FIELD THEORIES OF CONDENSED MATTER SYSTEMS

Field Theories of Condensed Matter Systems

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ADDISON-WESLEY PUBLISHING COMPANY
The Advanced Book Program
Redwood City, California • Menlo Park, California • Reading, Massachusetts
New York • Don Mills, Ontario • Workingham, United Kingdom • Amsterdam
Bonn • Sydney • Singapore • Tokyo • Madrid • San Juan

Publisher: *Allan M. Wylde*
Physics Editor: *Barbara Holland*
Marketing Manager: *Laura Likely*
Production Manager: *Jan V. Benes*
Production Assistant: *Karl Matsumoto*

Library of Congress Cataloging-in-Publication Data

Fradkin, Eduardo.
Field theories of condensed matter systems/ Eduardo Fradkin.
p. cm. -- (Frontiers in physics; no. 82)
Includes bibliographical references.
1. High temperature superconductivity 2. Hubbard model.
3. Antiferromagnetism. I. Title. II. Series: Frontiers in physics; v. 82.
QC611.98.H54F73 1991 537.6'23--dc20 91-34574
ISBN 0-201-52253-5

This book was prepared by the author, using the T_EX typesetting language.

1 2 3 4 5 6 7 8 9 10-MA-95 94 93 92 91

Frontiers in Physics

DAVID PINES/Editor

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Editor's Foreword

The problem of communicating in a coherent fashion recent developments in the most exciting and active fields of physics seems particularly pressing today. The enormous growth in the number of physicists has tended to make the familiar channels of communication considerably less effective. It has become increasingly difficult for experts in a given field to keep up with the current literature; the novice can only be confused. What is needed is both a consistent account of a field and the presentation of a definite "point of view" concerning it. Formal monographs cannot meet such a need in a rapidly developing field, and, perhaps more important, the review article seems to have fallen into disfavor. Indeed, it would seem that the people most actively engaged in developing a given field are the people least likely to write at length about it.

FRONTIERS IN PHYSICS has been conceived in an effort to improve the situation in several ways. Leading physicists today frequently give a series of lectures, a graduate seminar, or a graduate course in their special fields of interest. Such lectures serve to summarize the present status of a rapidly developing field and may well constitute the only coherent account available at the time. Often, notes on lectures exist (prepared by lecturers themselves, by graduate students, or by postdoctoral fellows) and are distributed on a limited basis. One of the principle purposes of the FRONTIERS IN PHYSICS Series is to make such notes available to a wider audience of physicists. A second principal purpose which has emerged is the concept of an *informal monograph*, in which authors would feel free to describe the present status of a rapidly developing field of research, in full knowledge, shared with the reader, that further developments might change aspects of that field in unexpected ways.

The physics of strongly correlated electron systems is arguably the most rapidly developing field of research in condensed matter physics, and quite possibly, in all of physics. It includes not only the recently discovered high temperature superconductors, but also heavy electron systems and quantum antiferromagnetism. The underlying mathematical description of many of the phenomenon of interest is closely related to that required for the integral and fractional quantum Hall effect, and is closely related to much current research in field theory, most notably Chern-Simons theories. Eduardo Fradkin is a leading researcher in this interface between statistical physics and elementary particle physics.

The present informal monograph on the field-theoretic description of strongly correlated electron systems provides graduate students and senior researchers alike with the first detailed account of this important sub-field of physics. It is a pleasure to welcome him to "Frontiers in Physics".

David Pines
Urbana, Illinois
February 1991

Preface

This volume is an outgrowth of the course "Physics of Strongly Correlated Systems" which I taught at the University of Illinois at Urbana-Champaign during the Fall of 1989. The goal of my course was to present the field-theoretic picture of the most interesting problems in Condensed Matter Physics, in particular those relevant to High Temperature Superconductors. The contents of the first six chapters is roughly what I covered in that class. The remaining four chapters were developped after January 1, 1990. Thus, that material is largely the culprit for this book being one year late! During 1990 I had to constantly struggle between finalizing the book and doing research that I just could not pass on. The result is that the book is one year late and I was late on every single paper that I thought was important! Thus, I have to agree with the opinion voiced so many times by other people who made the same mistake I did and say, don't ever write a book! Nevertheless, although the experience had its moments of satisfaction, none was like today's when I am finally done with it.

This book exists because of the physics I learned from so many people, but it is only a pale reflection of what I learned from them. I must thank my colleage Michael Stone, from whom I have learned so much. I am also indebted with Steven Kivelson, Fidel Schaposnik and Xiao-Gang Wen who not only informed me on many of the subjects which are discussed here but, more importantly, did not get too angry with me for not writing the papers I still owe them.

This book would not have existed either without the extraordinary help of Christopher Mudry, Carlos Cassanelo and Ana Lopez who took time off their research to help me with this crazy project. They have done an incredible job in reading the manuscript, finding my many mistakes (not just typos!), making very useful comments and helping me with the editing of the final version. I am particularly indebted to Christopher who made very important remarks and comments concerning the presentation of very many subjects discussed here. He also generated the figures. Mrs. Phyllis Shelton-Ball typeset the first six chapters. My wife, Claudia, made this project possible by learning TEX at great speed and typesetting the last four chapters, correcting some of my very boring and awkward writing style.

This book was also made possible by the love and help of my children Ana, Andres and Alejandro, who had to live with a father who became a ghost for a while. Ana and Andres helped in the proofreading, took care of

their little brother, who helped by keeping everybody happy.

Finally, I must acknowledge the support of the Department of Physics and the Center for the Advanced Study of the University of Illinois. The help and understanding of the staff at Addison Wesley is also gratefully acknowleged.

Eduardo Fradkin
Urbana, Illinois
January 1991

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CHAPTER 1

Introduction

1.1 Field Theory and Condensed Matter Physics

Condensed Matter Physics is a very rich and diverse field. If we are to define it as being "whatever gets published in the Condensed Matter section of a physics journal", we would conclude that it ranges from problems typical of material science to subjects which are as fundamental as particle physics or cosmology. Because of its diversity, it is sometimes hard to figure out where the field is going, particularly if you do not work in this field. Unfortunately, this is the case for people who have to make decisions about funding, grants, tenure and other unpleasant aspects in the life of a physicist. They have a hard time figuring-out where to put this subject which is neither applied science nor dealing with the smallest length scales or the highest energies. However, the richness of the field comes precisely from its diversity.

The past two decades have witnessed the development of two areas of Condensed Matter Physics which best illustrate the strengths of this field: Critical Phenomena and the Quantum Hall Effect. In both cases, it was the ability to produce extremely pure samples which allowed the discovery and the experimental study of the phenomenon. Its physical explanation required the use of concepts and the development of new theoretical tools, such as the renormalization group, conformal invariance and fractional statistics.

While the concept of conformal invariance was well known in field theory before Critical Phenomena was recognized as a field, its importance to the complete structure of the field theory was not understood. The situation changed with the development of the Renormalization Group (RG). For Condensed Matter Physics, the RG is the main tool for the interpretation of the experimental data, the conceptual framework and the computational algorithm which has allowed the theory to make powerful predictions. In Particle Physics, the RG is also a tool for the interpretation of the data. But, more importantly, the concept of infrared unstable fixed point has become the *definition* of the field theory itself.

Similarly, the Chern-Simons theories, which are field theories which describe systems exhibiting fractional statistics were known before the Quantum

Hall Effect (QHE) was discovered (actually they were discovered at about the same time) but were regarded as a curiosity of field theories below four dimensions: in other words, a beautiful piece of Mathemetical Physics but without relevance to "the world". We have come to recognize that Chern-Simons theories are the natural theoretical framework to describe the Quantum Hall Effect.

Another case to this point is superconductivity. Viable mechanisms for superconductivity have been known for the thirty-some years that have passed since the theory of Bardeen, Cooper and Schrieffer (BCS). This theory has successfully explained superconductivity, and a variety of related phenomena, in very diverse areas of Physics. This theory has been applied to such diverse areas of physics ranging from superconductivity in metals and superfluidity of liquid He_3 in condensed matter physics, to neutron stars and nuclear matter in nuclear physics, to dynamical symmetry breaking and grand unification mechanisms (such as technicolor) in elementary particle physics.

The origin of this constant interplay between Field Theory and Condensed Matter (or Statistical) Physics is that, despite their superficial differences, both fields deal with problems which involve a large number (macroscopic) of degrees of freedom which interact with each other. Thus, it should be of no surprise that the *same techniques* be used in both fields. The traditional trend was that field theory provided the tools (and the "sexy" terms) which were later adapted to a condensed matter problem. In turn, condensed matter models were used as "toy models" in which to try new techniques. For the most part, this is still the case. However, as the examples of the RG and the QHE show, the "toy models" can provide a framework for the understanding of much more general phenomenon. The *experimental accessibility* of condensed matter systems is just as important. The MOSFETS and heterojunctions, in which the Quantum Hall Effect is studied, have given us the surprisingly exact quantization of the Hall conductance whose understanding has required the use of Topology and Fiber Bundles.

The importance of Condensed Matter Physics to Field Theory, and viceversa, has been recognized at least since the 1950's. Landau and Feynman are perhaps the two theorists who best understood this deep connection. They worked in both fields and used their ideas and experience from one field in the other and then back.

1.2 What Has Been Included In This Book

This volume is an outgrowth of the course "Physics of Strongly Correlated Systems" which I taught at the University of Illinois at Urbana-Champaign during the Fall of 1989. Much of the material covered here has been the subject of intense research by a lot of people during the past four years. Most of what I discuss here has never been presented in a book, with the possible exception of some reprint volumes. While the *choice* of the material is motivated by current work on High Temperature Superconductors, the methods and ideas have a wide range of applicability.

This book is not a textbook. Many of the problems, ideas and methods which are discussed here have become essential to our current understanding of Condensed Matter Physics. I have made a considerable effort to make the material largely self-contained. Many powerful methods, which are necessary for the study of condensed matter systems in the strong fluctuation limit, are discussed and explained in some detail within the context of the applications. Thus, although the theoretical apparatus is not developed systematically and in its full glory, this material may be useful to many graduate students, to learn both the subject and the methods. For the most part I have refrained myself from just quoting results without explaining where they come from. So, if a particular method happens to be appropriate to the study of a particular subject, I present a more or less detailed description of the method itself. Thus, a number of essential theoretical tools are discussed and explained. Unfortunately, I only was able to cover part of the material that I wanted to include. Perhaps the biggest omission is a description of Conformal Field Theory. This will have to wait for a second edition, if and when I ever get crazy enough to come back to this nightmare.

The material covered in this book deals with the theories of the three most fundamental problems in contemporary Condensed Matter Physics: Quantum Antiferromagnetism, the Quantum Hall Effect and High Temperature Superconducivity. The reader will find a detailed presntation of the modern theories of quantum magnetism and of the Quantum Hall effect but not an explicit treatment of the theories of High Temperature Superconductors.

In chapter 2 the symmetries of the Hubbard model are studied. The relation between the Hubbard model and quantum magnetism is also discussed. In chapter 3 I develop the theory of the magnetic instability of Fermi systems . As in the rest of the book, I move back and forth between the path integral and the Hamiltonian appoaches. The non-linear sigma model for the antiferromagnetic state is derived directly from the Hubbard model. In chapter 4 I give a detailed discussion of the physics of one-dimensional antiferromagnets. In chapter 5 I give a derivation of the path integral method for spin systems. The main applications here are quantum ferromagnets and antiferromagnets. The non-linear sigma model is derived within the semiclassical limit. I also present a detailed diuscussion of the role of topological excitations and of topological terms in the effective action for both one and two dimensional systems. In chapter 6 the reader will find the (so far) most current theories of disordered spin states. It includes the Resonating Valence Bond theories, flux phases, dimer or column states and the chiral spin liquid. The language of gauge theories is used throughout this chapter.

Theories of Anyons with applications in chiral spin liquids and Anyon Superconductivity, are presnted in chapters 7 and 8. The reader will find here a pedagogical presentation of the Chern-Simons theory of Fractional Statistics. Special attention is given to the connections between this problem and the topological theory of knots. A two dimensional version of the Jordan-Wigner