATIONAL SERIES in the PHYSICAL and CHEMICAL ENGINEERING SCIENCES

An introduction to functional analysis and the spectral theory of linear operators for engineers, applied scientists, and mathematicians.

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with Applications to Transport and Chemical Reaction Systems

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To Ponnu and M. R. Doraiswami
and
To Shirley D

Preface

In recent years there has been growing consciousness of the role of functional analysis in the application of mathematics to the solution of scientific and engineering problems. It has manifested in the appearance of several expositions, differing in degree of rigor, to an audience whose affinity for abstraction is tempered by concern for its functional attributes. While this atmosphere should prove conducive for another text on the subject, one is left with a sense of apprehension that treatments in the area have been content with dwelling more on the expository features of functional analysis than on demonstrations of its special capabilities. Thus many applications have generally been confined to traditional problems albeit in the elegant garb of functional analysis. In writing this book our motive has been to demonstrate that the deductions of a general theory play their most important role in expanding the class of solvable problems and that elegance is merely a way of life.

Our interest in the area was spurred by a course in the middle sixties taught in the mathematics department of Minnesota by Professor G. R. Sell, who subsequently coauthored a book with A. W. Naylor, titled *Linear Operator Theory in Engineering and Science* and published by Holt, Rinehart and Winston in 1971. We would also like to record our special appreciation to the faculty of the Mathematics Department at the University of Minnesota, whose extraordinary interest in the applications of mathematics and interaction with colleagues in engineering fostered a unique mathematical culture among the engineering students and faculty there. An early book that has influenced us is B. Friedman's *Principles and Techniques of Applied Mathematics*, published by John Wiley & Sons in 1956.

In that abstraction is apparently antithetical to the art of practicality, it is but natural that the engineering community would view askance any lengthy exposition of the abstract elements of a subject such as linear operator theory. On the other hand, to establish that the cause of practicality can in fact be served most profitably by generalizations perceived through abstraction, it calls for some indulgence in the abstract theory. A happy medium seemed to us not possible given the heterogeneity of our audience, ranging from the (non-empty) set of the mathematically sophisticated to those that have little more than cursory contact with conventional mathematical techniques in engineering curricula. After teaching several versions of a graduate course in different chemical engineering departments such as the Indian Institute of Technology at Kanpur, the University of Minnesota, Purdue University, and the University of Houston, we have converged on a treatment with a proposal for different modes of coverage depending on the background of our audience. Although the problems governing applications are generally in the analysis of transport and chemical reaction systems and are of primary interest to chemical and mechanical engineers, we expect that the book may be of interest not only to other engineers but to scientists and applied mathematicians as well. Broadly, we have viewed our audience in three classes, with at least the background of an elementary course on calculus, matrix algebra, and differential equations.

Class I: Seeking detailed understanding of the elements of linear operator theory, and innovative skills.

Class II: Aspiring for a non-leisurely introduction to the benefits of the operator-theoretic method.

Class III: Seeking a quick appraisal of the special attributes of the book or solutions of certain types of problems.

It is not suggested that the foregoing sets be disjoint. It is quite conceivable that one in class I may choose to be in either II or III for a first reading. A course addressed to class I would last a year while that for class II could be given in a (hectic) semester. Class III is viewed as a heterogeneous group consisting of those that are either well-informed about operator-theoretic methods and/or those that have a particular interest in solving various types of linear boundary value problems.

A diligent coverage of the book in sequence is meant for class I. Chapter 0 covers rather elementary aspects of set theory. Particular attention is called to the Index of Symbols at the end of the book. The purpose of Chapter 1 is to provide a non-constructive development of real numbers with focus on certain properties of the real number system that carry over to the so-called Banach and Hilbert spaces on which the linear operators of interest are defined. Chapter 2 deals with the algebraic features of linear vector spaces. Chapter 3 introduces topological aspects in the setting of a metric space. Chapter 4 discusses the elements of Lebesgue integration. The treatment of linear spaces in which algebraic features are combined with topological properties is covered in Chapters 5 and 6. Spectral theory of self-adjoint operators has been treated separately in Chapter 7. The applications of linear operators have been covered in Chapters 8–11; Chapter 8 deals with applications in infinite dimensional space while Chapter 9, the longest chapter in the book, discusses differential operators and applications to a diverse variety of boundary value problems. In Chapter 10, partial differential operators and their applications are treated. Chapter 11 is an introduction to the theory of non-self-adjoint operators. We ask for repeated readings of the Foreword at the beginning of each chapter and the Concluding Remarks at the end to maintain perspective throughout the book. Special emphasis is placed on the exercises, which frequently introduce extensions and new applications.

For class II, we recommend beginning with Chapter 5 and working through Chapter 11 with ad hoc referrals to Chapters 0–4 as and when it becomes necessary.

For class III, Chapter 8, Sections 9.3–9.5 in Chapter 9, Chapter 10, with special emphasis in Sections 10.2–10.5, and Chapter 11 are recommended.

In classifying our readership and making alternative recommendations for coverage of the text material our motivation has been to prevent frustration in an enquiring reader more enthusiastic about applications than about the methods of mathematics. But we are unable to contain our pleasure when Bertrand Russell's dictum that "the best of mathematics is not merely to be learned as a task but to be assimilated as a part of daily thought," evokes a sparkling response of approval from a student.

An effort such as this is based liberally on the support of several faculty colleagues and students. Without their persistent encouragement, and occasionally embarassing enquiries about the status of our manuscript, we could well have been perpetually in the stage of rewriting, modifying, and cleaning up. Particular mention must be made of Professor James M. Caruthers, whose observations were truly most stimulating and whose continued encouragement to complete the manuscript provided the most impetus to its conclusion. The direct or indirect contributions of many of our students, Terry Papoutsakis (now at Rice), Brian Turner, and Shankar Nataraj, to mention only a few, are gratefully acknowledged. Mrs. Christa Van Etten deserves special commendation for her excellent typing of the manuscript. We gratefully acknowledge a grant from the Education Development Center at Kanpur for typing a very early version of this manuscript, the Purdue School of Chemical Engineering for free typing facilities, and the University of Houston for travel grants to enable us to meet for frequent discussions on the manuscript.

Finally, the forbearance of our wives, Geetha (who also helped compile the author index) and Shirley, made a subtle contribution too important not to receive special mention.

D.R., N.R.A.

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Contents

Sets, Mappings, and Other Preliminaries **0**

0.0 Foreword

The present chapter is concerned with certain preliminaries by way of mathematical language used to lay out the material in succeeding chapters. In this sense it contains a glossary of terminology that will be used throughout the book. It must be noted that set theory represents, in fact, the very fabric of modern mathematics. In addition, set-theoretic concepts and the algebra of sets, known as Boolean algebra, have a number of useful applications. For example, electrical network synthesis draws richly from Boolean algebra. It stands to reason that the usefulness of set-theoretic concepts must be felt in situations where "structure" or "configuration" is an important consideration. Our motivation for this chapter, however, is not to develop its contents to any degree of completeness required for its applications per se but rather because of its implications to subsequent material.

0.1 Introduction

In this chapter we present the rudiments of mathematical abstraction. Fundamentally, mathematics deals with abstract entities, called *objects* or *elements*, which invariably arise from a collection, class, or family of objects called a *set*. The objects are therefore referred to as members or elements of the set from which they arise. The concept of an object has a high degree of abstraction, since "object" does not necessarily have to be perceived as a stone or a tree; indeed, it may be anything, such as a particular sound or a smell or an algebraic symbol. Perceptions of any kind relating to objects are only necessary for us to be able to talk about them. Also, in some circumstances *an* object may well refer to several entities or a collection of entities. Thus in one context, we may refer to sets whose elements are themselves sets from a different context.

The reader is forewarned that the term "object" will be used frequently in this book. It is not relevant to ask what these objects are, for, as we had observed earlier, such nonspecificity is a part of mathematical abstraction. The set to which the objects belong will be identified by *assertions* of *properties* of its members. There is no more (or no less) to the identity of these objects than their properties, which define them. In order to talk about them, we will represent them by algebraic symbols, and assign them names, such as *numbers*, *scalars*, *vectors*, and so on.

0.2 Sets, Elements, and Operation in Sets

We have, in an intuitive way, defined a set and its constituent elements. We have deliberately refrained from using specific examples of sets and elements in order to promote a bit of abstract thinking on the part of the student. It is essential to make a symbolic representation of the statement "*this* object belongs to *that* set," in other words, to provide a notational framework in which mathematical statements such as the one in quotes may be expressed conveniently and concisely. Thus *that* set may be denoted by the letter A and *this* object by the letter a and the foregoing statement may be expressed by the concise representation "$a \in A$," which is read "a is an element of A." The negation of this statement, "a is not an element of A," is represented by "$a \notin A$."

Often the qualification is made that a set is nonempty; that is, it contains at least one element. This may sound irksome to the engineering student, who might quip: "Why would anyone be interested in talking about a set that contains nothing?" It should be realized, however, that a set is frequently defined by one or more mathematical propositions involving the elements of the set and that the *empty* or *null* set is a convenient instrument for describing situations under which no elements exist satisfying the propositions stated. Capital letters such as $S, A, B, \ldots$ will be used to denote sets. The definition of a set by propositions will be represented as follows:

$$A = \{a\text{: propositions involving } a\}$$

The letter a in braces above is a typical element of the set and the propositions involving a are one or more mathematical statements concerning a. Some-